MOMENT DISTRIBUTION METHOD
Various Methods of Application
by Cliff Leung

In this set of notes, three methods of applying the moment distribution method are presented:

<u>Method 1</u> – Release one joint in each iteration step. Unmodified stiffness factors are used. (The original method)

<u>Method 2</u> – Release multiple joints in each iteration step. Unmodified stiffness factors are used.

<u>Method 3</u> – Release multiple joints in each iteration step. Modified stiffness factors are used.

The moment distribution method essentially involves <u>four steps</u> **to find the end moments** of all members:
1. Find fixed end moments (FEM)
2. Find stiffness factors (K)
3. Find distribution factors (DF)
4. Proceed with the distribution or iteration process.

Before proceeding to the example problem, let's take a look at the difference between the unmodified and modified stiffness factors.

Unmodified and modified stiffness factors

The following structure is used as an example for determining unmodified and modified stiffness factors.

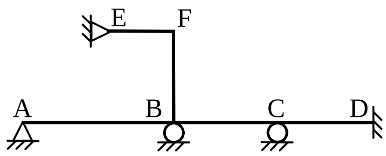


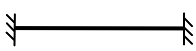
Figure 1.

<u>Unmodified stiffness factor</u>

The unmodified stiffness factor is $\boldsymbol{K = 4EI/L}$. If you decide to use unmodified stiffness factor throughout, then

$K_{AB} = 4EI/L \qquad K_{BA} = 4EI/L$
$K_{BC} = 4EI/L \qquad K_{CB} = 4EI/L$
$K_{CD} = 4EI/L \qquad K_{DC} = 4EI/L$
$K_{BF} = 4EI/L \qquad K_{FB} = 4EI/L$
$K_{EF} = 4EI/L \qquad K_{FE} = 4EI/L$

and you need to treat all members as fixed-fixed when determining FEMs, i.e.

For all members, use  to determine FEMs accordingly

Modified stiffness factor

There are several types of modification to the stiffness factor. This set of notes presents one of the most commonly used modifications as follows.

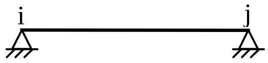
The modification applies to **member that has one of its ends being an <u>absolute end</u> of structure**. What are the absolute ends in the structure above? Ans: Joint A, Joint D and Joint E. Note that Joint F is not an absolute end.

The modified stiffness factor for this type of member is $\boldsymbol{K = 3EI/L}$ and the application is as follows:

Use unmodified K, i.e. $\boldsymbol{K = 4EI/\mathrm{L}}$, **for <u>far end that is not an absolute end (or far end fixed)</u>**

Use modified K, i.e. $\boldsymbol{K = 3EI/\mathrm{L}}$, **for <u>far end that is an absolute end (or far end pinned/roller supported)</u>**

What does far end mean? For example,



If you are finding $\boldsymbol{K_{ij}}$, then j is the far end
If you are finding $\boldsymbol{K_{ji}}$, then i is the far end

In other words, the second letter of the subscript of K indicates the far end joint.

Let's consider the structure in Figure 1 and determine the stiffness factors with modifications considered.

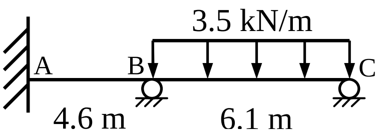
Stiffness factor	Far end is	Is far end the absolute end?	Is the absolute far end pinned or roller supported?	∴ K =
K_{AB}	B	No	-	$4EI/L$
K_{BA}	A	Yes	Yes	$3EI/L$
K_{BC}	C	No	-	$4EI/L$
K_{CB}	B	No	-	$4EI/L$
K_{CD}	D	Yes	No	$4EI/L$
K_{DC}	C	No	-	$4EI/L$
K_{BF}	F	No	-	$4EI/L$
K_{FB}	B	No	-	$4EI/L$
K_{EF}	F	No	-	$4EI/L$
K_{FE}	E	Yes	Yes	$3EI/L$

You need to treat the members which modified stiffness factor is used as fixed-pinned when determining FEMs. ie.,

For Members AB and EF, use 

For Members BC, CD and BF use 

Example problem



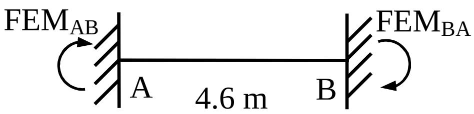
I_{AB} = 1.249×10^{-4} m^4
I_{BC} = 2.497×10^{-4} m^4
E is constant for all members

Method 1

Step 1. Find the fixed end moments for all members.

