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75th Anniversary Paper

75-plus years of anisotropy in exploration and reservoir seismics: A historical review of concepts and methods

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ABSTRACT

The idea that the propagation of elastic waves can be anisotropic, i.e., that the velocity may depend on the direction, is about 175 years old. The first steps are connected with the top scientists of that time, people such as Cauchy, Fresnel, Green, and Kelvin. For most of the 19th century, anisotropic wave propagation was studied mainly by mathematical physicists, and the only applications were in crystal optics and crystal elasticity. During these years, important steps in the formal description of the subject were made.

At the turn of the 20th century, Rudzki stressed the significance of seismic anisotropy. He studied many of its aspects, but his ideas were not applied. Research in seismic anisotropy became stagnant after his death in 1916. Beginning about 1950, the significance of seismic anisotropy for exploration seismics was studied, mainly in connection with thinly layered media and the resulting transverse isotropy. Very soon it became clear that the effect of layer-induced anisotropy on data acquired with the techniques of thattime was negligible, so for the next few decades the subject was studied only by a handful of researchers.

In the last two decades of the 20th century, anisotropy changed from a nuisance to a valuable asset. Gupta and especially Crampin pointed out that cracks in a rock mass lead to observable effects from which, in principle, the orientation and density of the cracks could be deduced. Since this information has direct significance for the reservoir properties of the rock, the interest in seismic anisotropy increased considerably. Improvements in acquisition technology, with well-designed approximations that made the complicated theory manageable and with efficient algorithms running on more powerful computers, have turned the theoretical ideas of the early times into an important exploration and production tool. Although seismic anisotropy is usually weak, it nevertheless has important consequences in our modern data. Thus, today anisotropy is an important issue in exploration and reservoir geophysics, and it belongs in every exploration geophysicist's toolkit.

EARLY ELASTIC ANISOTROPY

The theory of elastic-wave propagation was formulated early in the 19th century, and very soon elastic anisotropy was taken into account. This is surprising to physicists who grew up in the 20th century, when elastic anisotropy was regarded as a subject that interested only a minority. But the importance for the pioneers was obvious — they were interested in the

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propagation of light, and to them, light was a wave phenomenon in an invisible, intangible, but nevertheless elastic ether.

The fact that light is transversely polarized posed a difficulty. In isotropic elastic media, one always observed longitudinal waves in addition to transverse waves. Since theory predicted — and observations verified — that some of the features of waves are different in anisotropic media (e.g., double refraction and nonspherical wavefronts), it was tempting to blame the absence of longitudinal optical waves also on anisotropy (of the ether). Thus, the first articles on elastic-wave propagation already took anisotropy into account. For example, Green (1838) was the first to use strain energy, and he strongly supported the notion that there could be as many as 21 elastic constants.

In 1856, Lord Kelvin published “Elements of a mathematical theory of elasticity,” which exclusively discussed solids. This is not to be taken as an indication that he did not believe in the elastic ether, but only that he was interested in metals at that time and thus needed a solid foundation of the theory of elasticity. For this purpose, he invented concepts that became common only much later, such as vectors and vector spaces (in 6D space!), tensors, and eigensystems. With these tools, he could describe the elastic tensor in coordinate-free form. His ideas were so much ahead of his time that his paper — and a re-publication (in the “Elasticity” listing of the 1886 edition of the *Encyclopedia Britannica*) — were regarded by some of his contemporaries as scientifically unsound (despite his stature) and thus made no impact on the development of the theory of anisotropy. Only in the last quarter of the 20th century, when these ideas had been rediscovered, was Kelvin’s achievement recognized.

Kelvin was also the first to formulate the elastic-wave equation for anisotropic media. (He solved it for a simple case.) Since this achievement was published as part of his “no impact” papers, it was also overlooked. Hence, today the solution of the wave equation is attributed to Christoffel (1877).

SEISMIC ANISOTROPY

Anisotropy entered seismology in the last years of the 19th century with the first official appointment of a professor of geophysics. Maurice Rudzki assumed this position in 1895 at the Jagiellonian University of Cracow (former capital of Poland, but at that time under Austrian administration). Shortly afterward, he presented what was to become his scientific program to the Cracow Academy of Sciences (Rudzki, 1895, 520); we quote from a reprint published in German in *Gerlands Beiträge zur Geophysik*, the journal in which many of the early milestones of seismology have been published:

> If we have said that rocks must be treated as homogenous media, we did not mean to imply that these media would be isotropic. Many rocks can, of course, be regarded as isotropic, but in layered rocks one observes often an orientation of the grains — one should think of the orientation of mica flakes in gneiss — and moreover the structure of layered media is generally different parallel and perpendicular to the layers. The dependence of the physical properties is shown by the well-known fact that the conductivity of heat in layered media is different in directions perpendicular and parallel to the layers. We have still another reason to regard some rocks as anisotropic media. Rocks, in particular those at greater depth, are subject to large, and by far not always uniform [isotropic] pressure. But it is known that an isotropic body under uniaxial pressure can and will behave as a birefringent one.

