

Disequazioni goniometriche

$$\frac{\operatorname{tg}^2 x - 1}{2 \cos x + 1} \geq 0 \quad 0 < x \leq 2\pi$$

disequazione goniometrica
fratta

Studiamo il segno del
numeratore, il segno del
denominatore e stabiliamo
il segno del rapporto
con la regola dei segni.

$$\begin{cases} \text{tg}^2 x - 1 \geq 0 & \textcircled{1} \\ 2\cos x + 1 > 0 & \textcircled{2} \end{cases}$$

RisolviAMO separatamente la
 $\textcircled{1}$ e la $\textcircled{2}$.

$$\textcircled{1} \quad \text{tg}^2 x - 1 \geq 0$$

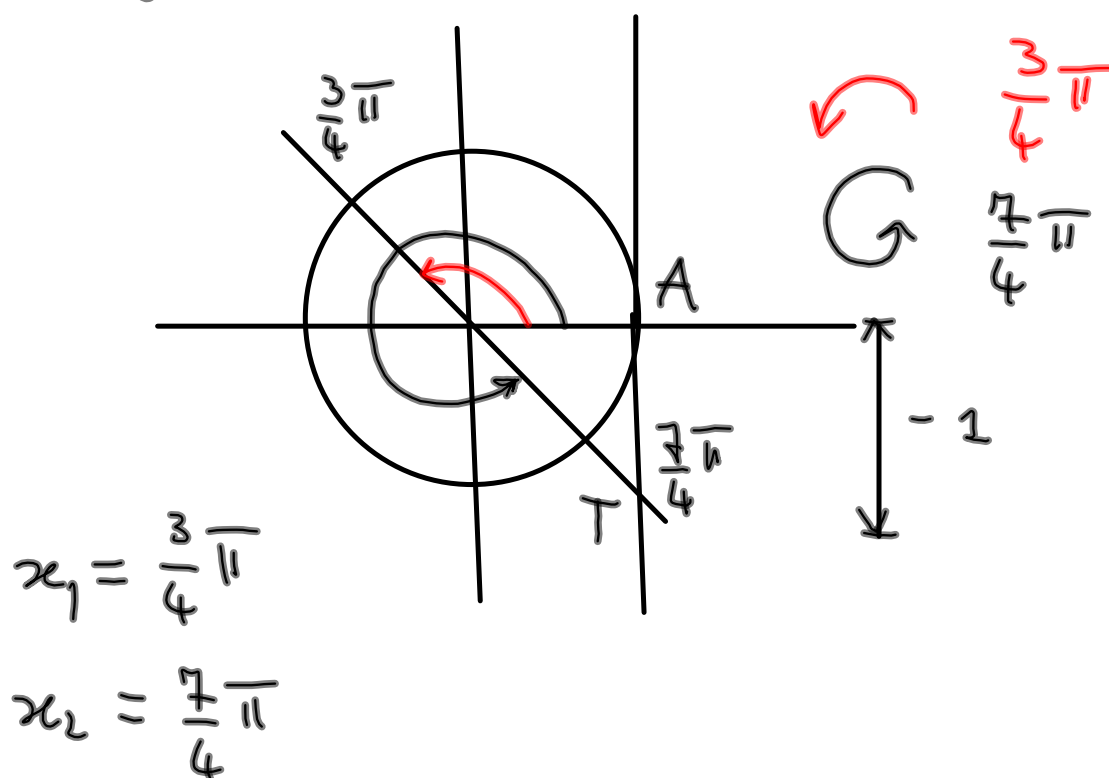
$$(\text{tg} x + 1)(\text{tg} x - 1) \geq 0$$

$$\left[\begin{array}{ll} \text{tg} x + 1 \geq 0 & \textcircled{A} \\ \text{tg} x - 1 \geq 0 & \textcircled{B} \end{array} \right.$$

