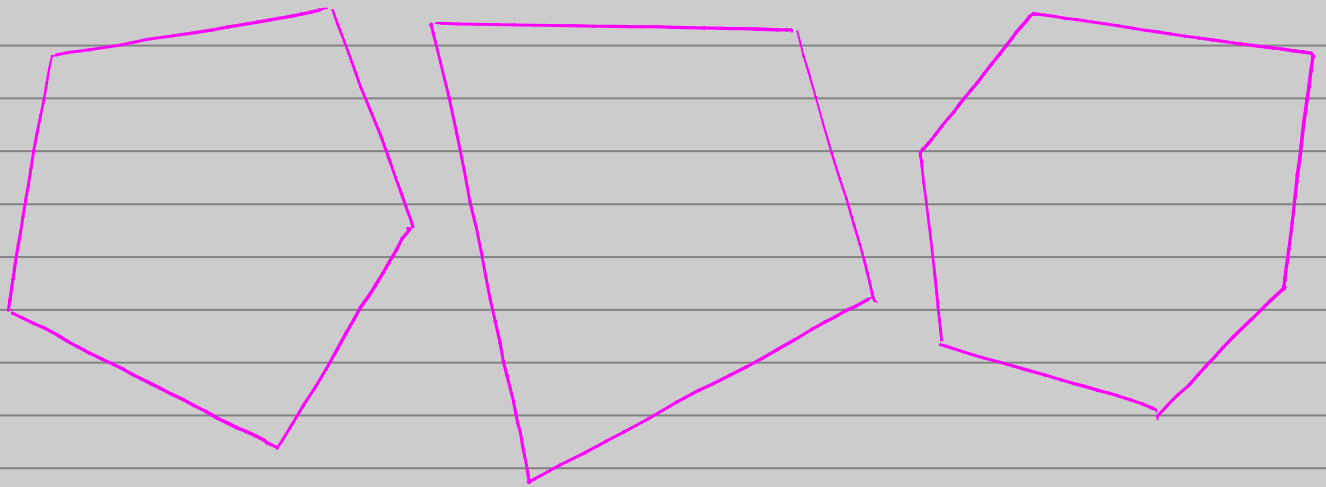
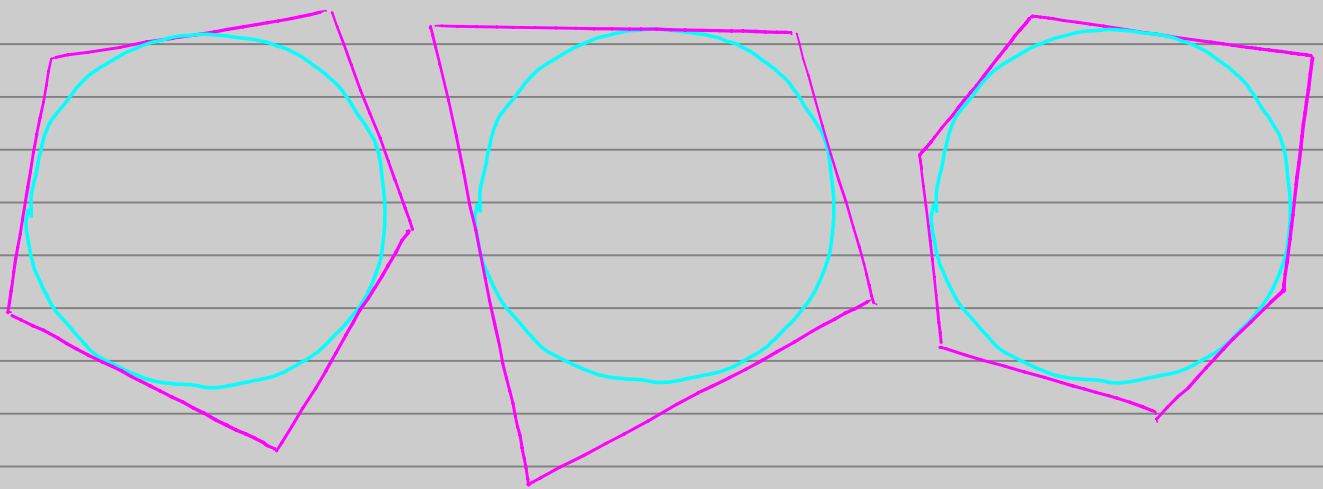


Hey! What do these polygons all have in common?



They all have the same ratio of Area to Perimeter!

Here's how you can tell...

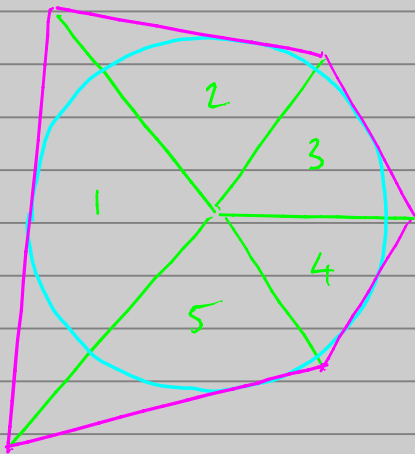
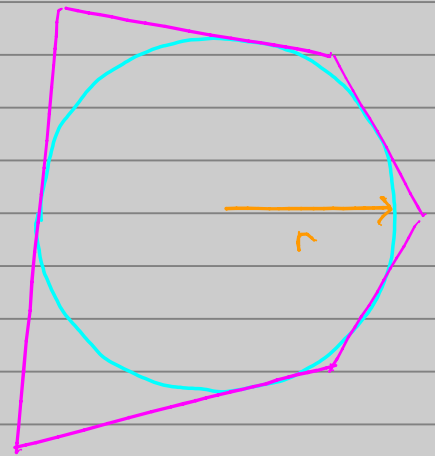


The same circle fits perfectly within the boundaries of each polygon's edges. This necessarily implies that their ratios of Area to Perimeter are equal.