In a later paper Rudzki (1911, 535) writes “Since in seismology, there exists the deplorable habit to regard anisotropic materials as isotropic, . . .” For him there was no doubt that rocks were anisotropic, and he marshaled a long list of reasons for this. In Rudzki (1898) he had gone beyond the plane-wave solution and attempted the determination of the wavefront for a transversely isotropic (TI) medium (one with a single axis of rotational symmetry; hence, also called polar anisotropy), but because of heavy numerical difficulties, he had to be content with only a few points — too few to get an impression of its shape. In Rudzki (1911) he had found a way to overcome the numerical difficulties and thus was the first to realize the possibility of triplication in the SV front (Figure 1). In the same paper, he solved the problem for orthorhombic media, but he regarded this as a purely mathematical exercise because he could not think of a reason for anisotropy of geological bodies that was more complicated than transverse isotropy.

Although in 1905 he had to assume the directorship of the astronomical observatory, seismic anisotropy remained the main interest throughout his life. He wrote on surface waves in a transversely isotropic half space (Rudzki, 1912) and on Fermat’s principle in anisotropic media (Rudzki, 1913). His last paper (Rudzki, 1915) was an attempt to make his ideas known to a large scientific audience. With his sudden death in 1916, research on seismic anisotropy virtually came to a standstill.

After a lapse of about 60 years, the importance of anisotropy for global seismics has increased significantly. Tomographic studies indicate that large parts of the earth’s mantle are anisotropic, associated with the flow of material accompanying global tectonics. Recently, anisotropy of the inner core has been established. For a concise description of recent research in this field see U. S. National Report to IUGG, 1991–1994, and Song and Richards (1996). And of course Rudzki’s arguments

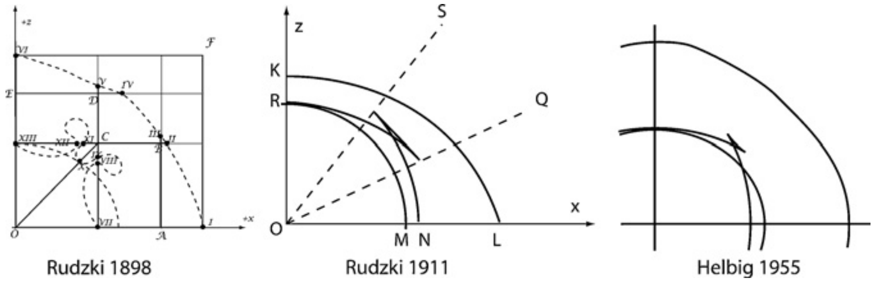

Figure 1. (left) Rudzki’s wavefront of 1898 was based on a few points only (redrawn). (Center) Rudzki’s wavefront of 1911 shows prominent cusps in the SV-sheet.

about crustal anisotropy are as valid now as they were in his time. All this was summed up in a three-day symposium organized by Kendall and Karato (1999).

ANISOTROPY IN EXPLORATION SEISMOLOGY

Anisotropy as an unwanted complication

In the first three decades (1920–1950) of exploration seismology, anisotropy played no significant role — there are only a few papers devoted to the subject. For example, McCollum and Snell (1932) reported on velocities measured on outcrops of Lorraine Shale (Quebec), where the bedding planes were vertical. Direct measurements of velocities along the bedding turned out to be 40% higher than those across the bedding. However, this paper seems to have had no impact on explorationists in the United States or Europe. A series of three papers by Zisman (1933a–c) reported on laboratory measurements of rocks. One contains the word "anisotropy" in the title. These papers had no follow-up by western explorationists either. There were investigations in other parts of the world (Oks, 1938; Gurvich, 1940, 1944; Riznichenko, 1948, 1949), but they seemed to have been noticed in the West only later.

The next field observations of seismic anisotropy were published in the early 1950s. The observation that refraction velocities (along the layers) were consistently higher than the corresponding velocities determined in boreholes (across the layers) could be explained by anisotropy (Cholet and Richard, 1954; Hagedoorn, 1954; Uhrig and van Melle, 1955; Kleyn, 1956). A more direct observation was made by Helbig in early 1954: During seismic work in iron mines in Devonian schists, velocities along the foliation were observed to be 20% higher than those across the foliation. These different observations led to two independent studies of layer-induced anisotropy by Postma (1955) and by Helbig (1956); the full text of Helbig's thesis (Helbig, 1958) was published two years later. The idea underlying these investigations is simple: Fluctuations of the elastic parameters in a sequence of isotropic layers on a scale much shorter than the wavelength lead to long-wave propagation that follows the equations of an anisotropic replacement medium. A replacement medium corresponding to a sequence of isotropic layers is, of course, transversely isotropic, a term first used by Love (1892). Some technical details of the theory of layered media are described in Appendix A.