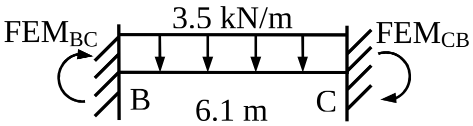
If we are using the unmodified stiffness factors, we need to determine the fixed end moments based on the scenario where the ends of all members are fixed.

Member AB



$FEM_{AB} = 0$
$FEM_{BA} = 0$

Member BC



w = 3.5kN/m; L = 6.1m
$FEM_{BC} = -wL^2/12 = -10.85$ kNm
$FEM_{CB} = wL^2/12 = 10.85$ kNm

Step 2. Find the stiffness factors

$K_{AB} = K_{BA} = 4EI/L = 4E(1.249\times10^{-4}) / 4.6 = 4E(2.715\times10^{-5})$
$K_{BC} = K_{CB} = 4EI/L = 4E(2.497\times10^{-4}) / 6.1 = 4E(4.093\times10^{-5})$

Step 3. Find the distribution factors

$$DF_{AB} = \frac{K_{AB}}{\infty + K_{AB}} = 0$$

$$DF_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC}} = 0.4$$

$$DF_{BC} = \frac{K_{BC}}{K_{BA} + K_{BC}} = 0.6$$

$$DF_{CB} = \frac{K_{CB}}{K_{CB} + 0} = 1$$

Note:
Since left of A is a fixed support, therefore the stiffness left of A is infinite.
Since right of C has no member, therefore the stiffness right of C is 0.

Step 4. Distribution process

Cycle	Joint	A	B		C	Remarks
		AB	**BA**	**BC**	**CB**	
	DF	0	0.4	0.6	1	
1	FEM	0	0	-10.85	10.85	Unlock Joint C to make Joint C in equilibrium
	Dist.	0	0	0	-10.85	
	CO	0	0	-5.425	0	
2	Total	0	0	-16.275	0	Unlock Joint B to make Joint B in equilibrium
	Dist.	0	6.51	9.765	0	
	CO	3.255	0	0	4.882	
3	Total	3.255	6.51	-6.51	4.882	Unlock Joint C to make Joint C in equilibrium
	Dist.	0	0	0	-4.882	
	CO	0	0	-2.44	0	
4	Total	3.255	6.51	-8.95	0	Unlock Joint B to make Joint B in equilibrium
	Dist.	0	0.976	1.464	0	
	CO	0.488	0	0	0.732	
5	Total	3.743	7.486	-7.486	0.732	Unlock Joint C to make Joint C in equilibrium
	Dist.	0	0	0	-0.732	
	CO	0	0	-0.366	0	
6	Total	3.743	7.486	-7.852	0	Unlock Joint B to make Joint B in equilibrium
	Dist.	0	0.146	0.220	0	
	CO	0.073	0	0	0.110	
	Total	3.816	7.632	-7.632	0.11	Good enough

Here are some explanations on the distribution process:

- In this method, we are unlocking each joint <u>one at a time</u> to make the joint in equilibrium.

- At each distribution step (i.e. Dist.), we need to apply an opposite moment to make the unlocked joint in equilibrium. If you look at the table above, we are trying to make the sum of all moments in the bold black box zero.

- <u>Cycle 1 - Distribution</u>
  Let's take a look at Cycle 1. Since Joint C is a roller, the moment for CB should be zero but there is currently a moment of 10.85 kNm. Therefore, we need to apply an opposite moment of -10.85 kNm to make it in equilibrium. So now, the numbers in the bold black box is zero (i.e. 10.85 + (-10.85) = 0).

- <u>Cycle 1 - Carry Over</u>
  After applying the moment of -10.85 kNm at CB, that moment applied will induce moment to the other end of the member (i.e. BC). How much will it induce? It will induce 1/2 of the applied moment at CB. This 1/2 is known as the carry-over (CO) factor. The arrow indicates that 1/2 of 10.85 kNm at CB is carried over to BC.

