

8.6

3) $(\underbrace{\forall \text{ ppl in the world}}_S, R) \quad (a, b) \in R$

a) $\forall a \forall b \{ (a, b) \mid a \text{ taller } b \}$

reflexive

false

Anti symmetric

$\forall a \forall b \{ (a, b) \mid a \text{ taller } b \wedge b \text{ taller } a \} \rightarrow a = b$

$\forall a \forall b \{ (a, b) \mid a \neq b \rightarrow a \text{ not taller } b \vee b \text{ not taller } a \}$

true

transitive

$\forall a \forall b \{ (a, b) \mid a \text{ taller } b \wedge b \text{ taller } c \} \rightarrow a \text{ taller } c$

true

therefore, (S, R) is not a poset.

b) a is not taller than b

reflexive

$\forall a (a \text{ is not taller than } a)$

true

antisymmetric

$$\forall a \forall b \{ (a, b) \mid a \text{ is not taller } b \wedge b \text{ is not taller } a \rightarrow a = b \}$$

true

Transitive:

$$\forall a \forall b \{ (a, b) \mid a \text{ is not taller } b \wedge b \text{ is not taller } c \rightarrow a \text{ is not taller } c \}$$

true

(S, R) is a poset

c) $a = b$ or a is an ancestor of b

reflexive

true

antisymmetric

$$\forall a \forall b ((a, b) \mid a = b \vee a \text{ ancestor } b) \wedge (b = a \vee b \text{ ancestor } a) \rightarrow a = b \}$$

true

Transitive

$$\forall a \forall b \{ (a, b) \mid (a = b \vee a \text{ ancestor } b) \wedge (b = c \vee b \text{ ancestor } c) \rightarrow (a = c \vee a \text{ ancestor } c) \}$$

true

d) a & b have a common friend.

reflexive (true)

antisymmetric

$\forall a \forall b \{ (a, b) \mid a \& b \text{ common friend} \wedge b \& a \text{ common friend} \rightarrow a = b \}$

(false)

transitive

$\forall a \forall b (a \& b \text{ common friend} \wedge b \& c \text{ common friend} \rightarrow a \& c \text{ common friend})$

(false)

(S, R) not a poset

5) a) $(\mathbb{Z}, =)$

reflexive

$\forall a (a = a)$

(T)

antisymmetric

$\forall a \forall b (a = b \wedge b = a \rightarrow a = b)$

(T)

Transitive

$\forall a \forall b (a = b \wedge b = c \rightarrow a = c)$

(T)

b) $(\mathbb{Z}, \neq)$

reflexive

$\forall a (a \neq a)$

(F)

c) $(\mathbb{Z}, \geq)$

reflexive

$\forall a (a \geq a)$

(T)

antisymmetric

$\forall a \forall b (a \geq b \wedge b \geq a \rightarrow a = b)$

(T)

transitive

$\forall a \forall b (a \geq b \wedge b \geq c \rightarrow a \geq c)$

(T)

d) $(\mathbb{Z}, \times)$
reflexive $\forall a (a \times a)$ $\textcircled{F}$

9) reflexive: $\textcircled{T}$

Antisymmetric: $\textcircled{T}$

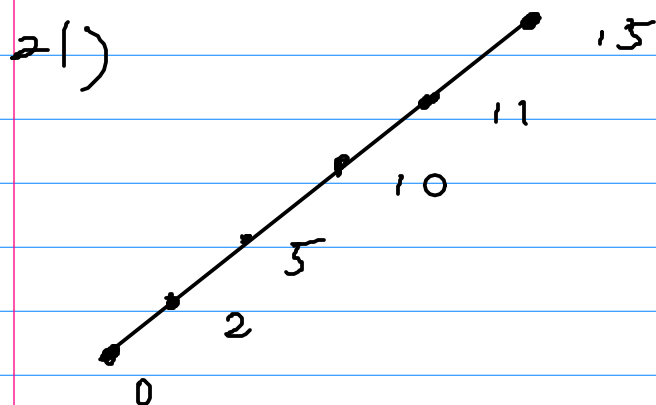
Transitive: $\textcircled{F}$ $a \rightarrow b \rightarrow d$ need $a \rightarrow d$

11) reflexive: $\textcircled{T}$
 Antisymmetric: $\textcircled{T}$
 Transitive: $\textcircled{T}$ $>$ $\textcircled{T}$

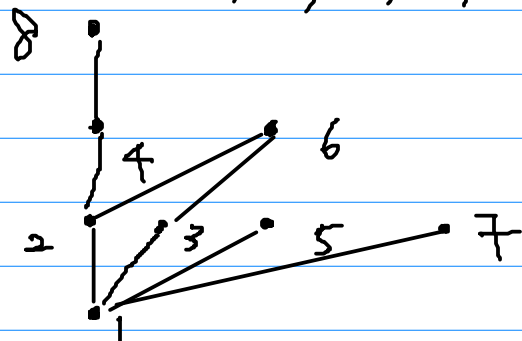
15) a) $\{\emptyset, \{0\}, \{1\}, \{2\}, \{0,1\}, \{0,2\}, \{1,2\}, \{0,1,2\}\}$
 $0 \& 1$

b) $4 \& 6$

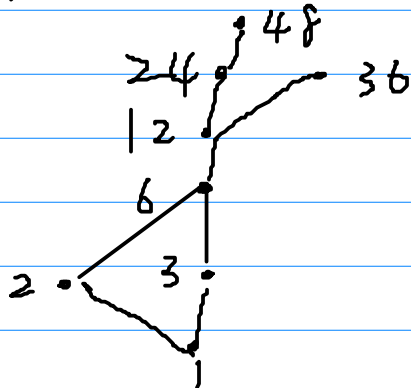
19) ~~0, 01, 11, 001, 010, 011, 0001, 0101~~
 $0 < 0001 < 001 < 01 < 010 < 0101 < 011 < 11$



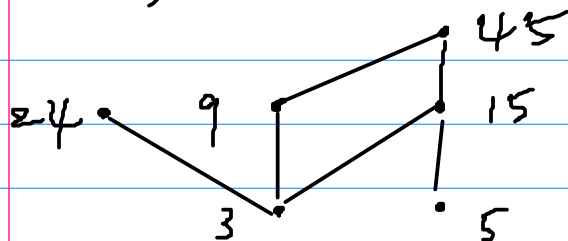
23) a) $(\{1, 2, 3, 4, 5, 6, 7, 8\}, |)$



c) $(\{1, 2, 3, 6, 12, 24, 36, 48\}, |)$



33) $(\{3, 5, 9, 15, 24, 45\}, |)$



a) maximal: 24, 45

b) minimal: 3, 5

c) greatest: no, several max.

d) least: no, several min.

e) upper bound of $\{3, 5\}$: 15, 45

f) least upper bound of $\{3, 5\}$: 15

g) $\vee$ lower bound of $\{15, 45\}$: $\{3, 5, 15\}$

h) greatest lower bound of $\{15, 45\}$: $\{15\}$