1 Simplify $\dfrac{x^4 - 10x^2 + 9}{(x^2 - 2x - 3)(x^2 + 8x + 15)}$. **[4]**

2 Find the unit vector in the direction of $\begin{pmatrix} 2 \\ -3 \\ \sqrt{12} \end{pmatrix}$. **[3]**

3 **(i)** Find the quotient when $3x^3 - x^2 + 10x - 3$ is divided by $x^2 + 3$, and show that the remainder is x. **[4]**

(ii) Hence find the exact value of

$$\int_0^1 \frac{3x^3 - x^2 + 10x - 3}{x^2 + 3}\,dx.$$ **[4]**

4 Use the substitution $x = \frac{1}{3}\sin\theta$ to find the exact value of

$$\int_0^{\frac{1}{6}} \frac{1}{(1 - 9x^2)^{\frac{3}{2}}}\,dx.$$ **[6]**

5 The lines l_1 and l_2 have equations

$$\mathbf{r} = \begin{pmatrix} 4 \\ 6 \\ 4 \end{pmatrix} + s\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

respectively.

(i) Show that l_1 and l_2 are skew. **[3]**

(ii) Find the acute angle between l_1 and l_2. **[4]**

(iii) The point A lies on l_1 and OA is perpendicular to l_1, where O is the origin. Find the position vector of A. **[3]**

6 Find the coefficient of x^2 in the expansion in ascending powers of x of

$$\sqrt{\frac{1 + ax}{4 - x}},$$

giving your answer in terms of a. **[8]**

7 The gradient of a curve at the point (x, y), where $x > -2$, is given by

$$\frac{dy}{dx} = \frac{1}{3y^2(x + 2)}.$$

The points $(1, 2)$ and $(q, 1.5)$ lie on the curve. Find the value of q, giving your answer correct to 3 significant figures. **[7]**

8 A curve has parametric equations

$$x = \frac{1}{t+1}, \qquad y = t - 1.$$

The line $y = 3x$ intersects the curve at two points.

(i) Show that the value of t at one of these points is -2 and find the value of t at the other point. **[2]**

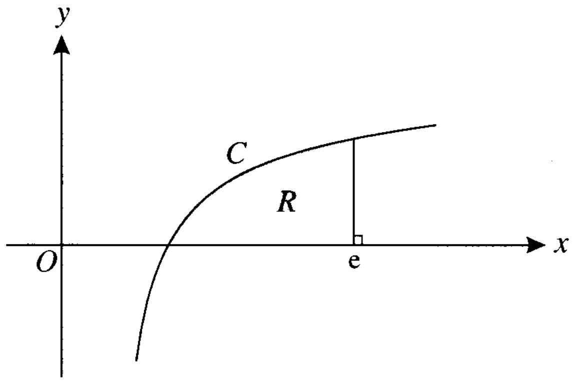
(ii) Find the equation of the normal to the curve at the point for which $t = -2$. **[6]**

(iii) Find the value of t at the point where this normal meets the curve again. **[2]**

(iv) Find a cartesian equation of the curve, giving your answer in the form $y = \mathrm{f}(x)$. **[3]**

9 **(i)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}(x \ln x - x) = \ln x$. **[3]**

(ii)



In the diagram, C is the curve $y = \ln x$. The region R is bounded by C, the x-axis and the line $x = \mathrm{e}$.

(a) Find the exact volume of the solid of revolution formed by rotating R completely about the x-axis. **[6]**

(b) The region R is rotated completely about the y-axis. Explain why the volume of the solid of revolution formed is given by

$$\pi \mathrm{e}^2 - \pi \int_0^1 \mathrm{e}^{2y} \, \mathrm{d}y,$$

and find this volume. **[4]**

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