- Cycle 1 - Total
  At the end of Cycle 1, we total the moments at each side of the joints. You can see that now the moment at CB is zero, therefore it Joint C is in equilibrium. Now we deal with Joint B in Cycle 2.

- Cycle 2 - Distribution
  Let's take a look at Cycle 2. Moment BA is 0 and Moment BC is -16.275 kNm, therefore Joint B is not in equilibrium. We need to apply an opposite moment of 16.275 kNm to make the joint in equilibrium.

  BA and BC will share the 16.275 kNm. So, how much of 16.275 kNm is distributed to BA and BC? The amount of distribution will be based on the distribution factors (DF). i.e.,
  The amount distributed to BA is $DF_{BA}\times 16.275 = 0.4\times 16.275 = 6.51$ kNm.
  The amount distributed to BC is $DF_{BC}\times 16.275 = 0.6\times 16.275 = 9.765$ kNm.

  So now, the sum of moments in the bold black box is zero (i.e. 0 + (-16.275) + 6.51 + 9.765 = 0) to indicate that the joint is in equilibrium.

- Cycle 2 – Carry Over
  The moment applied at BA (6.51 kNm) and at BC (9.765 kNm) will induce moment to the adjacent ends, i.e. AB and CB respectively.
  The amount induced on (or carried over to) AB = $1/2\times 6.51 = 3.255$ kNm.
  The amount induced on (or carried over to) CB = $1/2\times 9.765 = 4.882$ kNm.

- Cycle 2 – Total
  At the end of Cycle 2, Joint B is in equilibrium, i.e. Moment BA = - Moment BC. However, Joint C is now not in equilibrium due to the carry over moment from BC. So in the next cycle, we deal with Joint C again using the same procedures as Cycle 1.

- The moments at the end of Cycle 6 are deemed good enough. You can treat the moment at CB as zero.

Method 2

In Method 1, we unlock one joint in each cycle to make that joint in equilibrium. In Method 2, we will look at unlocking multiple joints that are not in equilibrium in each cycle to expedite the iteration process.

Step 1. FEMs – same as Method 1
Step 2. Stiffness factors – same as Method 1
Step 3. Distribution factors – same as Method 1

Step 4. Distribution process

Cycle	Joint	A	B		C	Remarks
		AB	BA	BC	CB	
	DF	0	0.4	0.6	1	
	FEM	0	0	-10.85	10.85	
1	Dist.	0	4.34	6.51	-10.85	Unlock B and C
	CO	2.170	0	-5.425	3.255	
2	Dist.	0	2.17	3.255	-3.255	Unlock B and C
	CO	1.085	0	-1.628	1.628	
3	Dist.	0	0.651	0.977	-1.628	Unlock B and C
	CO	0.326	0	-0.814	0.488	
4	Dist.	0	0.326	0.488	-0.488	Unlock B and C
	CO	0.163	0	-0.244	0.244	
5	Dist.	0	0.098	0.146	-0.244	Unlock B and C
	CO	0.049	0	-0.122	0.073	
	Total	3.793	7.585	-7.707	0.073	Good enough

Here are some explanations on the distribution process:

- In this method, we do not need to total the moments at each side of the joints after each cycle as in Method 1.

- Instead, we apply an opposite moment in each Dist. step to counter the carry over moment. In other words, after CO is done, we apply a moment opposite to the total moment due to CO at each joint and distribute that opposite moment to each side of the joint.

  For example, at the end of Cycle 1, the total moment at Joint B after CO is 0 + (-5.425) = -5.425 kNm. Therefore, we apply an opposite moment of 5.425 kNm to make the joint in equilibrium. This moment of 5.425 kNm is distributed to BA and BC in the Dist. step of Cycle 2 so that Moment BA is 2.17 kNm and Moment BC is 3.255 kNm.

- At the end, the moments calculated are similar to those obtained in Method 1. Note that the moment-distribution method is an approximate method only and the more iteration you perform the more accurate the results you should get.

Method3

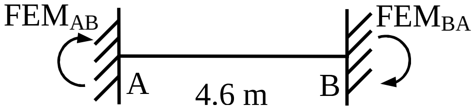
In Method 1 and 2, we use the unmodified stiffness factors throughout our calculation. In this method, we will try using the modified stiffness factors so as to further expedite the iteration process.

