

PROBABILITY

I. FUNDAMENTAL PRINCIPLE (also called multiplication of choices)

If a selection consists of k steps, the first can be made in n_1 ways, for each of these the second can be made in n_2 ways, ..., and for each of these the k^{th} can be made in n_k ways, then the whole selection can be made in $(n_1)(n_2)\dots(n_k)$ ways.

EXAMPLE: If there are 3 female candidates for governor and 5 male candidates for mayor, in how many ways can the two offices be filled?

$$(n_1)(n_2) =$$

The ways are as follows:

II. n FACTORIAL

n factorial, denoted as $n!$, is defined as follows:

$$n! = (n)(n-1)(n-2)\dots 1$$

EXAMPLE: $5! =$

EXAMPLE: $4!3! =$

EXAMPLE: $\frac{100!}{98!} =$

NOTE: It is convenient to define $0! = 1$

III. PERMUTATIONS

A permutation of n different objects taken x at a time is an arrangement of x out of the n objects with attention given to the order of arrangement. The number of permutations of n objects taken x at a time is denoted by ${}_nP_x$ and is given by the following formula:

$${}_nP_x = \frac{n!}{(n-x)!}$$

In particular, the number of permutations of n objects taken n at a time is as follows:

$${}_nP_n = \frac{n!}{(n-n)!} \frac{n!}{0!} = \frac{n!}{1} = n!$$

EXAMPLE: The number of permutations of the letters a, b, and c taken 2 at a time is .

$${}_nP_x = \frac{n!}{(n-x)!} =$$

The permutations are as follows:

EXAMPLE: The number of permutations of the letters a, b, and c taken 3 at a time is .

$${}_nP_n = n! =$$

The permutations are as follows:

The number of permutations of n objects consisting of groups of which n_1 are alike, n_2 are alike...is:

$$\frac{n!}{n_1!n_2!\dots} \quad \text{where } n = n_1 + n_2 + \dots$$

EXAMPLE: The number of permutations of letters in the word "ADD" is .

$$\frac{n!}{n_1!n_2!} =$$

The permutations are as follows:

IV. COMBINATIONS

GIVEN

A combination of n different objects taken x at a time is a selection of x out of the n objects with no attention to the order of arrangement. The number of combinations of n objects taken x at a time is denoted by $\binom{n}{x}$ and is given by the following formula:

$$\binom{n}{x} = \frac{n!}{x!(n-x)!} = \frac{n!}{x!}$$

In particular, the number of combinations of n objects taken n at a time is as follows:

$$\binom{n}{n} = \frac{n!}{n!(n-n)!} = \frac{n!}{n!0!} = \frac{n!}{n!1} = \frac{n!}{n!} = 1$$

EXAMPLE: The number of combinations of the letters a, b, and c taken 2 at a time is .

$$\binom{n}{x} = \frac{n!}{x!(n-x)!} =$$

The combinations are as follows:

EXAMPLE: The number of combinations of the letters a, b, and c taken 3 at a time is 1.

$$\binom{n}{n} = 1$$

The 1 combination is as follows:

The number of combinations of n objects taken 1 or 2, ..., or n at a time is as follows:

$$\binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n} = 2^n - 1$$

EXAMPLE: The number of combinations of the letters a, b, and c taken 1, 2, or 3 at a time is .

$$2^n - 1 =$$

The combinations are as follows:

V. DEFINITION OF PROBABILITY

Suppose an event E can happen in S ways out of a total of n possible, equally likely ways. Then the probability of occurrence of the event (called its success) is denoted by:

$$p = P(E) = \frac{S}{n}$$

The probability of non-occurrence of the event (called its failure) is denoted by:

$$P(E') = 1 - \frac{S}{n} = 1 - p$$

$$\text{Thus, } P(E) + P(E') = 1$$

The probability of an event is a number between 0 and 1, inclusive (i.e., $0 \leq P(E) \leq 1$). If the event cannot occur, its probability is 0. If the event must occur (i.e., its occurrence is certain), its probability is 1.

EXAMPLE: What is the probability of getting a 3 in a single toss of a fair die?

$$p = P(E) = \frac{S}{n} =$$

EXAMPLE: What is the probability of not getting a 3 in a single toss of a fair die?

$$P(E') = 1 - p =$$

VI. ODDS

If p is the probability that an event will occur, and p does not equal 0 or 1, then the odds in favor of its happening are p to $1 - p$; the odds against its happening are $1 - p$ to p .

EXAMPLE: What are the odds in favor of a 3 ████ in a single toss of a fair die?

p to $1 - p =$

EXAMPLE: What are the odds against a 3 ████ in a single toss of a fair die?

$1 - p$ to $p =$

VI. CONDITIONAL PROBABILITY; JOINT PROBABILITY; INDEPENDENT AND DEPENDENT EVENTS

If E_1 and E_2 are two events, the probability that E_2 occurs given that E_1 has occurred is denoted by $P(E_2|E_1)$ and is called the conditional probability of E_2 given that E_1 has occurred.

If the occurrence of E_1 does not affect the probability of occurrence of E_2 , then $P(E_2|E_1) = P(E_2)$ and we say the E_1 and E_2 are independent events; otherwise, they are dependent events.