PROOF:

(In order to visually demonstrate the phenomenon, an arbitrary polygon will be used as an example, though all calculations will be generalized)

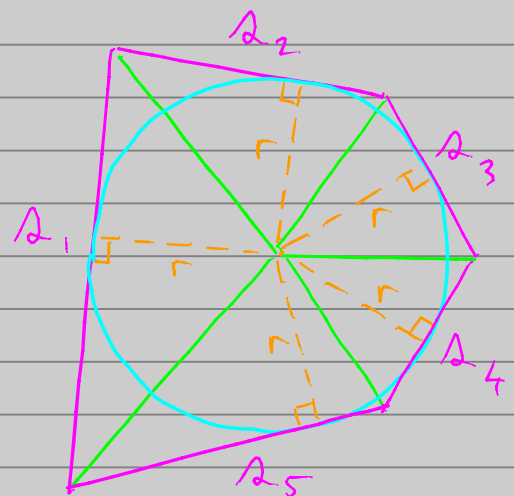
Let r represent the radius of a circle surrounded by an n -sided polygon.



By connecting each vertex to the centre of the circle, the polygon can be exhaustively divided into n -many triangles.

Each outer edge is perpendicular to the radius of the circle at the point of intersection (see Lemma), $\therefore$ for triangle K ,

$$\begin{aligned} A_K &= \frac{1}{2} b h \\ &= \frac{1}{2} \Delta_K r \end{aligned}$$



So, to calculate the area of the polygon, we may simply add up the areas of each of the triangles

$$\therefore A = \sum_{i=1}^n A_i$$

$$= A_1 + A_2 + A_3 + \dots + A_n$$

$$= \frac{1}{2} \Delta_1 r + \frac{1}{2} \Delta_2 r + \frac{1}{2} \Delta_3 r + \dots + \frac{1}{2} \Delta_n r$$

$$= \frac{1}{2} r (\Delta_1 + \Delta_2 + \Delta_3 + \dots + \Delta_n)$$

$$= \frac{r}{2} \left(\sum_{i=1}^n \Delta_i \right)$$

And naturally, the perimeter of the polygon is just the sum of the outer edges

$$\therefore P = \sum_{i=1}^n \Delta_i$$

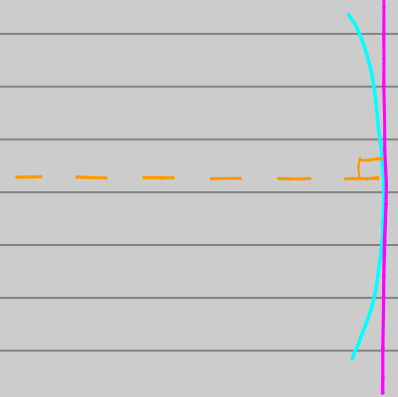
We may now calculate the polygon's ratio of Area to Perimeter:

$$\frac{A}{P} = \frac{\frac{r}{2} \left(\sum_{i=1}^n \Delta_i \right)}{\sum_{i=1}^n \Delta_i}$$

$$= \frac{r}{2} \quad \left(\text{since } \sum_{i=1}^n \Delta_i \neq 0 \right)$$

Note that the ratio depends only upon r , so for a circle with fixed radius, any surrounding polygons will always have the same ratio of Area to Perimeter. $\square$

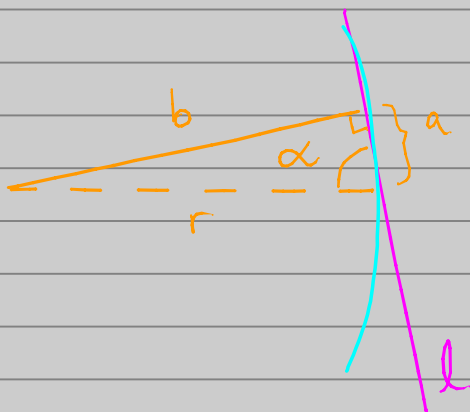
Lemma: A line segment intersecting the edge of a circle at a single point will be perpendicular to the circle's radius at that point.



$\triangleleft$ (Proof by contradiction)

Let l represent the line segment.

Suppose that line segment l is not perpendicular to the circle's radius; thus, they intersect at some angle, α , where $\alpha < 90^\circ$.



Then, there exists a line segment, b , which connects the centre of the circle to line segment l at a 90° angle.

By Pythagorean Theorem,

$$a^2 + b^2 = r^2$$

$$\therefore b^2 = r^2 - a^2$$

$$b = \sqrt{r^2 - a^2}$$

$$\therefore b < r$$

Since b is perpendicular to l , it represents the shortest distance between l and the centre of the circle.

$\therefore$ There is a point where the distance between l and the centre of the circle is less than r , the radius of the circle.

i.e.: l has penetrated the circle's outer edge, and will thus intersect it at more than one point $\downarrow$

Thus, l must be perpendicular to the radius of the circle at the point of intersection $\triangleright$