$$\textcircled{A} \quad \text{tg} x \geq -1$$

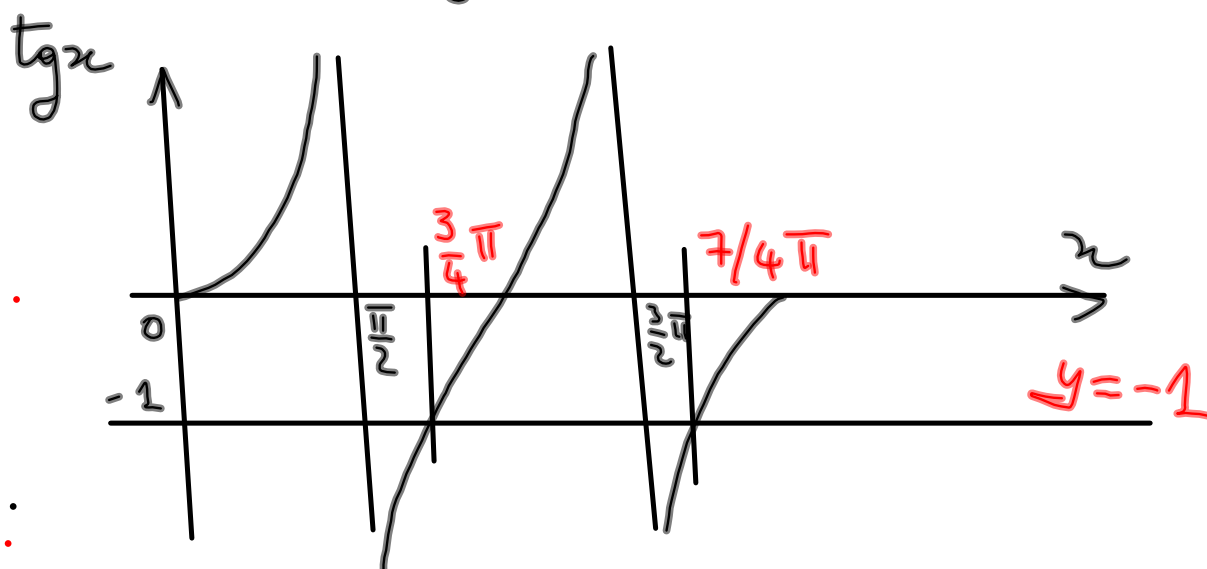
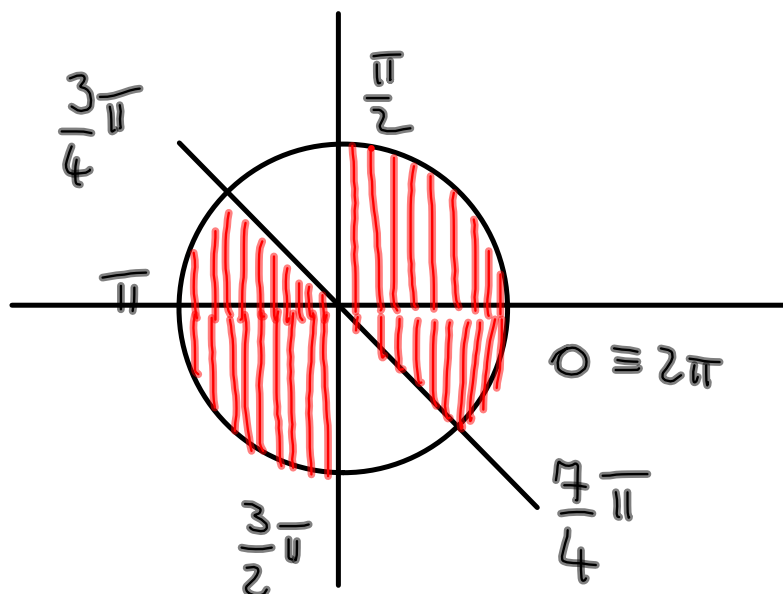
Risoliamo dapprima
l'equazione associata alla $\textcircled{A}$

$$\operatorname{tg} x = -1 \quad 0 \leq x < 2\pi$$



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Risoliamo ora la disequazione