If we denote by E_1, E_2 the event that "both E_1 and E_2 occur," called a compound or joint event, then:

$$P(E_1, E_2) = P(E_1)P(E_2|E_1)$$

NOTE: if $P(E_1, E_2) = P(E_1)P(E_2|E_1)$
then $\frac{P(E_1, E_2)}{P(E_1)} = P(E_2|E_1)$

In particular:

$$P(E_1, E_2) = P(E_1)P(E_2) \text{ for independent events}$$

If we denote by E_1, E_2, E_3 the event that " E_1 and E_2 and E_3 occur," then:

$$P(E_1, E_2, E_3) = P(E_1)P(E_2|E_1)P(E_3|E_1, E_2)$$

NOTE: if $P(E_1, E_2, E_3) = P(E_1)P(E_2|E_1)P(E_3|E_1, E_2)$

$$\text{then } \frac{P(E_1, E_2, E_3)}{P(E_1)P(E_2|E_1)} = P(E_3|E_1, E_2)$$

$$\frac{P(E_1, E_2, E_3)}{P(E_1) \left[\frac{P(E_1, E_2)}{P(E_1)} \right]} = P(E_3|E_1, E_2)$$

$$\frac{P(E_1, E_2, E_3)}{P(E_1, E_2)} = P(E_3|E_1, E_2)$$

In particular:

$$P(E_1, E_2, E_3) = P(E_1)P(E_2)P(E_3) \text{ for independent events}$$

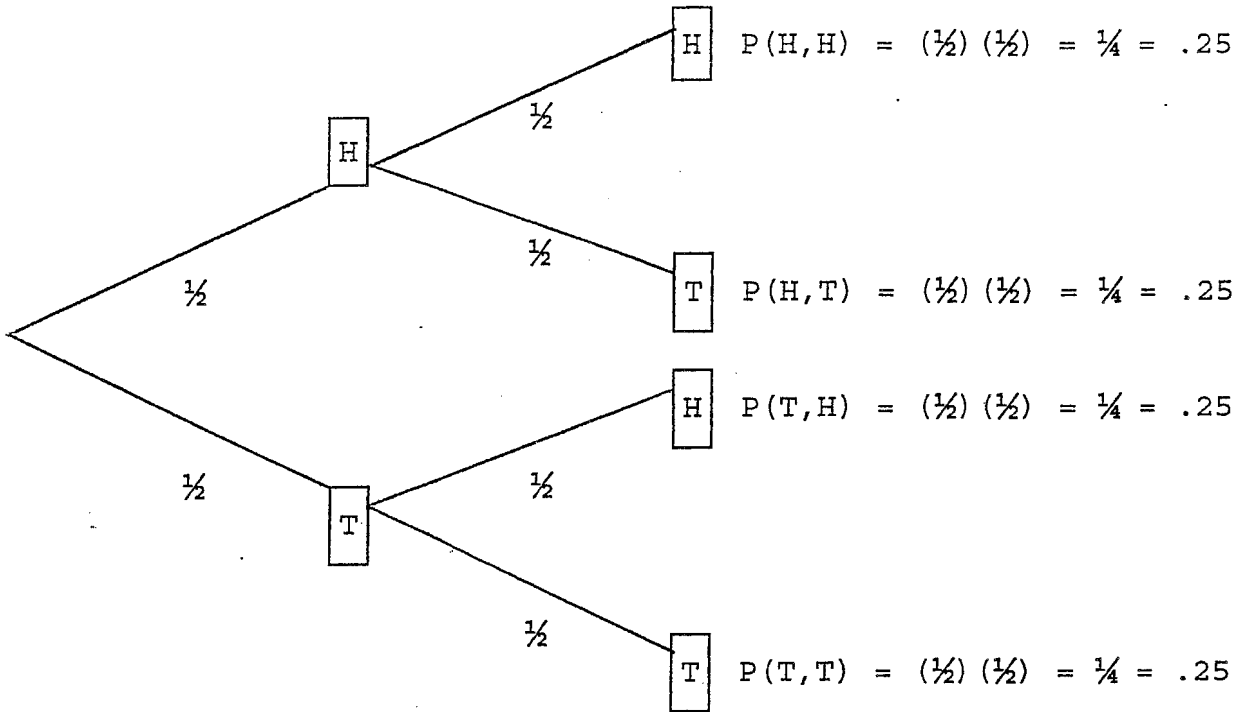
As an extension of this relationship:

$$P(E_1, E_2, E_3, \dots, E_n) = P(E_1)P(E_2)P(E_3) \dots P(E_n) \text{ for independent events}$$

EXAMPLE: Let E_1 and E_2 be the events "heads on the first toss" and "heads on the second toss" of a fair coin, respectively. What is the probability of heads on both the first and second toss of a fair coin?

$$P(E_1, E_2) = P(E_1)P(E_2) =$$

TREE DIAGRAM



$P(2 \text{ HEADS}) = \boxed{.25}$

$P(1 \text{ HEAD}) = .25 + .25 = .50$

$P(\text{NO HEADS}) = .25$
[I.E. 2 TAILS]

EXAMPLE: Suppose a box contains 3 white balls and 2 black balls. Let E_1 be the event "first ball drawn is black" and E_2 the event "second ball drawn is black" where the balls are not replaced after being drawn. What is the probability that the first ball drawn is black?

$P(E_1) = \frac{S}{n} =$

What is the probability that the second ball drawn is black given the first ball drawn is black?

$P(E_2|E_1) =$

What is the probability that both balls drawn are black?

$$P(E_1, E_2) = P(E_1)P(E_2|E_1) =$$

What is the probability that both balls drawn are black if replacement is allowed?

$$P(E_1, E_2) = P(E_1)P(E_2) =$$

EXAMPLE: A market research firm is surveying 125 households within a community. The following data is ascertained:

	HOUSEHOLDS WITH ANNUAL INCOME OF \$80,000 OR LESS		HOUSEHOLDS WITH ANNUAL INCOME ABOVE \$80,000	
	PHONE	NO PHONE	PHONE	NO PHONE
TV	27	20	18	10
NO TV	18	10	12	10

MATRIX

	COLUMN 1	COLUMN 2	COLUMN 3	COLUMN 4	COLUMN 5
ROW 1	*	*	*	*	@
ROW 2	*	*	*	*	@
ROW 3	@	@	@	@	1

* denotes a joint probability

@ denotes a simple probability

NOTE: A simple probability is also called a marginal probability since it is located in the margin of the matrix.