The interest in layer-induced anisotropy waned quickly. Krey and Helbig (1956) showed that under conditions close to those of the standard surveys of the time (observation of P-waves only; the v_S/v_P ratio is about the same for all constituent layers, with the largest offset about equal to the depth of the reflectors), the anisotropy induced by isotropic layers has practically no effect on the data.

Figure 2 shows the slowness surfaces (the slowness vector has the direction of the wave normal and the magnitude $1/v$). The leftmost panel corresponds to a thin-bedded sequence of two rock-types with a velocity ratio of 1:1.4, the central panel to a velocity ratio of 1:2, and the rightmost panel to a (highly unrealistic) ratio of 1:2.8. In all three cases, the velocity ratio v_S/v_P of the two rock-types is identical. With today's knowledge, this appears to be unrealistic, but 50 years ago this seemed like a good approximation. In all three cases, the slowness surface (and thus also the wavefront) of the P-wave is almost spherical for angles within about 30° from the vertical, although it deviates substantially for near-horizontal angles.

This paper had several consequences:

1) It explained why disregarding anisotropy in standard surveys with restricted offsets was possible.
2) It showed why vertical and horizontal velocities could nevertheless be significantly different.
3) It indicated that anisotropy would have to be taken into account for wide-aperture surveys, for proper time-depth conversion, and for shear-wave surveys.
4) It lowered the urgency of further research.

Anisotropy remained important for layer sequences with fluctuating v_S/v_P ratio (e.g., for coal-bed sequences), and for intrinsic anisotropy, as in shales. For refraction seismics, the situation was different because of point 2) above. For example, Krey (1957) had suggested the use of low-frequency geophones (1–3 Hz) to overcome shadowing of the subsurface by high-velocity intercalations. Standard refraction surveys rely exclusively on first arrivals, and in the situation considered by Krey, these were caused by refractions from the intercalation instead of refraction from the target layer. The proposed method specifically relies on an anisotropic overburden. If the wavelength exceeds the thickness of the high-velocity intercalation sufficiently, no propagating wave exists in — and, therefore, no refraction arrivals can be observed from — the intercalation. The adjacent layers (of lower velocity) and the high-velocity intercalation are then replaced by an anisotropic layer. This compound layer has an effective horizontal velocity that is lower than the velocity of the target layer. Thus, for this frequency/wavelength range, the first arrivals are refracted from the target layer, and refraction seismics is possible again, albeit with an anisotropic overburden.

Two complete algorithms for refraction seismics with an anisotropic overburden were published (Gassmann, 1964;

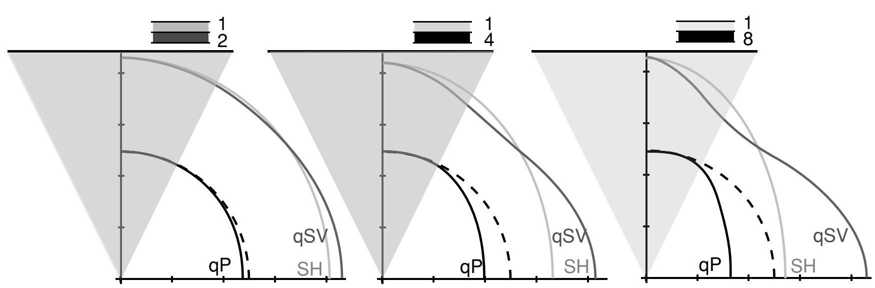

Figure 2. The slowness surfaces (solid curves) in media with layer-induced anisotropy compared with the slowness surface of P-waves in an isotropic medium (dashed curves) with identical slowness along the vertical. The assumption is that two kinds of isotropic layers contribute equally to the thin-bed sequence. In each layer, the ratio of S- to P-velocity is identical. The numbers in the upper right of each panel are the ratio of the stiffnesses of the two layers (i.e., for identical density, the squares of the velocities). The inner sheet corresponds to the slowness of the P-waves; the transparent triangles indicate the standard survey aperture of the 1950s. The left-hand panel is fairly realistic for many geological situations; the others are not.

Helbig, 1964). The first is based on wavefronts, the second on slowness surfaces. However, these algorithms require a priori knowledge of the wave surface or the slowness surface. Gassman's algorithm had the advantage that it could easily be incorporated into the wavefront method of refraction interpretation (Thornburgh, 1930; Ansel, 1931). Helbig's algorithm was simpler both in its analytical and its graphical form. However, refraction seismics was rarely applied during this time, and no application of the method has been reported.

All these arguments were based exclusively on the kinematics of arrivals, involving traveltimes only; the day had not yet arrived when exploration geophysicists would take amplitudes seriously; and so the large effects on reflection amplitudes, even from small degrees of anisotropy, were not yet noticed.

Anisotropy remained a specialty. Only a handful of exploration geophysicists worldwide were active, and technical journals carried, on average, fewer than one article per year (see Figure 3).