$$\textcircled{A} \quad 0 \leq x < \frac{\pi}{2} \quad \cup \quad \frac{3\pi}{4} \leq x < \frac{3\pi}{2}$$

$$\frac{7\pi}{4} \leq x \leq 2\pi$$

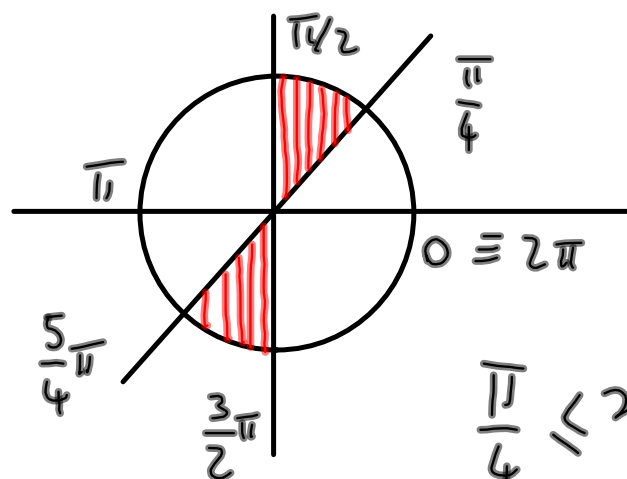
$$\textcircled{B} \quad \operatorname{tg} x \geq 1$$

