

Corollary: $f_n = \frac{\varphi^n - (-\varphi)^{-n}}{\sqrt{5}}$ (Binet's formula)

$$\triangleleft f_n = \varphi^{n-1} - \varphi^{n-3} + \dots + (-1)^n \varphi^{-n+3} + (-1)^{n+1} \varphi^{-n+1}$$

$$\varphi^2 f_n = \varphi^2 \varphi^{n-1} - \varphi^2 \varphi^{n-3} + \dots + (-1)^n \varphi^2 \varphi^{-n+3} + (-1)^{n+1} \varphi^2 \varphi^{-n+1}$$

$$= \varphi^{n+1} - \varphi^{n-1} + \dots + (-1)^n \varphi^{-n+5} + (-1)^{n+1} \varphi^{-n+3}$$

$$= \varphi^{n+1} - \varphi^{n-1} + \dots - (-1)^{n+1} \varphi^{-n+5} - (-1)^n \varphi^{-n+3}$$

$$\varphi^2 f_n + f_n = \varphi^{n+1} - \cancel{\varphi^{n-1}} + \cancel{\varphi^{n-3}} - \dots - \cancel{(-1)^{n+1} \varphi^{-n+5}} - \cancel{(-1)^n \varphi^{-n+3}} \\ + \cancel{\varphi^{n-1}} - \cancel{\varphi^{n-3}} + \dots + \cancel{(-1)^{n+1} \varphi^{-n+5}} + \cancel{(-1)^n \varphi^{-n+3}} + (-1)^{n+1} \varphi^{-n+1}$$

$$\varphi^2 f_n + f_n = \varphi^{n+1} + (-1)^{n+1} \varphi^{-n+1}$$

$$(\varphi^2 + 1) f_n = \varphi(\varphi^n) + (-1)(-1)^n \varphi(\varphi^{-n})$$

$$(\varphi + 1 + 1) f_n = \varphi(\varphi^n - (-1)^n (\varphi^{-n}))$$

$$(\varphi + 2) f_n = \varphi\left(\varphi^n - \frac{(-1)^n}{\varphi^n}\right)$$

$$\left(\frac{1+\sqrt{5}}{2} + 2\right) f_n = \varphi\left(\varphi^n - \left(\frac{-1}{\varphi}\right)^n\right)$$

$$\left(\frac{1+\sqrt{5}}{2} + \frac{4}{2}\right) f_n = \varphi\left(\varphi^n - \left(\frac{1}{-\varphi}\right)^n\right)$$

$$\left(\frac{1+\sqrt{5}+4}{2}\right) f_n = \varphi(\varphi^n - (-\varphi)^{-n})$$

$$\left(\frac{\sqrt{5}+5}{2}\right) f_n = \varphi(\varphi^n - (-\varphi)^{-n})$$

$$\sqrt{5} \left(\frac{1+\sqrt{5}}{2}\right) f_n = \varphi(\varphi^n - (-\varphi)^{-n})$$

$$\sqrt{5} \varphi f_n = \varphi(\varphi^n - (-\varphi)^{-n})$$

$$\therefore f_n = \frac{\varphi(\varphi^n - (-\varphi)^{-n})}{\sqrt{5}\varphi}$$

$$f_n = \frac{\varphi^n - (-\varphi)^{-n}}{\sqrt{5}}$$