MATRIX

(CONVERSION PROCESS)

27/125	20/125	18/125	10/125	75/125
18/125	10/125	12/125	10/125	50/125
45/125	30/125	30/125	20/125	1 (I.E., 125/125)

$P(TV) =$

75/125

 $=$

.60

 (I.E., the answer is located in
 ROW

 COLUMN

)

MATRIX

(PARTITION PROCESS)

$P(\text{INCOME} > \$80,000) = 30/125 + 20/125 =$

50/125

 $=$

.40

 (I.E., the

answer is the summation
of the numbers located

IN ROW

 COLUMN

AND ROW

 COLUMN

)

MATRIX

(PARTITION PROCESS)

$$P(\text{TV}, \text{INCOME} > \$80,000) = 18/125 + 10/125 =$$

$$\boxed{28/125} = \boxed{.22} \quad (\text{I.E., the}$$

answer is the summation
of the numbers located

IN ROW

COLUMN

AND ROW

COLUMN

)

MATRIX

(PARTITION PROCESS)

Evaluate the following probability:

$$P(\text{TV} | \text{INCOME} > \$80,000, \text{PHONE})$$

$$\text{Let } E_1 = \text{INCOME} > \$80,000$$

$$E_2 = \text{PHONE}$$

$$E_3 = \text{TV}$$

$$P(E_3 | E_1, E_2) = \frac{P(E_1, E_2, E_3)}{P(E_1, E_2)}$$

$$P(TV | INCOME > \$80,000, PHONE) = \frac{P(INCOME > \$80,000, PHONE, TV)}{P(INCOME > \$80,000, PHONE)} =$$

_____ =

$$\frac{\boxed{}}{\boxed{}} = \boxed{} = \boxed{}$$

$\boxed{}$

VII. MUTUALLY EXCLUSIVE EVENTS

Two or more events are called mutually exclusive if the occurrence of any one of them excludes the occurrence of the others. Thus, if E_1 and E_2 are mutually exclusive events, $P(E_1, E_2) = 0$.

If E_1 or E_2 denotes the event that "either E_1 or E_2 or both occur," then:

$$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) - P(E_1, E_2)$$

In particular:

$$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) \text{ for mutually exclusive events}$$

As an extension of this relationship:

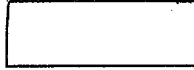
$$P(E_1 \text{ or } E_2 \text{ or } \dots \text{ or } E_n) = P(E_1) + P(E_2) + \dots + P(E_n) \text{ for mutually exclusive events}$$

EXAMPLE: If E_1 is the event "drawing an ace" from a deck of cards and E_2 is the event "drawing a king," what is the probability of drawing either an ace or a king in a single draw?



$$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) =$$

EXAMPLE: If E_1 is the event "drawing an ace" from a deck of cards and E_2 is the event "drawing a spade," what is the probability of drawing either an ace or a spade or both in a single draw?



$$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) - P(E_1, E_2) =$$

VIII. MUTUALLY EXCLUSIVE EVENTS; INDEPENDENT EVENTS; DEPENDENT EVENTS

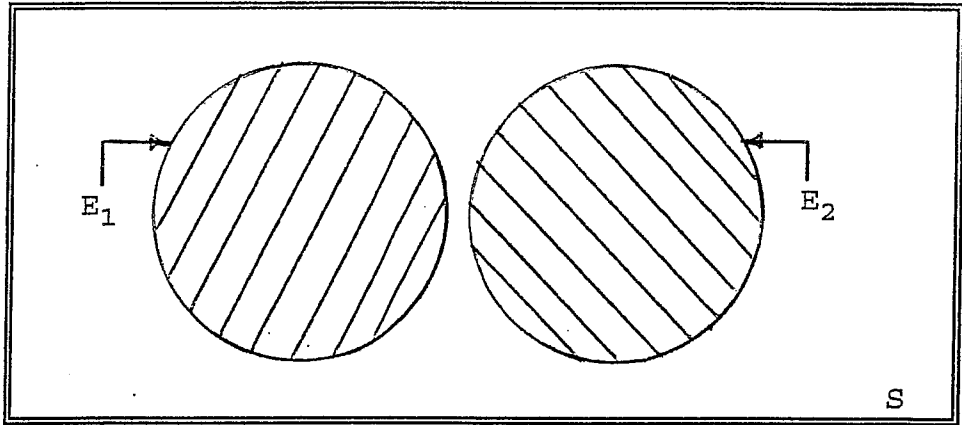
Two events are either mutually exclusive or they are not mutually exclusive. If the two events are mutually exclusive, then $P(E_1, E_2) = 0$. If the two events are not mutually exclusive, then $P(E_1, E_2) \neq 0$ and the two events are either independent or they are not independent (I.E., they are dependent). If $P(E_1, E_2) = P(E_1)P(E_2)$, then the two events are independent. If $P(E_1, E_2) \neq P(E_1)P(E_2)$, then the two events are not independent (I.E. they are dependent). If the two events are dependent, then $P(E_1, E_2) = P(E_1)P(E_2|E_1)$. Extensions to more than two events can be made.

