

$$\min_{\underline{x}} x_1^2$$

$$P.O \quad \begin{cases} (x_1 - 1)^2 + x_2 = 5 \\ x_1 + x_2 = 0 \end{cases} \quad \begin{cases} 5 - (x_1 - 1)^2 + x_2 = 0 \\ -x_1 - x_2 = 0 \end{cases} \quad \begin{cases} 5 - (x_1^2 - 2x_1 + 1) - x_2 = 0 \\ -x_1 - x_2 = 0 \end{cases} \quad \begin{cases} 5 - x_1^2 + 2x_1 + 1 - x_2 = 0 \\ -x_1 - x_2 = 0 \end{cases}$$

$$L(x, \lambda) = x_1^2 + \lambda_1(5 - x_1^2 + 2x_1 + 1 - x_2) + \lambda_2(-x_1 - x_2)$$

$$\frac{\partial L}{\partial x_1} = 2x_1 - 2\lambda_1 x_1 + 2\lambda_1 - \lambda_2$$

$$\frac{\partial L}{\partial \lambda_1} = 5 - x_1^2 + 2x_1 + 1 - x_2$$

$$\frac{\partial L}{\partial x_2} = -\lambda_1 - \lambda_2$$

$$\frac{\partial L}{\partial \lambda_2} = -x_1 - x_2$$

$$\begin{cases} 2x_1 - 2\lambda_1 x_1 + 2\lambda_1 - \lambda_2 = 0 \\ -\lambda_1 - \lambda_2 = 0 \\ 5 - x_1^2 + 2x_1 + 1 - x_2 = 0 \\ -x_1 - x_2 = 0 \end{cases}$$

$$-\lambda_1 = \lambda_2$$

$$2x_1 - 2\lambda_1 x_1 + 2\lambda_1 + \lambda_1 = 0$$

$$2x_1 - 2\lambda_1 x_1 + 3\lambda_1 = 0$$

$$2x_1 - \lambda_1(2x_1 - 3) = 0$$

$$\frac{2x_1}{2x_1 - 3} = \lambda_1 \quad \lambda_2 = -\frac{2x_1}{2x_1 - 3}$$

$\lambda_1$  i  $\lambda_2$  - nieodmowne funkcje \*

$$L(\lambda) = \lambda_1(5 - x_1^2 + 2x_1 + 1 - x_2) + \lambda_2(-x_1 - x_2)$$

$$\frac{\partial L}{\partial x_1} = 2x_1 \lambda_1 + 2\lambda_1 - \lambda_2$$

$$\frac{\partial L}{\partial \lambda_1} = 5 - x_1^2 + 2x_1 + 1 - x_2 \quad \begin{cases} \lambda_1(2x_1 + 3) = 0 \\ \lambda_2 = -\lambda_1 \end{cases}$$

$$\frac{\partial L}{\partial x_2} = -\lambda_1 - \lambda_2$$

$$\frac{\partial L}{\partial \lambda_2} = -x_1 - x_2$$

$$\begin{cases} 2x_1 \lambda_1 + 2\lambda_1 - \lambda_2 = 0 \\ -\lambda_1 - \lambda_2 = 0 \\ 5 - x_1^2 + 2x_1 + 1 - x_2 = 0 \\ -x_1 - x_2 = 0 \end{cases}$$

$$2x_1 \lambda_1 + 2\lambda_1 + \lambda_1 = 0$$

$$\lambda_2 = -\lambda_1 \quad 2x_1 \lambda_1 + 3\lambda_1 = 0$$

$$\lambda_1(2x_1 + 3) = 0$$

$$\begin{cases} \lambda_1(2x_1 + 3) = 0 \\ \lambda_2 = -\lambda_1 \\ 5 - x_1^2 + 2x_1 - x_2 + 1 = 0 \\ x_1 = -x_2 \end{cases}$$