The significance of anisotropy increased only marginally when shear-wave surveys became practical. The review book by Danbom and Domenico (1987) contains most of the papers given at the Shear-wave Exploration Symposium in Midland, Texas (March, 1984) and of the shear-oriented papers presented at the 1984 SEG Annual Meeting in Atlanta. According to the index, the term anisotropy is mentioned in four of the fifteen papers — in the two introductory overview papers [Danbom and Domencio (1987a) and Helbig (1987)], in a case history (Justice et al., 1987), and in a seminal article that described a new model for the generation of anisotropy in an originally isotropic background medium and a new exploration concept (Crampin, 1987).

Anisotropy as a source of information

The situation changed with the arrival of this new concept. Gupta (1973a, b, 1974) and Crampin (e.g., 1978, 1981, 1983, 1987) pointed out that azimuthal anisotropy, caused by oriented cracks and stress (even in an otherwise isotropic rock mass) led to anisotropic effects that were easily measurable in a properly designed experiment. Crampin developed a unified theory denoted as extensive dilatational anisotropy (EDA) comprising the following points: (1) cracks in a rock mass line up preferentially with their flat faces perpendicular to the direction of the smallest compressional stress; (2) at reservoir depth, the largest compressional stress is the overburden pressure and the smallest compressional stress is horizontal, so that the cracks preferentially line up with a vertical plane; (3) this results in azimuthal anisotropy (in the simplest case, TI with horizontal axis); (4) that two- and three-component observations are suitable to measure the corresponding shear-wave splitting; and (5) that such measurements are sensitive indicators of the state of stress and microfracturing in rocks. Since all rocks contain nuclei of cracks (grain boundaries, microtectonic cracks, cracks caused by thermal expansion and contraction), the phenomenon was expected to be widespread. Crampin and Peacock (2005) give a modern review of this phenomenon.

Azimuthal anisotropy means even greater complications than does vertical TI, but the information one might obtain by inverting the observations — the orientation and the intensity of cracking — is related to the permeability tensor of the rock. Suddenly, anisotropy changed from a nuisance to an opportunity, for which special multicomponent surveys were undertaken. While in surface seismics the high cost of three-component acquisition and interpretation restricts the number of applications, for VSP and crossed-dipole sonic logging analysis, including azimuthal anisotropy, has become practically routine. In addition, with the current expanded interest in converted-wave (C-wave) surveys [frequently conducted using ocean-bottom seismics (OBS)], it is also seen to be crucial for adequate handling of the data.

Since the arrival of this new exploration concept, the number of active researchers, the number of publications on the subject, the number of presentations at meetings, and — most important — the number of field applications has increased markedly. Today, there is hardly an issue of a geophysical journal without at least one article on anisotropy, and exploration meetings often have several sessions devoted entirely to seismic anisotropy. Beginning in 1982, biannual international workshops on seismic anisotropy were held at places all over Europe and North America (11IWSA took place in Saint John's, Newfoundland, in the summer of 2004).

WHAT MAKES ANISOTROPY DIFFICULT?

Conceptual difficulties

At first sight, seismic anisotropy means only that the wave-propagation velocity depends on direction, i.e., a wavefront emanating from a point is not a sphere, even in a homogeneous medium. This means that many quantities that are isotropic scalars become anisotropic vectors or tensors. The full tensor description of the elastic anisotropic stress-strain ($\sigma - \varepsilon$) relation is

$$\sigma_{ij} = \sum_{k,l=1}^{3} c_{ijkl}\varepsilon_{kl}, \tag{1}$$

introducing the fourth-rank elastic tensor c_{ijkl}. This is a daunting object to contemplate, for one may not easily write down (on 2D paper) any specific case, because of the four indices. However, that problem was eased by Voigt (1910), who pointed out that because of elementary symmetries, the $3 \times 3 \times 3 \times 3$ tensor c_{ijkl} could be mapped into a 6×6 matrix $C_{\alpha\beta}$, which may be used to describe any anisotropic relationship (although, since it is not a tensor, it introduces awkwardness into any analysis).

Another complication is that strictly longitudinal and transverse waves occur only in particular directions. However, in transversely isotropic media, the wave polarized in planes perpendicular to the axis of symmetry is always strictly transverse. The three polarization directions of the three

	1951	1952	1953	1954	1955	1956	1957	1958	1959	1960	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970
GEOPHYSICS					2			1		1	2					2				
Geophysical Prospecting				2		3			1	1			1	1						

Figure 3. Number of articles on seismic anisotropy in the two leading journals — 1950–1970.

waves existing for a given direction of the wave normal are mutually perpendicular. For weak anisotropy — where the ray direction deviates but little from the direction of the wave normal — this is approximately true also for waves corresponding to a given ray direction.

It is strongly tempting to assume that — at least for moderate anisotropy — the wavefront is (or, at least, can be approximated by) an ellipsoid, the simplest generalization of a sphere. Moreover, optical wavefronts in single-axis crystals are either ellipsoids or spheres.

