

$$x = \frac{\pi}{3} - 2k\pi$$

$$x = \frac{\pi}{3} + 2k\pi$$

↓

$$k \in \mathbb{Z}$$

$$k = \{0, \pm 1, \pm 2, \dots\}$$

$$x = \frac{\pi}{3} + 2k\pi$$

$$k=0 \quad x_0 = \frac{\pi}{3}$$

$$k=1 \quad x_1 = \frac{\pi}{3} + 2\pi = \frac{7}{3}\pi$$

$$k=-1$$

$$k \in \mathbb{Z}^+ \quad (+)$$

$$k \in \mathbb{Z}^- \quad (-)$$

$$x_{-1} = \frac{\pi}{3} - 2\pi = -\frac{5}{3}\pi$$

$$x = \frac{\pi}{3} - 2k\pi$$

$$k=0 \quad x_0 = \frac{\pi}{3}$$

$$k=-1 \quad x_{-1} = \frac{\pi}{3} - 2\pi = -\frac{5}{3}\pi$$

$$k=1 \quad x_1 = \frac{\pi}{3} - 2\pi = -\frac{5}{3}\pi$$

Equazioni lineari⁽¹⁾ in seno
e coseno *omogenee*

→ $\sin x - \sqrt{3} \cos x = 0$
lineare

$\sin^2 x - \sqrt{3} \cos x = 0$
*non lineare (2° grado in seno
e 1° grado in coseno)*

(1) Seno e coseno figurano nell'equazione elevati al più alla potenza
↑

$$\sin x - \sqrt{3} \cos x = 0$$

$$\frac{\sin x}{\cos x} = \sqrt{3} \frac{\cos x}{\cos x}$$

$$\cos x \neq 0 \quad x \neq \frac{\pi}{2} + k\pi$$

$$\tan x = \sqrt{3} \Rightarrow x = \frac{\pi}{3} + k\pi$$

$$a \sin x + b \cos x = 0 \quad (1)$$
$$a, b \neq 0$$

Per risolvere la (1)

si può dividere per $\cos x$, imponendo $\cos x \neq 0$,
cioè $x \neq \frac{\pi}{2} + k\pi$.

Si ottiene $\operatorname{tg} x = -\frac{b}{a} \quad (2) \Rightarrow$

$$\Rightarrow x = \operatorname{Arctg}\left(-\frac{b}{a}\right) + k\pi$$

(2)

$$a \sin x + b \cos x = 0$$

$$a \sin x = -b \cos x$$

$$\frac{\sin x}{\cos x} = - \frac{b}{a} \frac{\cos x}{\cos x}$$

$$\operatorname{tg} x = - \frac{b}{a}$$

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Equazioni lineari non
omogenee in seno e coseno

Metodo algebrico

$$\sin x + \cos x - 1 = 0$$

$$\sin x + \cos x = 1$$

Formule parametriche !

$$\sin x = \frac{2t}{1+t^2} \quad \cos x = \frac{1-t^2}{1+t^2}$$

$$t = \tan \frac{x}{2}$$

$$\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} = 1j \quad \frac{2t + 1 - t^2}{1+t^2} = \frac{1+t^2}{1+t^2}$$

$$\cancel{(1+t^2)}^1 \frac{2t+1-t^2}{\cancel{1+t^2}^1} = \frac{(1+t^2)}{(\cancel{1+t^2})^1} \cancel{(1+t^2)}^1$$

