

$$\min_{\underline{x}} x_1^2$$

$$P.O \begin{cases} (x_1-1)^2 + x_2 = 5 \\ x_1 + x_2 = 0 \end{cases} \quad \begin{cases} 5 - (x_1-1)^2 + x_2 = 0 \\ -x_1 - x_2 = 0 \end{cases} \quad \begin{cases} 5 - (x_1^2 - 2x_1 + 1) - x_2 = 0 \\ -x_1 - x_2 = 0 \end{cases} \quad \begin{cases} 5 - x_1^2 + 2x_1 + 1 - x_2 = 0 \\ -x_1 - x_2 = 0 \end{cases}$$

$$L(x, \lambda) = x_1^2 + \lambda_1(5 - x_1^2 + 2x_1 + 1 - x_2) + \lambda_2(-x_1 - x_2)$$

$$\frac{\partial L}{\partial x_1} = 2x_1 - 2\lambda_1 x_1 + 2\lambda_1 - \lambda_2$$

$$\frac{\partial L}{\partial \lambda_1} = 5 - x_1^2 + 2x_1 + 1 - x_2$$

$$\frac{\partial L}{\partial x_2} = -\lambda_1 - \lambda_2$$

$$\frac{\partial L}{\partial \lambda_2} = -x_1 - x_2$$

$$\begin{cases} 2x_1 - 2\lambda_1 x_1 + 2\lambda_1 - \lambda_2 = 0 \\ -\lambda_1 - \lambda_2 = 0 \\ 5 - x_1^2 + 2x_1 + 1 - x_2 = 0 \\ -x_1 - x_2 = 0 \end{cases}$$

$$-\lambda_1 = \lambda_2$$

$$2x_1 - 2\lambda_1 x_1 + 2\lambda_1 + \lambda_1 = 0$$

$$2x_1 - 2\lambda_1 x_1 + 3\lambda_1 = 0$$

$$2x_1 - \lambda_1(2x_1 - 3) = 0$$

$$\frac{2x_1}{2x_1 - 3} = \lambda_1 \quad \lambda_2 = -\frac{2x_1}{2x_1 - 3}$$

λ_1, λ_2 - множители Лагранжа *

$$L(\lambda) = \lambda_1(5 - x_1^2 + 2x_1 + 1 - x_2) + \lambda_2(-x_1 - x_2)$$

$$\frac{\partial L}{\partial x_1} = 2x_1 \lambda_1 + 2\lambda_1 - \lambda_2$$

$$\frac{\partial L}{\partial \lambda_1} = 5 - x_1^2 + 2x_1 + 1 - x_2$$

$$\frac{\partial L}{\partial x_2} = -\lambda_1 - \lambda_2$$

$$\frac{\partial L}{\partial \lambda_2} = -x_1 - x_2$$

$$\begin{cases} 2x_1 \lambda_1 + 2\lambda_1 - \lambda_2 = 0 \\ -\lambda_1 - \lambda_2 = 0 \\ 5 - x_1^2 + 2x_1 + 1 - x_2 = 0 \\ -x_1 - x_2 = 0 \end{cases}$$

$$2x_1 \lambda_1 + 2\lambda_1 + \lambda_1 = 0$$

$$\lambda_2 = -\lambda_1 \quad 2x_1 \lambda_1 + 3\lambda_1 = 0$$

$$\lambda_1(2x_1 + 3) = 0$$

$$\begin{cases} \lambda_1(2x_1 + 3) = 0 \\ \lambda_2 = -\lambda_1 \\ 5 - x_1^2 + 2x_1 - x_2 + 1 = 0 \\ x_1 = -x_2 \end{cases}$$

I

$$\lambda_1 = 0$$

$$\lambda_2 = 0$$

$$5 + x_1^2 - x_2^2 - x_1 + 1 = 0$$

$$x_1^2 - 3x_2 + 6 = 0$$

$$\Delta = 9 - 12 = -3$$

$$\Delta = 9 - 24 = -15$$

$$\sqrt{\Delta} = \sqrt{-15} = i\sqrt{15}$$

$$x_2 =$$

$$x_2 = \frac{2}{3 - i\sqrt{15}} \quad \vee \quad x_2 = \frac{2}{3 + i\sqrt{15}}$$

$$\text{II } \lambda_1 \neq 0$$

$$2x_1 + 3 = 0$$

$$x_1 = -1\frac{1}{2}$$

$$x_2 = 1\frac{1}{2}$$

$$x^* = \begin{bmatrix} -1\frac{1}{2} \\ 1\frac{1}{2} \end{bmatrix}$$

$$f(x^*) = (-1\frac{1}{2})^2 = \frac{4}{9} = 2\frac{1}{4}$$

II