Risolviameo l'equazione

$$\operatorname{tg} x = 1$$

$$x = \frac{\pi}{4}$$

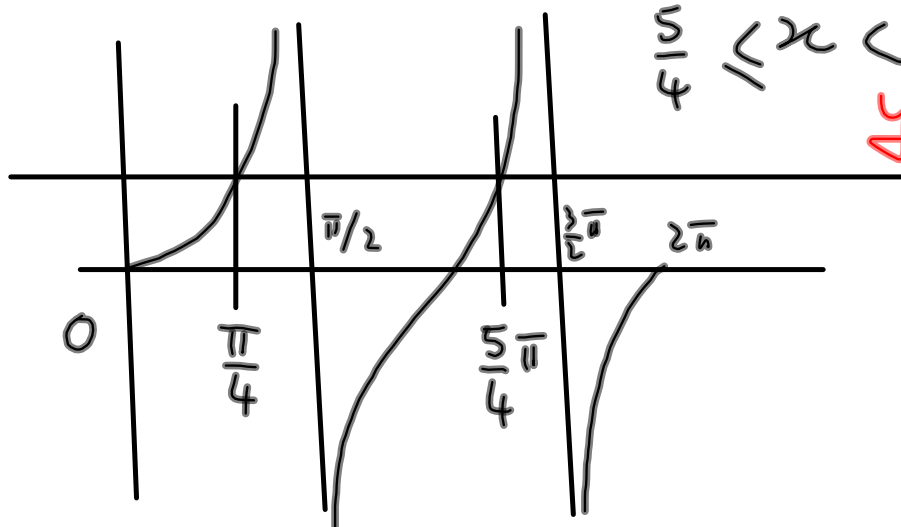
$$x = \frac{5}{4}\pi$$

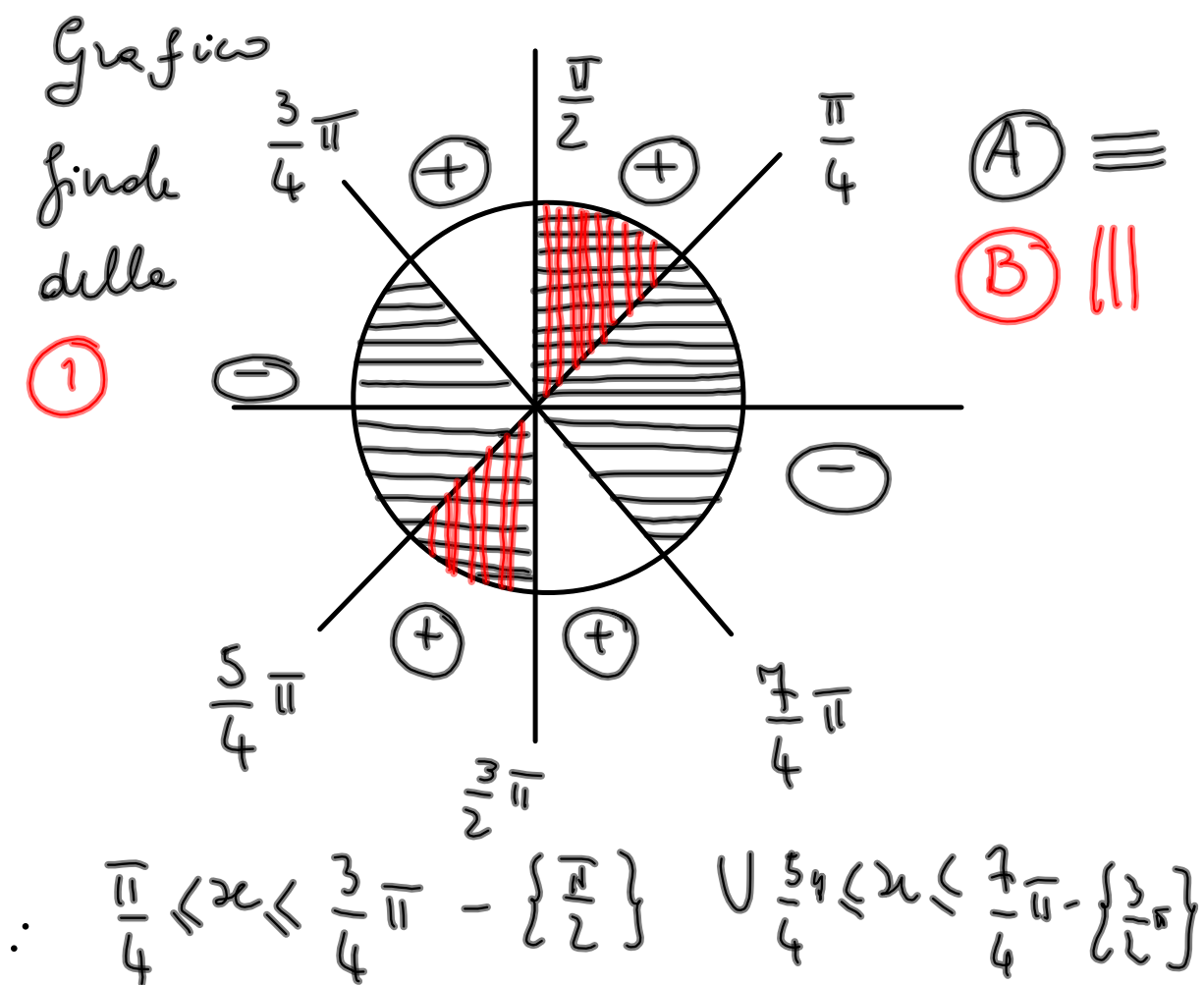


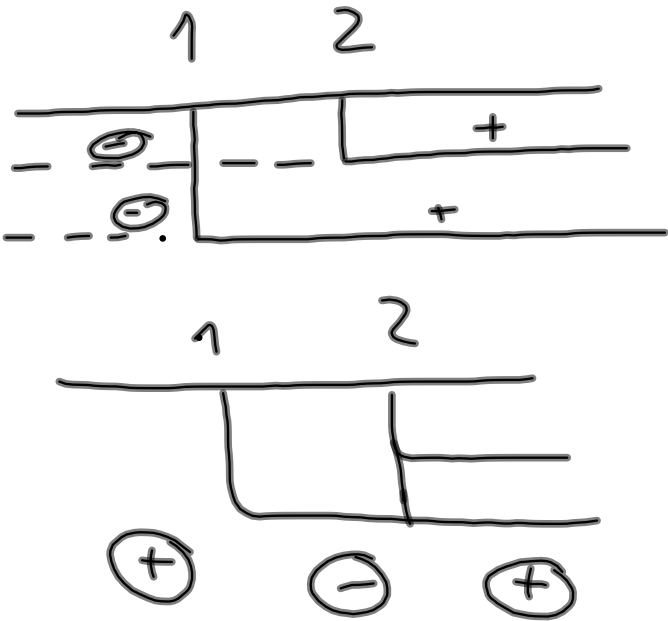
$$\frac{\pi}{4} \leq x < \frac{\pi}{2}$$