$$1+t^2 \neq 0 \quad \text{sempre!}$$

$$2t + \cancel{1} - t^2 = t^2 + \cancel{1}$$

$$2t^2 - 2t = 0$$

$$2t = 0 \Rightarrow t = 0$$

$$2t(t-1) = 0 \Rightarrow t-1=0 \Rightarrow t=1$$

$$\operatorname{tg} \frac{\alpha}{2} = 1$$

$$\frac{\alpha}{2} = \frac{\pi}{4} + k\pi$$
$$\alpha = \frac{\pi}{2} + 2k\pi$$

$$\operatorname{tg} \frac{\alpha}{2} = 0$$

$$\frac{\alpha}{2} = k\pi$$

$\Downarrow$

$$\alpha = 2k\pi$$

Le soluzioni sono

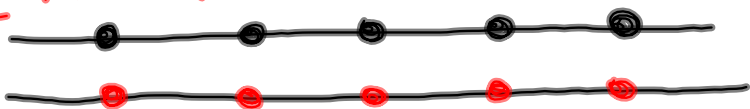
$$\alpha = \frac{\pi}{2} + 2k\pi$$

$$\cup \alpha = 2k\pi$$

Metodo geometrico analitico

$$\begin{cases} \sin x + \cos x = 1 \\ \sin^2 x + \cos^2 x = 1 \end{cases}$$

$$\begin{cases} Y + X = 1 \\ X^2 + Y^2 = 1 \end{cases}$$



$$X = \cos x$$

$$Y = \sin x$$

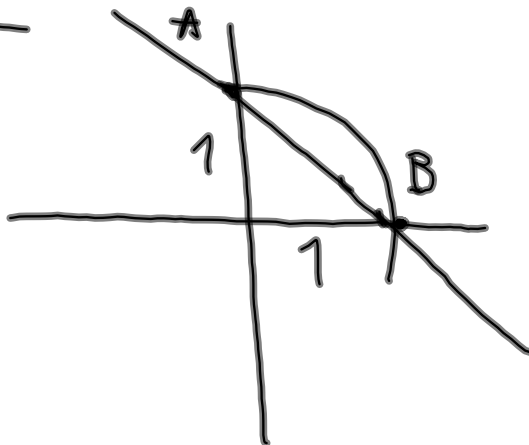
$$\begin{cases} Y + X = 1 \\ X^2 + Y^2 = 1 \end{cases} \Rightarrow \begin{cases} Y = 1 - X \\ X^2 + (1 - X)^2 - 1 = 0 \end{cases}$$

$$2X^2 - 2X = 0; \quad 2X(X - 1) = 0$$

$$\begin{array}{l} A \quad X = 0 \quad Y = 1 \\ \hline B \quad X = 1 \quad Y = 0 \end{array}$$

$$A(0, 1)$$

$$B(1, 0)$$



∴

$$X=0 \quad Y=1$$

$$\begin{cases} \cos x = 0 \\ \sin x = 1 \end{cases}$$

$$x = \frac{\pi}{2} + 2k\pi$$

$$X=1 \quad Y=0$$

$$\begin{cases} \cos x = 1 \\ \sin x = 0 \end{cases}$$

$$x = 2k\pi$$

Equazioni omogenee
di 2° grado

$$\frac{\sin^2 x}{\cos^2 x} - (1 + \sqrt{3}) \frac{\sin x \cos x}{\cos^2 x} + \sqrt{3} \frac{\cos^2 x}{\cos^2 x} = 0$$

Divido primo e secondo
numero per $\cos^2 x$

$$\cos^2 x \neq 0 \Rightarrow x \neq \frac{\pi}{2} + k\pi \quad k \in \mathbb{Z}$$

$$\operatorname{tg}^2 x - (1 + \sqrt{3}) \operatorname{tg} x + \sqrt{3} = 0$$

$$\operatorname{tg} x = t$$

$$t^2 - (1 + \sqrt{3})t + \sqrt{3} = 0$$

$$\Delta = (1 + \sqrt{3})^2 - 4\sqrt{3} =$$

$$= 1 + 2\sqrt{3} + 3 - 4\sqrt{3} =$$

$$= 1 - 2\sqrt{3} + 3 = (1 - \sqrt{3})^2$$

$$t = \frac{1 + \sqrt{3} \pm (1 - \sqrt{3})}{2} = \begin{matrix} / \\ \backslash \end{matrix} \begin{matrix} \sqrt{3} \\ 1 \end{matrix}$$

$$A = 1$$

$$B = -(1 + \sqrt{3})$$

$$C = \sqrt{3}$$

$$\Delta = B^2 - 4AC$$

$\therefore$

$$\operatorname{tg} x = \sqrt{3}$$

$$x = \frac{\pi}{3} + k\pi$$

$$\operatorname{tg} x = 1$$

$$x = \frac{\pi}{4} + k\pi$$

$$k \in \mathbb{Z}$$

Equazioni di 2° grado
non omogenee

$$2\sqrt{3}\sin^2 x - \sin x \cos x + \sqrt{3}\cos^2 x = \sqrt{3}$$

$$2\sqrt{3}\sin^2 x - \sin x \cos x + \sqrt{3}\cos^2 x = \sqrt{3}(\cos^2 x + \sin^2 x)$$

$$\underline{2\sqrt{3}\sin^2 x} - \sin x \cos x + \cancel{\sqrt{3}\cos^2 x} - \cancel{\sqrt{3}\cos^2 x} - \underline{\sqrt{3}\sin^2 x} = 0$$

$$\sqrt{3}\sin^2 x - \sin x \cos x = 0 \quad x \neq \frac{\pi}{2} + k\pi$$

$$\sqrt{3}\tan^2 x - \tan x = 0 \Rightarrow \begin{cases} \tan x = 0 \\ \tan x = \frac{\sqrt{3}}{3} \end{cases}$$

∴

Per il compito

1) equazioni
goniometriche
elementari

2) risoluzione
grafica di
disuguaglianze

3) dominio
segno
immaginario

Es. pag. 33 Q

n° 20, 22, 25,
29, 42,
54, 64, 66, 67,
68

$n = 106, 107, 108$

147, 149, 151, 152
153

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