

Mathe Klausur Nr. 1 Ketten/Produktregel, Ober/Untersumme und Integralrechnung

Verketten von Funktionen

Beispiel:

$$\begin{array}{ll} F(x) = \sqrt{x} & F(G(x)) = \sqrt{2x+1} \\ G(x) = 2x+1 & G(F(x)) = 2\sqrt{x} + 1 \end{array}$$

$$\begin{array}{ll} F(x) = 1-3x & F(G(x)) = 1-3(1+2x^2) \\ G(x) = 1+2x^2 & G(F(x)) = 1+2(1-3x)^2 \end{array}$$

$$\begin{array}{ll} F(x) = (1-x)^2 & F(G(x)) = (1-(2x+1))^2 \\ G(x) = 2x+1 & G(F(x)) = 2((1-x)^2)+1 \end{array}$$

Kettenregel

$$\begin{array}{l} K(x) = F(G(x)) \\ K'(x) = F'(G(x)) \cdot G'(x) \end{array}$$

$$\begin{array}{l} F(x) = (x^5 + x^2 + 123)^{1000111} \\ U(x) = x^5 + x^2 + 123 \\ V(x) = U^{1000111} \\ F'(x) = 1000111(x^5 + x^2 + 123)^{100110} \cdot 5x^4 + 2x \end{array}$$

Produktregel

$$\begin{array}{l} K(x) = F(x) \cdot G(x) \\ K'(x) = F'(x) \cdot G(x) + F(x) \cdot G'(x) \end{array}$$

$$\begin{array}{ll} K(x) = (x^2-4) \cdot (x^3+1) & \\ F(x) = x^2-4 & F'(x) = 2x \\ G(x) = x^3+1 & G'(x) = 3x^2 \\ K'(x) = (2x \cdot (x^3+1)) + ((x^2-4) \cdot 3x^2) & \\ K'(x) = 5x^4 + 2x - 12x^2 & \end{array}$$

Unter/Obersumme

$$F(x) = x^2 + 1 \quad I = 0; 2$$

U8

$$\frac{1}{4} \cdot ((F(0/4)) + (F(1/4)) + (F(2/4)) + (F(3/4)) + (F(4/4)) + (F(5/4)) + (F(6/4)) + (F(7/4)))$$

O8

$$\frac{1}{4} \cdot ((F(1/4)) + (F(2/4)) + (F(3/4)) + (F(4/4)) + (F(5/4)) + (F(6/4)) + (F(7/4)) + (F(8/4)))$$

Integralrechnung

$$F(x)=x^2+1$$

$$l=0;2$$

$$\int^2_0(x^2+1)*dx=[1/3x^3+x]^2=1/3(2)^3+2=4,66666667$$

Kurvendiskussion

Symmetrie

$f(x)=f(-x)$ =Achsensymmetrie zur y-Achse

$f(-x)=-f(x)$ =Punktsymmetrie zum Ursprung

Nullstellen

$$F(x)=x^2+56$$

$$F(x)=x^2+56=0 \quad //p.q.$$

$$x_1=7,4833$$

$$x_2=-7,4833$$

Extrempunkte

Notwendig: $F'(x)=0$

Hoch/Tiefpunkt

$$F''(x) \neq 0$$

$$F''(x) > 0 \quad \text{Tiefpunkt}$$

$$F''(x) < 0 \quad \text{Hochpunkt}$$

$$F(x_1)=(x_1/F(x_1))$$

$$F(x_2)=(x_2/F(x_2))$$

Wendepunkt

$$F'(x)=0$$

$$F''(x)=0$$

$$F'''(x) \neq 0$$

$$x_1=F''(x)$$

$$W(x_1/F(x_1))$$

$F'''(x) > 0$ Rechts-Links Wendepunkt

$F'''(x) < 0$ Links-Rechts Wendepunkt