IX. RELATIONSHIP OF PROBABILITY TO POINT SET THEORY

In modern probability theory, we think of all possible outcomes or results of an experiment, game, etc., as points in a space called a sample space (denoted as S). If S contains a finite number of points, then with each point we can associate a non-negative number such that the sum of all numbers corresponding to all points in S add to 1. An event is a set or collection of points in S such as those depicted in the following diagrams [called Venn diagrams]:

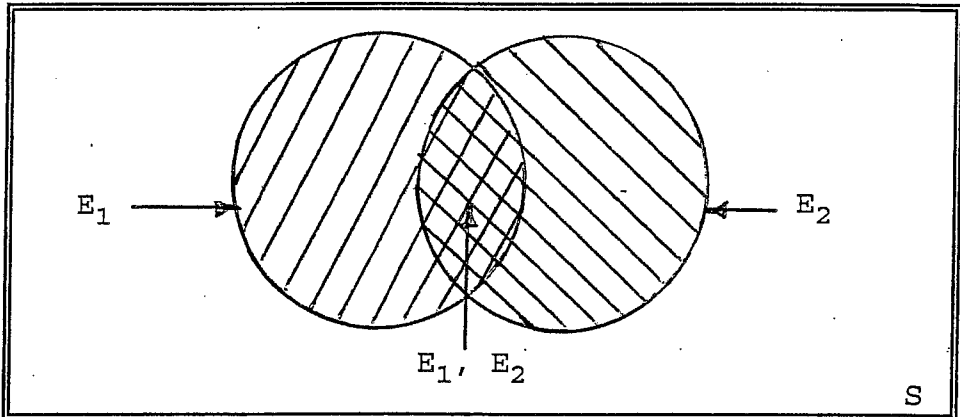
VENN DIAGRAM

(TWO EVENTS WHICH ARE MUTUALLY EXCLUSIVE)



VENN DIAGRAM

(TWO EVENTS WHICH ARE NOT MUTUALLY EXCLUSIVE)



The event E_1 or E_2 is the set of points which are either in E_1 or E_2 or both, while the event E_1, E_2 is the set of points common to both E_1 and E_2 . The probability of an event such as E_1 is the sum of the probabilities associated with all points contained in the set E_1 . Similarly, the probability of E_1 or E_2 is the sum of the probabilities associated with all points contained in the set E_1 and the set E_2 . If E_1 and E_2 have no points in common (i.e., the events are mutually exclusive), then $P(E_1 \text{ or } E_2) = P(E_1) + P(E_2)$. If E_1 and E_2 have points in common, then $P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) - P(E_1, E_2)$.

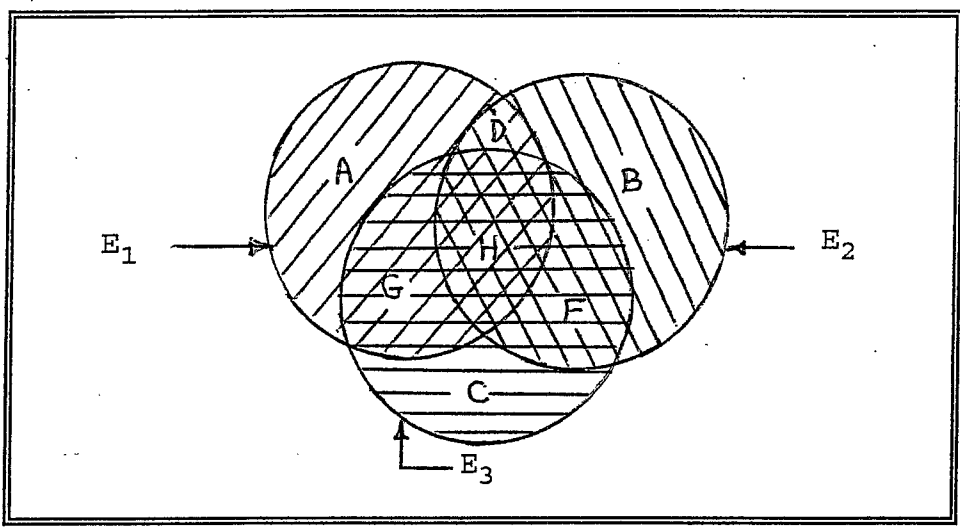
The set E_1 or E_2 is often denoted as $E_1 \cup E_2$, and is called the union of the two sets. The set E_1, E_2 is often denoted as $E_1 \cap E_2$, and is called the intersection of the two sets. Extensions to more than two sets can be made. Thus, instead of E_1 or E_2 or E_3 and E_1, E_2, E_3 we could use the notation $E_1 \cup E_2 \cup E_3$ and $E_1 \cap E_2 \cap E_3$, respectively.

The symbol $\emptyset$ is often used to denote a set with no points in it (called the null set or the empty set). The probability associated with an event corresponding to this set is zero. If E_1 and E_2 have no points in common, the corresponding events are mutually exclusive and $P(E_1, E_2) = 0$.

EXAMPLE: Assume three events (denoted as E_1, E_2 , and E_3) are not mutually exclusive. Develop an expression for the union of the three events using a Venn diagram.

VENN DIAGRAM

(THREE EVENTS WHICH ARE NOT MUTUALLY EXCLUSIVE)



$$P(E_1 \cup E_2 \cup E_3) =$$

NOTE: Alternative notation for the aforementioned expression is as follows:

$$P(E_1 \text{ or } E_2 \text{ or } E_3) =$$

XI. MATHEMATICAL EXPECTATION

If the probabilities of obtaining the amounts $a_1, a_2, a_3, \dots$, or a_k , are, respectively, $p_1, p_2, p_3, \dots$, and p_k , then the mathematical expectation (denoted as E) is as follows:

$$E = a_1p_1 + a_2p_2 + a_3p_3 + \dots + a_kp_k$$

EXAMPLE: A player pays \$2 to play a game of chance. If on a single draw from a deck of cards, the ace of spades is drawn, the player is paid \$15. The player is paid \$1 if any other spade is drawn. What is the expected value of the game for the player?

$$E = a_1p_1 + a_2p_2 + a_3p_3 =$$

That is, the expectation of the player is a

of \$ for each play of the game.