However, this is a case where analogy breaks down. In optical theory, the second-rank dielectric tensor connects two (first-rank) vectors (magnetic field and electric field), whereas in elastic theory, the fourth-rank elastic tensor connects two second-rank tensors (stress and strain). Moreover, even in optics, life is not that simple. Fresnel (1821) had already shown that in optically biaxial crystals (triclinic, monoclinic, orthorhombic), the wavefront is a fourth-order surface that intersects the coordinate planes in circles and ellipses but is neither circular nor elliptical in any other plane. Rudzki (1911) had found that in transversely isotropic media, the P-wavefront is elliptical (and SV-wavefront spherical) only under the condition

$$(c_{11} - c_{44})(c_{33} - c_{44}) - (c_{13} + c_{14})^2 = 0 \qquad (2)$$

This condition is rarely satisfied; for isotropic-layer-induced anisotropy, it is satisfied only if all layers have the same shear stiffness, but in this case, the medium is isotropic anyway. Strictly elliptical wavefronts occur only in symmetry planes for the shear wave that is polarized across such a plane, e.g., for SH-waves in transverse isotropy.

In spite of the previous injunction, many papers have investigated elliptical anisotropy. All exploration papers before Postma (1955) assume elliptical anisotropy, but so do several later papers (e.g., Levin, 1978). It turns out that under the assumption of elliptical anisotropy, all seismic processing and interpretation can be carried out under the assumption of isotropy, provided one applies afterward a graphical compression parallel to the axis of symmetry in the ratio v_V/v_H, where v_V and v_H are the vertical and horizontal velocity, respectively (Gurvich, 1940, 1944; Helbig, 1979, 1983).

One of the important steps in seismic data processing is determination of the rms velocity as the short-spread moveout velocity. Since moveout depends on the wavefront curvature, the method applied to anisotropic, nonspherical wavefronts yields erroneous results, even for short offsets.

In the forward problem, we determine the velocity of homogeneous media as $v = \sqrt{c/p}$, the square root of elastic stiffness divided by the density. This is the velocity v_n of plane waves (pointing, of course, along the normal to the plane wave), and in anisotropic media, this differs from the velocity v_r, which points along the ray (Figure 4). The two velocities have different magnitudes and different directions. For weak anisotropy, the differences will be small, but it remains a complication, in addition to the variation of both velocities with direction.

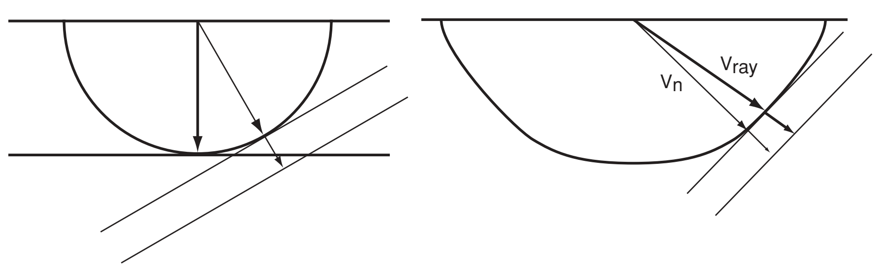

Figure 4. For a spherical wavefront (left) the ray is perpendicular to the wavefront, and thus there is only one velocity. If the wavefront is not spherical, the ray is not everywhere perpendicular to it; thus, one has to distinguish between the ray velocity v_r and the wavefront-normal velocity v_n (the velocity of plane waves).

What about Snell's law? In isotropic media it is generally expressed as: the ratio of the sines of the angles between the rays, and the normal to the interface is proportional to the ratio of the velocities. Which of the velocities should one use? One has to go back to the derivation to see that Snell's law has to do with plane waves (every wavefront can be decomposed into plane waves!). Thus, the correct formulation is: the ratio of the sines of the angles between the wave normals, and the interface normal is proportional to the ratio of the normal velocities.

It is easy to see that these complications have their consequences in all interpretation and processing steps that rely on the curvature of wavefronts and on ray geometry. This is particularly important for advanced concepts, since amplitude-versus-offset (AVO) interpretation has to be reformulated if one or both of the two layers separated by the interface are anisotropic.

Operational difficulties

The difficulties listed in the previous section have to do with basic understanding and can be dealt with through new formulation of rules and improved software. But to make practical use of seismic anisotropy, we have to observe parts of the wave field that under the assumption of isotropy were disregarded or treated as undesirable noise.

As mentioned before, one of the most important phenomena is the splitting of shear waves. To observe shear waves clearly one has to use — and record — two or three components instead of one. Moreover, to study the dependence on direction, one has to use reasonably wide apertures and many azimuths. A two- or three-component 3D survey requires a major effort and, thus, is not easily undertaken. It is no wonder that the first applications were done with three-component tools in boreholes. However, a considerable number of full surface-to-surface surveys have been undertaken, which speaks for the importance the exploration community now attaches to seismic anisotropy.