### Metoda Lagrange'a

$$1) \min_{\underline{x}} x_1^2 + x_2^2 + x_3^2$$

$$\begin{array}{l} \text{p.o.} \\ \quad x_1 + x_2 + x_3 = 2 \\ \quad x_1 - x_2 + 2x_3 = 3 \end{array} \quad \left\{ \begin{array}{l} 2 - x_1 - x_2 - x_3 = 0 \\ 3 - x_1 + x_2 - 2x_3 = 0 \end{array} \right.$$

$$L(x, \lambda) = f(x) + \lambda^\top (b - g(x)) = x_1^2 + x_2^2 + x_3^2 + \lambda_1 (2 - x_1 - x_2 - x_3) + \lambda_2 (3 - x_1 + x_2 - 2x_3)$$

$$\frac{\partial L}{\partial x_1} = 2x_1 - \lambda_1 - \lambda_2 \quad \frac{\partial L}{\partial x_3} = 2x_3 - \lambda_1 - 2\lambda_2 \quad \frac{\partial L}{\partial \lambda_2} = 3 - x_1 + x_2 - 2x_3$$

$$\frac{\partial L}{\partial x_2} = 2x_2 - \lambda_1 + \lambda_2 \quad \frac{\partial L}{\partial \lambda_1} = 2 - x_1 - x_2 - x_3$$

$$\begin{cases} 2x_1 - \lambda_1 - \lambda_2 = 0 \\ 2x_2 - \lambda_1 + \lambda_2 = 0 \\ 2x_3 - \lambda_1 + 2\lambda_2 = 0 \\ 2 - x_1 - x_2 - x_3 = 0 \\ 3 - x_1 + x_2 - 2x_3 = 0 \end{cases}$$

$$\begin{cases} \lambda_2 = \lambda_1 - 2x_2 \\ 2x_1 - \lambda_1 - \lambda_2 + 2x_2 \\ \lambda_2 = x_1 - x_2 \\ 2x_3 - x_1 - \lambda_1 + 2x_2 = 0 \\ 3 - x_1 + x_2 - 2x_3 = 0 \end{cases}$$

$$\begin{cases} \lambda_1 = x_1 + x_2 \\ \lambda_2 = x_1 - x_2 \\ 2x_1 - 2\lambda_1 + 2x_2 = 0 \\ 2x_3 - x_1 - \lambda_1 + 2x_2 = 0 \\ 3 - x_1 + x_2 - 2x_3 = 0 \end{cases}$$

$$\begin{cases} \lambda_1 = x_1 + x_2 \\ \lambda_2 = x_1 - x_2 \\ 2x_1 - 2\lambda_1 + 2x_2 = 0 \\ 2x_3 - x_1 - \lambda_1 + 2x_2 = 0 \\ x_1 = 2 - x_2 - x_3 \\ x_2 = x_1 - 3 + 2x_3 \end{cases}$$

$$\underline{2x_3 + 2 - x_2 - x_3} - 3x_1 + 9 - \underline{6x_3} = 0$$

$$-5x_3 + 11 - x_2 - 3x_1 = 0$$

$$x_2 = -3x_1 - 5x_3 + 11$$

$$x_1 = 2 + 3x_1 + 5x_3 - 11 - x_3$$

$$-2x_1 = 4x_3 - 9$$

$$x_1 = -2x_3 + 4\frac{1}{2}$$

$$x_2 = -2x_3 + 4\frac{1}{2} - 3 + 2x_3$$

$$I \quad \lambda_1 = 0 \quad 4$$

$$\lambda_2 = 0$$

$$5 + x_2^2 - 2x_2 - x_2 + 1 = 0$$

$$x_2^2 - 3x_2 + 6 = 0$$

DEG ~~D=3~~ D=3

$$\Delta = 9 - 24 = -15$$

$$\sqrt{\Delta} = \sqrt{-15} = i\sqrt{15}$$

EF

$$x_2 = \frac{3 - i\sqrt{15}}{2} \quad v \quad x_2 = \frac{3 + i\sqrt{15}}{2}$$

$$\left. \begin{array}{l} \text{II } \lambda_1 \neq 0 \\ 2x_1 + 3 = 0 \\ x_1 = -1 \frac{1}{2} \\ x_2 = 1 \frac{1}{2} \\ x^* = \begin{bmatrix} -1 \frac{1}{2} \\ 1 \frac{1}{2} \end{bmatrix} \\ f(x^*) = \left(-1 \frac{1}{2}\right)^2 = \frac{9}{4} = 2 \frac{1}{4} \end{array} \right\}$$