X. APPENDIX

A. SUPPLEMENTARY PROBLEMS

1. GIVEN: A and B are independent events; $P(A) = .25$; $P(B) = .40$

Find the following probabilities:

- A. $P(A|B)$
- B. $P(A \cap B)$
- C. $P(A \cup B)$
- D. $P(A' \cap B')$

2. GIVEN: $P(A) = .40$; $P(B|A) = .30$; $P(B'|A') = .20$

Find the following probabilities:

- A. $P(A')$
- B. $P(B|A')$
- C. $P(B)$
- D. $P(A \cap B)$
- E. $P(A|B)$

3. GIVEN: $P(A) = .56$; $P(B) = .43$; $P(A \cap B) = .18$

Find the following probabilities:

- A. $P(A')$
- B. $P(B')$
- C. $P(A \cup B)$
- D. $P(A' \cap B)$
- E. $P(A' \cup B)$
- F. $P(A \cap B')$

4. GIVEN: A and B are mutually exclusive events;
 $P(A) = .37$; $P(B) = .41$

Find the following probabilities:

- A. $P(A')$
- B. $P(B')$
- C. $P(A \cap B)$
- D. $P(A \cup B)$
- E. $P(A' \cup B')$
- F. $P(A' \cap B')$

5. GIVEN:

		Y		
		0	1	2
X	0	.30	.14	.02
	1	.18	.10	.06
	2	.12	.06	.02

GIVEN: The following events: EVENT A: $X = 0$
EVENT B: $Y \neq 0$
EVENT C: $X + Y > 0$

Find the following probabilities:

- A. $P(C)$
- B. $P(A, B)$
- C. $P(A \text{ or } B)$
- D. $P(A|B)$

NOTE: Let X refer to the number of replacement parts needed by machine #1. Let Y refer to the number of replacement parts needed by machine #2. Event A refers to the

probability that machine #

will need replacement

parts. Event B refers to the probability

that machine # will need

or replacement parts. Event C

refers to the probability that machine #1

and/or machine #2 will need or

replacement parts.

ASSIGNMENT:

Complete the five supplementary problems in the appendix of the lecture for probability.

6. Four objects (such as A, B, C, and D) taken 1 or 2 or 3 or 4 at a time will yield how many combinations?

6. Data compiled by the weather bureau in Utopia indicate that it has snowed there on December 25th 64 times in the last 80 years.

A. What is the probability that it will snow in Utopia on December 25th?

B. What are the odds that it will snow in Utopia on December 25th?

C. What are the odds that it will not snow in Utopia on December 25th?

7. A contractor is bidding on a road construction project which could yield a profit of \$60,000 (with a probability of .60) or a loss of \$22,500 (with a probability of .40). What is the contractor's mathematical expectation?

6. A market research firm is surveying 100 households within a community. The following data is ascertained:

	HOUSEHOLDS WITH ANNUAL INCOME ABOVE \$100,000		HOUSEHOLDS WITH ANNUAL INCOME OF \$100,000 OR LESS	
	PHONE NUMBER LISTED	NO PHONE NUMBER LISTED	PHONE NUMBER LISTED	NO PHONE NUMBER LISTED
BOAT	20	14	7	4
NO BOAT	15	12	18	10

Evaluate the following probability:

$P(\text{NO BOAT} \mid \text{INCOME ABOVE } \$100,000, \text{ PHONE NUMBER LISTED})$

7. Develop a formula for the union of four events (such as A, B, C, and D) which are not mutually exclusive.
8. Foster Enterprises is a corporation with many divisions, two of which are John's Department Store and Gordon's Petroleum Company. Both divisions are interested in getting students at Utopia College to use credit cards issued by each division. Assume you are hired by Foster Enterprises to obtain information from a sample of 200 students, and you are able to ascertain the following information:

		GORDON'S PETROLEUM COMPANY	
		STUDENTS POSSESS CREDIT CARD	STUDENTS DO NOT POSSESS CREDIT CARD
JOHN'S DEPARTMENT STORE	STUDENTS POSSESS CREDIT CARD	60	60
	STUDENTS DO NOT POSSESS CREDIT CARD	15	65

Let E_1 denote that a college student does possess a credit card issued by John's Department Store.

Let E_1' denote that a college student does not possess a credit card issued by John's Department Store.

Let E_2 denote that a college student does possess a credit card issued by Gordon's Petroleum Company.

Let E_2' denote that a college student does not possess a credit card issued by Gordon's Petroleum Company.

- Develop a table showing all simple and joint probabilities.
- Find the value of each of the following probabilities:
 $P(E_1)$; $P(E_1')$; $P(E_2)$; $P(E_2')$
- Find the value of each of the following probabilities:
 $P(E_1, E_2)$; $P(E_1, E_2')$; $P(E_1', E_2)$; $P(E_1', E_2')$
- What is the complementary event to possessing a credit card issued by John's Department Store?
- Why is possessing a credit card issued by John's Department Store and possessing a credit card issued by Gordon's Petroleum Company a joint event?
- Develop a Venn diagram.

- G. Find the value of each of the following probabilities:
 $P(E_1 \text{ or } E_2)$; $P(E_1' \text{ or } E_2)$; $P(E_1 \text{ or } E_1')$
- H. Find the value of each of the following probabilities:
 $P(E_2|E_1)$; $P(E_1|E_2')$
- I. Are the events possessing a credit card issued by John's Department Store and possessing a credit card issued by Gordon's Petroleum Company independent events?

9. Foster's Carousel Horses can be ordered in seven different colors, three different sizes, and three different styles. How many carousel horses would have to be ordered if the Utopian Amusement Park wanted to display one carousel horse of each type?

~~13. How many permutations can be made from the word "JOHN"?~~

10. "Pick First Three" at the Utopian Racetrack consists of picking the correct order of finish of the first three horses in the tenth race. If there are 12 horses entered in the tenth race, how many "Pick First Three" outcomes are possible?

11. "Flip One Two" at the Racetrack of Utopia consists of picking the greyhounds that will finish first and second in the tenth race irrespective of order. If 8 greyhounds are entered in the tenth race, how many "Flip One Two" outcomes are possible?