Here are several points that you need to note when using the modified stiffness factor that is presented in this set of notes (i.e. $K=3EI/L$):

1. The modified stiffness factor **can be used when a member has one of its ends an absolute end of the structure**.
2. When the modified stiffness factor is used for a particular member, **the member must be treated as fixed-pinned when determining the FEMs**.
3. **Moment does not need to be carried over to the pinned end (absolute end)** of a member for which modified stiffness factor is used.

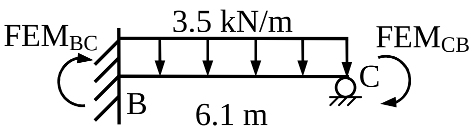
Step 1. Find the fixed end moments for all members.

Member AB



$FEM_{AB} = 0$
$FEM_{BA} = 0$

Member BC



w = 3.5kN/m; L = 6.1m
$FEM_{BC} = -wL^2/8 = -16.28$ kNm
$\mathbf{FEM_{CB} = 0}$

Notice that the FEMs of Member BC in this case is different from that in Method 1.

Step 2. Find the stiffness factors

Member AB
$K_{AB} = K_{BA} = 4EI/L = 4E(1.249\times10^{-4})\ /\ 4.6 = 1.086\times10^{-4}E$

Member BC
Since Member BC has one of its ends (i.e. C) an absolute end of the structure, therefore, we can use modified stiffness factor for this member.
$K_{BC} = 3EI/L = 3E(2.497\times10^{-4})\ /\ 6.1 = 1.228\times10^{-4}E$
$K_{CB} = 4EI/L = 4E(2.497\times10^{-4})\ /\ 6.1 = 1.637\times10^{-4}E$

Step 3. Find the distribution factors

$$DF_{AB} = \frac{K_{AB}}{\infty + K_{AB}} = 0$$

$$DF_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC}} = 0.47$$

$$DF_{BC} = \frac{K_{BC}}{K_{BA} + K_{BC}} = 0.53$$

$$DF_{CB} = \frac{K_{CB}}{K_{CB} + 0} = 1$$

Step 4. Distribution process

Cycle	**Joint**	**A**	**B**		**C**	**Remarks**
		AB	**BA**	**BC**	**CB**	
	DF	0	0.47	0.53	1	
	FEM	0	0	-16.28	0	
1	Dist.	0	7.652	8.628	0	Unlock B and C
	CO	3.826	0	0	0	
	Total	3.826	7.652	-7.652	0	

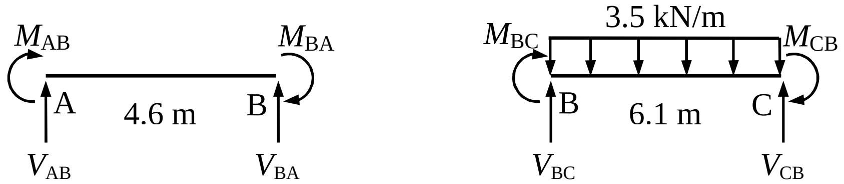
Here are some notes on the distribution process:

- Note that in the distribution process, you **do not need to carry the moment from BC over to CB** unlike in Methods 1 and 2, since by using the modified stiffness factor, it has already implied that moment will not be carried over to the pinned end.

- After Cycle 1, all joints are in equilibrium and therefore the solution is found in just one Cycle. Comparing to Methods 1 and 2, this method of using the modified stiffness factors can yield the solution quicker.

Bending Moment and Shear Force Diagrams

After determining the end moments using either Methods 1, 2 or 3, the question usually asks you to draw the bending moment and shear force diagrams. To do so, you need to determine the end forces of all members first.



From the results of the moment distribution:
M_{AB} = 3.256 kNm; M_{BA} = 7.652 kNm; M_{BC} = -7.652 kNm; M_{CB} = 0.

Determine V_{AB} and V_{BA} by say $\Sigma M_A=0$ and $\Sigma V=0$ for Member BA
Determine V_{BC} and V_{CB} by say $\Sigma M_B=0$ and $\Sigma V=0$ for Member BC

Therefore, V_{AB} = -2.371 kN; V_{BA} = 2.371 kN; V_{BC} = 11.93; and V_{CB} = 9.42 kN.

The shear force and bending moment diagrams of each member are