THE MODERN ERA

Anisotropy began to have a significant impact on seismic exploration during the last 25 years, when a series of trends converged: better data and new types of data were acquired; for example:

- longer offset P-wave data, which show nonhyperbolic moveout caused by polar anisotropy;
- 3C × 3C data, which clearly show the effects of shear-wave splitting;
- wide-azimuth 3D data, which show the effects of azimuthal anisotropy; and
- OBS data, which show all these effects.

At the same time, more powerful computers have enabled the use of more accurate algorithms, relaxing restrictive assumptions and increasingly reveal shortcomings in isotropic velocity fields. More appropriate approximations have been devised as well, enabling analysis of this new data with these new computers.

This last point opens an opportunity for philosophical rumination. The straightforward application of equation 1 in the wave equation produces (for the simplest geophysical case, polar anisotropy) expressions for the three plane-wave velocities, which, by the kindest description, are awkward:

$$v_P^2(\theta) = \frac{1}{2\rho}\left[c_{33} + c_{44} + (c_{11} - c_{33})\sin^2\theta + D\right]$$
$$v_{SV}^2(\theta) = \frac{1}{2\rho}\left[c_{33} + c_{44} + (c_{11} - c_{33})\sin^2\theta - D\right]$$
$$v_{SH}^2(\theta) = \frac{1}{2\rho}\left[c_{44}\cos^2\theta + c_{66}\sin^2\theta\right] \tag{3}$$

where θ is the polar angle, ρ is density, and the subscripts on the shear velocities (conventionally) imply that the symmetry axis is vertical. The difficulties are hidden inside the notation:

$$D \equiv \Big\{(c_{33} - c_{44})^2 + 2\big[2(c_{13} + c_{44})^2 - (c_{33} - c_{44}) \times (c_{11} + c_{33} - 2c_{44})\big]\sin^2\theta + \big[(c_{11} + c_{33} - 2c_{44})^2 - 4(c_{13} + c_{44})^2\big]\sin^4\theta\Big\}^{1/2}. \tag{4}$$

These results were already known to Rudzki (1911); a modern derivation is given, for example, by Tsvankin (2001).

The fourth power and the square root in equation 4 generally are the source of considerable complexity. Furthermore, equations 3 and 4 imply that four elastic tensor elements are required for a full description just of P-waves, and this is not feasible in most geophysical contexts. However, in geophysics we have little need for exact expressions like these, since (1) they already contain the approximation of polar symmetry, (2) in any case, the anisotropy is almost always weak (in some sense) in our problems, and (3) anisotropic effects always show up in our data as combinations of $c_{\alpha\beta}$, so we need not determine the individual $c_{\alpha\beta}$ themselves.

Hence, it makes sense to define new parameters that express the small departure of rocks from isotropy. But how should the isotropic reference be defined? Mathematically and physically, the purest reference is the isotropic average over all directions, since that approach shows no a priori preference over any particular direction. However, in geophysics we recognize that the vertical, in fact, is a special direction (because both the symmetry axis and the acquisition surface tend to be normal to it). It has therefore been found especially useful to define anisotropy in terms of the deviation from vertical velocities.

Further, it has been found that simple linearization offers no advantages and precludes some progress, compared to a linearization based on analysis of the exact equations. Thus, the combinations

$$v_{\mathbf{P0}} \equiv \sqrt{\frac{c_{33}}{\rho}} \qquad v_{S0} \equiv \sqrt{\frac{c_{44}}{\rho}}$$
$$\gamma \equiv \frac{c_{66} - c_{44}}{2c_{44}} \qquad \varepsilon \equiv \frac{c_{11} - c_{33}}{2c_{33}}$$
$$\delta \equiv \frac{(c_{13} + c_{44})^2 - (c_{33} - c_{44})^2}{2c_{33}(c_{33} - c_{44})} \approx \frac{c_{13} + 2c_{44} - c_{33}}{c_{33}} \tag{5}$$

lead to the simplification of equations 3 to

$$v_P(\theta) \approx v_{P0}[1 + \delta\sin^2\theta\cos^2\theta + \varepsilon\sin^4\theta]$$
$$v_{SV}(\theta) \approx v_{S0}\left[1 + \left(\frac{v_{P0}}{v_{S0}}\right)^2(\varepsilon - \delta)\sin^2\theta\cos^2\theta\right]$$
$$v_{SH}(\theta) = v_{S0}[1 + \gamma\sin^2\theta]. \tag{6}$$

Here, v_{P0} and v_{S0} are vertical velocities, and $\varepsilon, \delta, \gamma$ are three non-dimensional combinations which reduce to zero in the case of isotropy, and hence may be called anisotropy parameters. The second (linear) form of δ given in equation 5 comes from a simple linearization of equation 3, but leads to no further simplification and therefore is not needed. Since it is the combinations $\varepsilon, \delta, \gamma$ that appear in our data, it is often not necessary or even possible to determine individual tensor elements or to know with certainty the cause of the anisotropy (e.g., fine-layering, intrinsic shaliness, etc).