1. GIVEN: A and B are independent events; $P(A) = .25$;
 $P(B) = .40$

	B	B'	
A	.10	.15	.25
A'	.30	.45	.75
	.40	.60	1

NOTE: $P(A, B) = P(A) P(B)$ if the events are independent.

$$P(A, B) = (.25)(.40) = .10$$

$$A. \quad P(A | B) = \frac{P(B, A)}{P(B)} = \frac{.10}{.40} = \boxed{.25}$$

$$P(A | B) = P(A) = \boxed{.25} \quad \text{OR} \quad \text{since A and B are independent events.}$$

$$B. \quad P(A \cap B) = \boxed{.10}$$

$$C. \quad P(A \cup B) = P(A) + P(B) - P(A, B) = .25 + .40 - .10 = \boxed{.55}$$

$$\text{OR}$$

$$P(A \cup B) = .10 + .15 + .30 = \boxed{.55}$$

$$\text{OR}$$

$$P(A \cup B) = 1 - .45 = \boxed{.55}$$

$$D. \quad P(A' \cap B') = \boxed{.45}$$

$$2. \quad \text{GIVEN: } P(A) = .40; P(B | A) = .30; P(B' | A') = .20$$

	B	B'	
A	.12	.28	.40
A'	.48	.12	.60
	.60	.40	1

$$\text{NOTE: } P(B | A) = \frac{P(A, B)}{P(A)}$$

$$.30 = \frac{P(A, B)}{.40}$$

$$(.30)(.40) = P(A, B)$$

$$.12 = P(A, B)$$

$$\text{NOTE: } P(B' | A') = \frac{P(A', B')}{P(A')}$$

$$.20 = \frac{P(A', B')}{.60}$$

$$(.20)(.60) = P(A', B')$$

$$.12 = P(A', B')$$

$$A. \quad P(A') = \boxed{.60}$$

$$B. \quad P(B | A') = \frac{P(A', B)}{P(A')} = \frac{.48}{.60} = \boxed{.80}$$

$$C. \quad P(B) = \boxed{.60}$$

$$D. \quad P(A \cap B) = \boxed{.12}$$

$$E. \quad P(A | B) = \frac{P(B, A)}{P(B)} = \frac{.12}{.60} = \boxed{.20}$$

3. GIVEN: $P(A) = .56$; $P(B) = .43$; $P(A \cap B) = .18$

	B	B'	
A	.18	.38	.56
A'	.25	.19	.44
	.43	.57	1

A. $P(A') =$.44

B. $P(B') =$.57

C. $P(A \cup B) = P(A) + P(B) - P(A, B) = .56 + .43 - .18 =$.81

OR

$P(A \cup B) = .18 + .38 + .25 =$.81

OR

$P(A \cup B) = 1 - .19 =$.81

D. $P(A' \cap B) =$.25

E. $P(A' \cup B) = P(A') + P(B) - P(A', B) =$

$.44 + .43 - .25 =$.62

OR

$P(A' \cup B) = .18 + .25 + .19 =$.62

OR

$P(A' \cup B) = 1 - .38 =$.62

F. $P(A \cap B') =$.38

4. GIVEN: A and B are mutually exclusive events; $P(A) = .37$;
 $P(B) = .41$

	B	B'	
A	0	.37	.37
A'	.41	.22	.63
	.41	.59	1

NOTE: $P(A, B) = 0$ if the events are mutually exclusive.

A. $P(A') =$.63

B. $P(B') =$.59

C. $P(A \cap B) = \boxed{0}$

D. $P(A \cup B) = P(A) + P(B)$ if the events are mutually exclusive.

$P(A \cup B) = .37 + .41 = \boxed{.78}$
OR

$P(A \cup B) = 1 - .22 = \boxed{.78}$

E. $P(A' \cup B') = P(A') + P(B') - P(A', B') =$

$.63 + .59 - .22 = \boxed{1}$
OR

$P(A' \cup B') = .37 + .41 + .22 = \boxed{1}$
OR

$P(A' \cup B') = 1 - 0 = \boxed{1}$

F. $P(A' \cap B') = \boxed{.22}$

5. GIVEN:

		Y		
		0	1	2
X	0	.30	.14	.02
	1	.18	.10	.06
	2	.12	.06	.02

GIVEN: The following events: EVENT A: $X = 0$;
EVENT B: $Y \neq 0$; EVENT C: $X + Y > 0$

NOTE: EVENT A: $X = 0$

NOTE: EVENT B: $Y \neq 0$

NOTE: EVENT C: $X + Y > 0$

A. $P(C) = .14 + .02 + .18 + .10 + .06 + .12 + .06 + .02 =$

.70

OR

$P(C) = 1 - .30 =$ **.70**

B. $P(A,B) = .14 + .02 =$ **.16**

NOTE:

C. $P(A \text{ OR } B) = P(A) + P(B) - P(A,B) = [.30 + .14 + .02] +$
 $[.14 + .02 + .10 + .06 + .06 + .02] - [.14 + .02] =$

$.46 + .40 - .16 =$ **.70**

OR

$P(A \text{ OR } B) = .30 + .14 + .02 + .10 + .06 + .06 + .02 =$ **.70**

OR

$P(A \text{ OR } B) = 1 - [.18 + .12] = 1 - .30 =$ **.70**

$$D. \quad P(A | B) = \frac{P(B, A)}{P(B)} = \frac{.16}{.40} = \boxed{.40}$$

$$6. \quad 2^n - 1 = 2^4 - 1 = 16 - 1 = \boxed{15}$$

$$\binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n} = \binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4} =$$

$$\frac{4!}{1!(4-1)!} + \frac{4!}{2!(4-2)!} + \frac{4!}{3!(4-3)!} + 1 =$$

$$\frac{4!}{1!3!} + \frac{4!}{2!2!} + \frac{4!}{3!1!} + 1 =$$

$$\frac{(4)(3)(2)(1)}{(1)(3)(2)(1)} + \frac{(4)(3)(2)(1)}{(2)(1)(2)(1)} + \frac{(4)(3)(2)(1)}{(3)(2)(1)(1)} + 1 =$$

$$4 + 6 + 4 + 1 = \boxed{15}$$

ENUMERATION:

A, B, C, D
AB, AC, AD, BC, BD, CD
ABC, ABD, ACD, BCD,
ABCD

$$6. \quad A. \quad p = P(E) = \frac{s}{n} = \frac{64}{80} = \frac{4}{5} = \boxed{.80}$$

$$B. \quad \text{ODDS (IN FAVOR OF): } p \text{ to } 1 - p = \frac{4}{5} \text{ to } 1 - \frac{4}{5} =$$

$$\frac{4}{5} \text{ to } \frac{1}{5} = \boxed{4 \text{ to } 1}$$

$$C. \quad \text{ODDS (AGAINST): } 1 - p \text{ to } p = \frac{1}{5} \text{ to } \frac{4}{5} = \boxed{1 \text{ to } 4}$$

$$7. \quad E = a_1 \cdot p_1 + a_2 \cdot p_2 + \dots + a_k \cdot p_k =$$

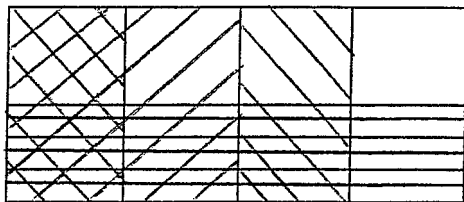
$$(\$60,000)(.60) + (-\$22,500)(.40) =$$

$$\$36,000 - \$9,000 = \boxed{\$27,000}$$

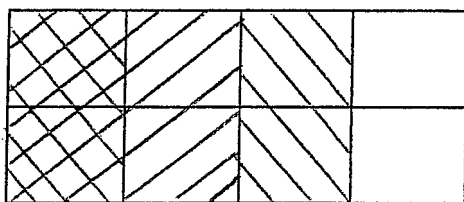
$$6. \quad P(E_3 | E_1, E_2) = \frac{P(E_1, E_2, E_3)}{P(E_1, E_2)}$$

$$P(\text{NO BOAT} | \text{INCOME ABOVE } \$100,000, \text{PHONE NUMBER LISTED}) =$$

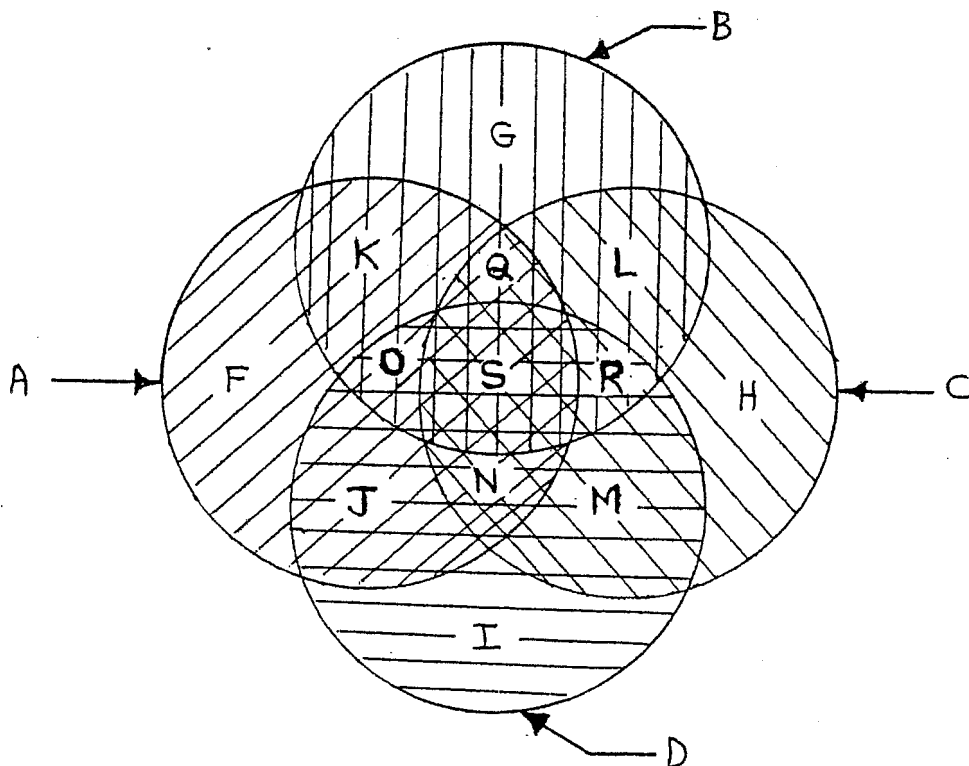
$$\frac{P(\text{INCOME ABOVE } \$100,000, \text{PHONE NUMBER LISTED}, \text{NO BOAT})}{P(\text{INCOME ABOVE } \$100,000, \text{PHONE NUMBER LISTED})}$$



$$= \frac{\frac{15}{100}}{\frac{35}{100}} = \frac{15}{35} = \frac{3}{7} = \boxed{.43}$$



7. VENN DIAGRAM FOR 4 EVENTS WHICH ARE NOT MUTUALLY EXCLUSIVE:



$$P(A \text{ OR } B \text{ OR } C \text{ OR } D) = \begin{array}{l} P(A) + P(B) + P(C) + P(D) \\ - P(A,B) - P(B,C) - P(C,D) \\ - P(A,D) + P(A,B,C,D) \end{array}$$