$$\frac{5\pi}{4} \leq x < \frac{3\pi}{2}$$

$y = 1$







$$2 \cos x + 1 > 0$$

$$\cos x > -\frac{1}{2}$$

Risolviamo l'equazione

$$\cos x = -\frac{1}{2}$$

$$x = \frac{2}{3}\pi \quad x = \frac{4}{3}\pi$$

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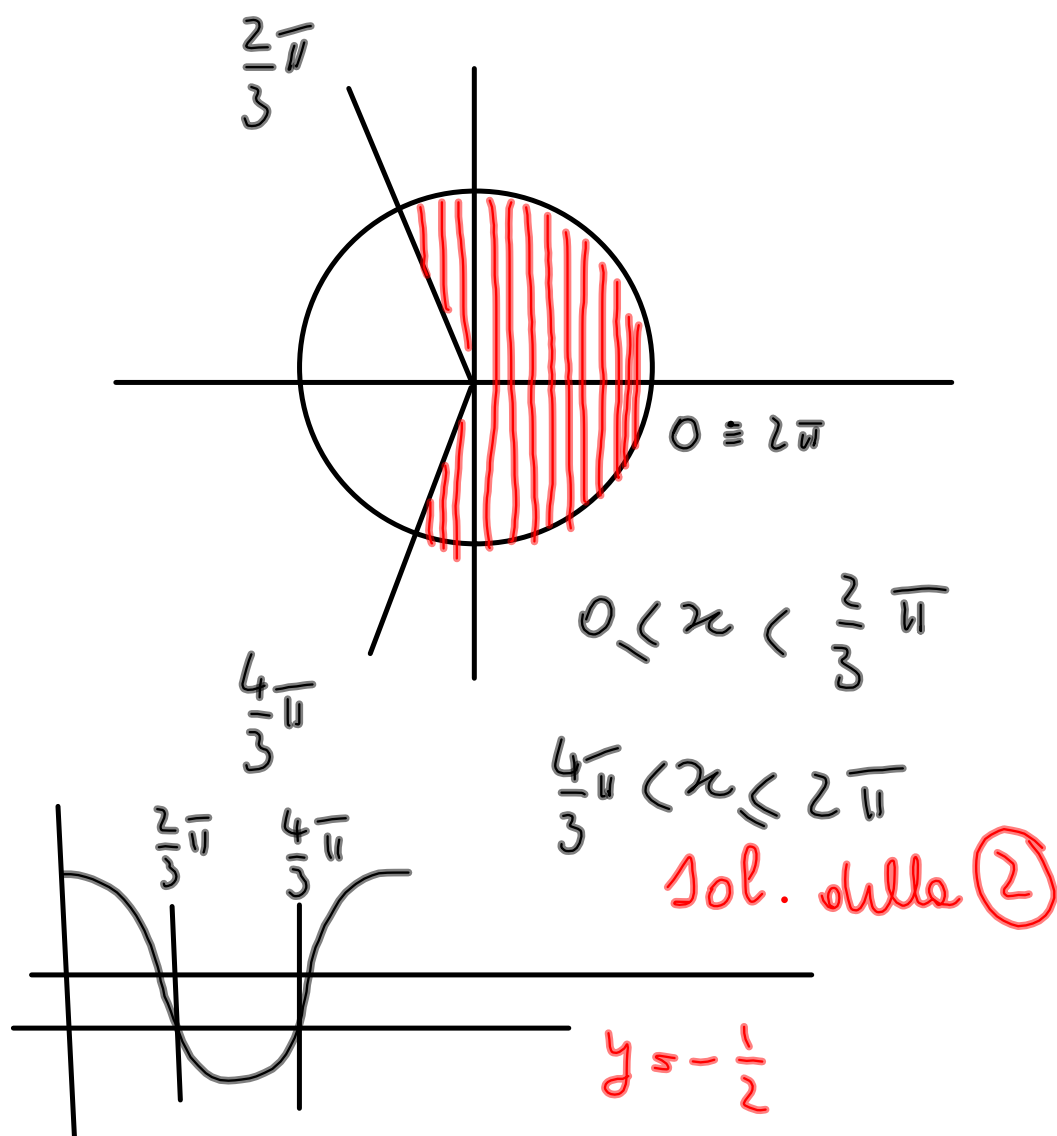
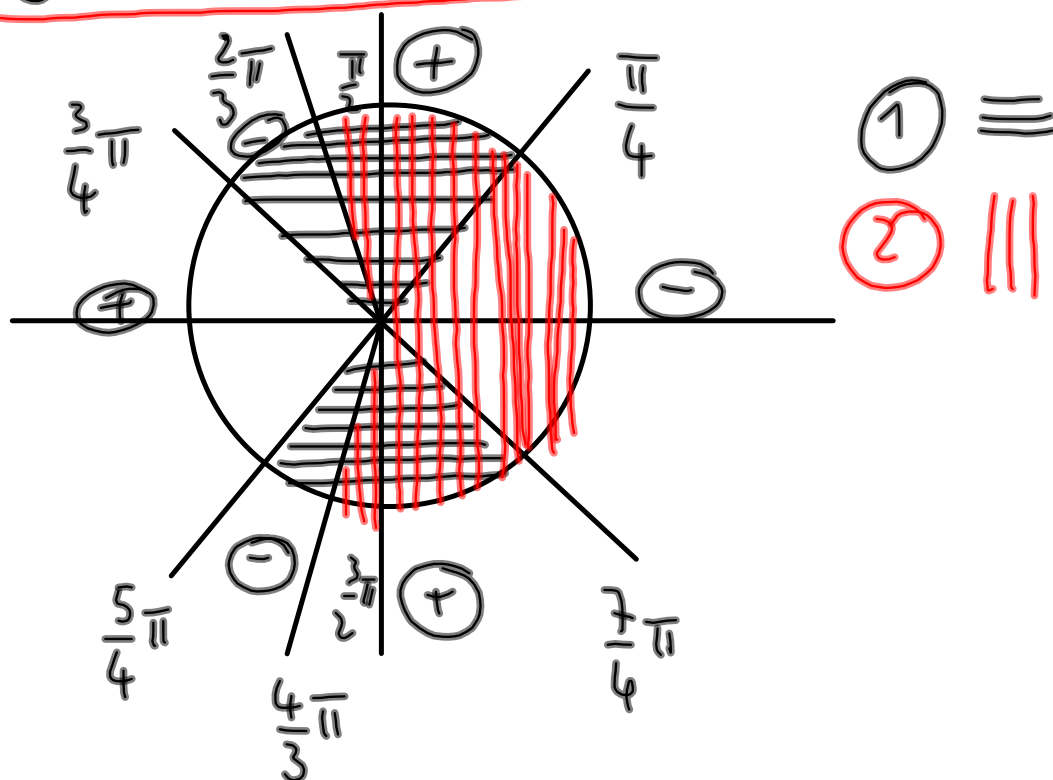


Grafico finale



$$\frac{\pi}{4} \leq x < \frac{2\pi}{3} - \left\{ \frac{\pi}{2} \right\} \cup \frac{3\pi}{4} \leq x < \frac{5\pi}{4}$$

$$\cup \frac{4\pi}{3} < x \leq \frac{7\pi}{4} - \left\{ \frac{3\pi}{2} \right\}$$

Convenzioni sui
segni nelle disequazioni
goniometriche

$\begin{cases} \text{entrambi i tratti} \\ \text{nessun tratto} \end{cases} \Rightarrow \textcircled{+}$

un solo tratto $\Rightarrow \textcircled{-}$