So, are the rocks of the sedimentary crust weakly anisotropic (in the sense that defined parameters such as $\varepsilon, \delta, \gamma$ are $\ll 1$)? The answer is not as clear as it might seem, since representative sampling is an issue. Wang (2002), gives a good survey of old and new laboratory measurements; including many measurements with $\varepsilon > 0.2$, which is not really small. However, it appears that most laboratory samples are better consolidated than many recent sediments (i.e., $v_{P0}/v_{S0} \approx 2$ instead of ≈ 3, as commonly observed in the field), so these high-quality data may not be representative of field occurrences. Further, since anisotropy is a scale-dependent phenomenon, anisotropy measured with high precision at the laboratory scale must be compounded with layering effects (e.g., following Backus, 1962) to estimate anisotropy in the seismic band. It is not often that all required data are known. Nonetheless, as an operational guide, it seems that the assumption of weak anisotropy is usually a good one.

The remarkable simplification in equation 6 (based not on artificial assumptions, such as elliptical anisotropy, but rather on the physical observation that seismic anisotropy is weak) has proved to be enormously useful for geophysical analysis. [In fact, the paper (Thomsen, 1986) which defined them is the single most-cited article in the history of GEOPHYSICS (Peltoniemi, 2005).] As an example, it is easy to show that for short spreads, the moveout is hyperbolic and that the short-spread P-wave moveout velocity for a single polar-anisotropic layer is given by

$$v_{P-NMO} \approx v_{P0}(1+\delta). \quad (7)$$

This explains (independent of any assumptions about layering) why the short-spread surface acquisition of SEG's first 50 years did not reveal the kinematic effects of anisotropy, since for such short spreads, there is no deviation from hyperbolic moveout.

Further, in the case where anisotropy is caused by thin isotropic layers, it is easy to show that δ depends on the variation (among the layers) of the velocity ratio v_p/v_s. If the elastic contrasts (among the layers) are weak, then (Thomsen, 2002)

$$\delta_{thin-iso-tyrs} = 2\left\langle \frac{\Delta\left(v_s^2/v_p^2\right)}{\rho v_s^2} \right\rangle \frac{1-\left\langle v_s^2/v_p^2\right\rangle}{\left\langle 1/\rho\, v_s^2\right\rangle - \left\langle 1/\rho v_p^2\right\rangle}. \quad (8)$$

Here, the first term contains the average $\langle\,\rangle$ of the variation Δ of the square of the velocity ratio. In a theoretical exercise, if one computes the anisotropy resulting from isotropic thin layers and casually assumes that the velocity ratio is uniform, one has already concluded that $\delta = 0$, thus eliminating the sought-after effect. This explains the null conclusion of Figure 2.

In reality, the variation of the vertical velocity ratio in a thin-bed sequence is usually small (and intrinsically anisotropic shales are included in the sequence), so that δ is usually small but not zero. This is a natural explanation for the ubiquity of time-depth mis-ties, since the velocity v_{P0} needed for converting arrival times to depths is not given by the moveout velocity v_{P-NMO} (see equation 7), even in this simple case. In a many-layered context, conversion of moveout velocity to interval velocity produces expressions like equation 7 for each layer.

The anisotropic nonhyperbolic moveout shown in Figure 2 at larger angles is governed by a different anisotropic parameter, $\eta \equiv (\varepsilon - \delta)/(1+2\delta)$ (Alkhalifah and Tsvankin, 1995). In fact, they show that v_{P-NMO} and η are sufficient parameters for all P-wave kinematic time processing (including, for example, DMO and time migration) in polar anisotropic media. [For kinematic depth processing, or for dynamic processing (e.g., AVO), a third parameter (e.g., δ) is required.] The power of these conclusions is apparent through an examination of the first equation of equation 3, which requires four parameters for its implementation; the reduction to three or two parameters is important for the practical realization of anisotropic analysis.

These points are well-illustrated in Figure 5 (from Grechka et al., 2001). The authors (and their colleagues) used the results of anisotropic 2.5D forward modeling on a complex structure with known polar anisotropic parameters. They then depth-migrated these data in four approximations (with results as shown in the figure):

1) With the correct parameters, known from the model, a good image is obtained; note the fault intersection (referenced below).
2) With parameters fitted from the data, v_{P-NMO} and η were fitted closely to the true values, but δ was undeterminable from surface data only; hence, it was set to 0. Thus, the image is well focused, but the fault intersection appears at the wrong depth (compare using the horizontal line).
3) With isotropically fitted v_{P-NMO}, but with $\eta = \delta = 0$, the image is distinctly less focused, and the fault intersection is at the wrong depth.
4) With the correct vertical velocity (as determinable through borehole information) instead of the optimal v_{P-NMO}, the image is poorly focused, but at the correct depth.