ALTERNATIVE NOTATION:

$$P(A \cup B \cup C \cup D) = \begin{array}{l} P(A) + P(B) + P(C) + P(D) \\ - P(A \cap B) - P(B \cap C) - P(C \cap D) \\ - P(A \cap D) + P(A \cap B \cap C \cap D) \end{array}$$

8. A.

	E_2	E_2'	
E_1	60/200	60/200	120/200
E_1'	15/200	65/200	80/200
	75/200	125/200	200/200

	E_2	E_2'	
E_1	.30	.30	.60
E_1'	.075	.325	.40
	.375	.625	1

B. $P(E_1) = .60$; $P(E_1') = .40$; $P(E_2) = .375$; $P(E_2') = .625$

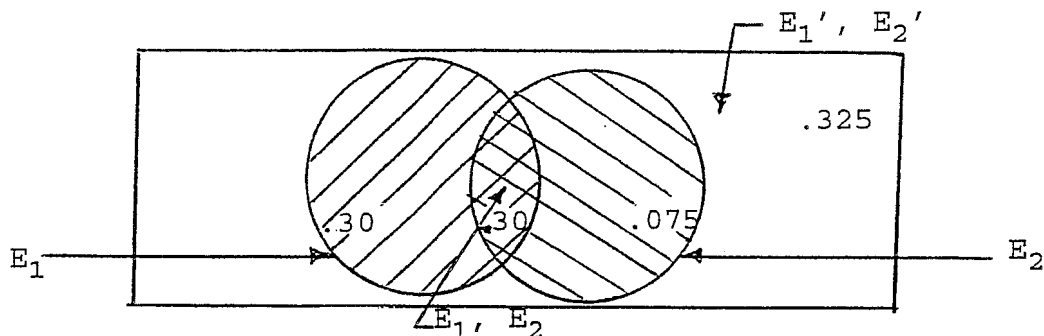
C. $P(E_1, E_2) = .30$; $P(E_1, E_2') = .30$; $P(E_1', E_2) = .075$;

$P(E_1', E_2') = .325$

D. The event that a college student does not possess a credit card issued by John's Department Store (i.e., E_1') is the complementary event to the event that a college student does possess a credit card issued by John's Department Store (i.e., E_1).

E. Both possessing a credit card issued by John's Department Store (i.e., E_1) and possessing a credit card issued by Gordon's Petroleum Company (i.e., E_2) can occur at the same time (i.e., E_1, E_2).

F. VENN DIAGRAM FOR 2 EVENTS WHICH ARE NOT MUTUALLY EXCLUSIVE:



$$\begin{aligned}
 G. \quad P(E_1 \text{ or } E_2) &= P(E_1) + P(E_2) - P(E_1, E_2) \\
 &= .60 + .375 - .30 \\
 &= .975 - .30 \\
 &= \boxed{.675}
 \end{aligned}$$

$$\begin{aligned}
 P(E_1' \text{ or } E_2) &= P(E_1') + P(E_2) - P(E_1', E_2) \\
 &= .40 + .375 - .075 \\
 &= .775 - .075 \\
 &= \boxed{.70}
 \end{aligned}$$

$$\begin{aligned}
 P(E_1 \text{ or } E_1') &= P(E_1) + P(E_1') \text{ since } E_1 \text{ and } E_1' \text{ are} \\
 &\quad \text{mutually exclusive events} \\
 &= .60 + .40 \\
 &= \boxed{1.00}
 \end{aligned}$$

$$\begin{aligned}
 H. \quad P(E_2 | E_1) &= \frac{P(E_1, E_2)}{P(E_1)} \\
 &= \frac{.30}{.60} \\
 &= \boxed{.50}
 \end{aligned}$$

$$\begin{aligned}
 P(E_1 | E_2') &= \frac{P(E_2', E_1)}{P(E_2')} \\
 &= \frac{.30}{.625} \\
 &= \boxed{.48}
 \end{aligned}$$

I. Are E_1 and E_2 independent events?

$$P(E_2|E_1) \stackrel{?}{=} P(E_2)$$

$$.50 \neq .375$$

Therefore, the events are not independent (i.e., the events are dependent).

ALTERNATIVE METHOD:

Are E_1 and E_2 independent events?

$$P(E_1) P(E_2) \stackrel{?}{=} P(E_1, E_2)$$

$$(.60)(.375) \stackrel{?}{=} .30$$
$$.225 \neq .30$$

Therefore, the events are not independent (i.e., the events are dependent).

9. FUNDAMENTAL PRINCIPLE:

$$(n_1)(n_2) \dots (n_k) = (7)(3)(3) =$$

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$$13. \frac{n!}{n_1!n_2! \dots} = \frac{4!}{1!1!1!1!} = \frac{(4)(3)(2)(1)}{(1)(1)(1)(1)} =$$

24 PERMUTATIONS

ALTERNATIVE METHOD:

$${}_nP_x = \frac{n!}{(n-x)!}$$

$${}_4P_4 = \frac{4!}{(4-4)!} = \frac{4!}{0!} = \frac{(4)(3)(2)(1)}{1} =$$

24 PERMUTATIONS

$$10. \quad {}_n P_x = \frac{n!}{(n - x)!}$$

$${}_{12} P_3 = \frac{12!}{(12 - 3)!} = \frac{12!}{9!} = \frac{(12)(11)(10)(9)\dots(1)}{(9)\dots(1)} =$$

1320 PERMUTATIONS

$$11. \quad {}_n C_x = \binom{n}{x} = \frac{n!}{x!(n - x)!}$$

$${}_8 C_2 = \binom{8}{2} = \frac{8!}{2!(8 - 2)!} = \frac{8!}{2!6!} = \frac{(8)(7)(6)\dots(1)}{(2)(1)(6)\dots(1)} = \frac{56}{2} =$$

28 COMBINATIONS