It is probably safe to say that improved P-wave imaging (with the polar-anisotropic assumption), as shown in the example above, has had the largest economic impact on our business. However, as the data, the computers, and the algorithms improve, other anisotropic features will probably become more important. As a simple example — if the layers are tilted, the polar-symmetry axis is also tilted, and the image point moves laterally (as well as vertically) because of the anisotropy (in addition to the ordinary migration effect) (Vestrum et al., 1999).

However, as we saw before, polar anisotropy itself is generally an idealization. Most rock formations have lower symmetry; often the simplest realistic symmetry is orthorhombic (to purists, orthotropic) — the symmetry of a brick. This corresponds to a single set of vertical fractures in an otherwise isotropic or polar-anisotropic medium, or to two orthogonal sets of such fractures. Both are common in lightly deformed flat-lying sediments; they correspond to extensional failure, normal to vertical and orthogonal stress directions.

Orthorhombic media usually have nine independent elastic tensor elements; it is clearly not feasible, in most geophysical contexts, to determine all nine. However, Tsvankin (1997) shows how to generalize the logic of equations 5 to this case, defining two vertical velocities and seven anisotropic parameters. Further, he shows why much of our P-wave analysis, designed for isotropic or polar-anisotropic cases, works tolerably well even when applied to data from orthorhombic rocks. For example, in a P-wave survey over such formations, the moveout velocity turns out to be

$$v_{P-NMO} \approx v_{P0}[1+\delta(\varphi)], \quad (9)$$

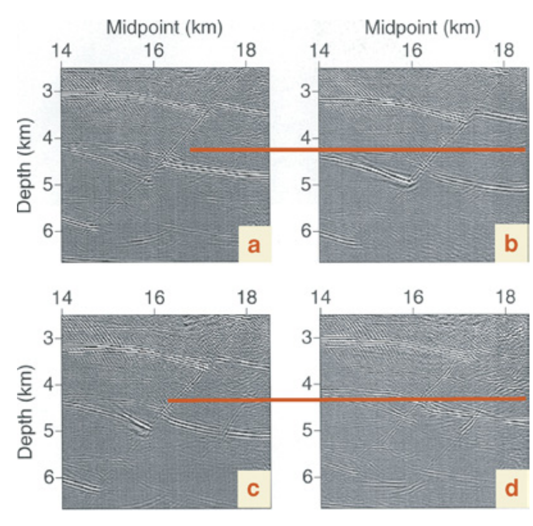

Figure 5. Four different images from four different anisotropic assumptions (Grechka et al., 2001).

where the variation of $\delta(\varphi)$ with azimuth ϕ is a defined function of the elastic tensor elements. In a wide-azimuth survey, this variation can be measured and taken into account. However, in a 2D or narrow-azimuth (e.g., towed-streamer) survey conducted at an angle to the (unknown) principal directions of orthorhombic symmetry, the azimuthal variation is not measured, but an analysis of the data based on equations 5 still succeeds. Because of the similarity in form between equations 9 and 7, the analyst can simply ignore the azimuthal variation of δ, as well as the rest of the orthorhombic variation.

This azimuthal variation of orthorhombic velocity over an orthorhombic ground is a special case of a more general feature of velocity, which was not well known prior to its exposition by Grechka and Tsvankin (1998). To the same (short-offset) approximation that velocities are hyperbolic in time, they are elliptical in azimuth; this is a mathematical consequence of Taylor's theorem and holds for all physical effects, including anisotropy and heterogeneity of any kind.

Small anisotropy can cause large effects

In deriving equations 6, the parameters $\varepsilon, \delta, \gamma$ were each assumed to be small compared to one. Wherever $\varepsilon, \delta, \gamma$ appear (without a large coefficient) in an equation where the leading term is 1 (such as equation 7), they make a correspondingly small, second-order variation from the isotropic case. That does not mean that these variations are negligible; with modern data and imaging algorithms, the neglect of even such small variations can significantly degrade the images.

However, there are cases where the (small) anisotropy parameters appear in contexts where all the terms are small compared to 1. Then, the (small) anisotropy causes a first-order effect, often not small compared to the isotropic effect. For example, the measured P-wave AVO gradient is governed by the plane-wave reflection coefficient. For a planar interface between two polar-anisotropic media, the gradient term in the reflection coefficient, in the linearized approximation, is given by (e.g. Rueger, 1998)

$$R_2 \equiv \frac{1}{2}\left[\frac{\Delta v_{P0}}{\overline{v_{P0}}} - \left(\frac{2v_{S0}}{v_{PO}}\right)^2 \frac{\Delta\mu_0}{\overline{\mu_0}} + \Delta\delta\right], \tag{10}$$

where μ_0 is the vertical shear modulus. Here, the leading (isotropic) terms are the fractional jumps across the interface of v_{P0} and μ_0, which are commonly small in practice, and assumed to be small in the analysis. In this context, the jump in anisotropy parameter δ is not necessarily small compared to these other terms (especially at a sand-shale interface, where $\delta_{\text{sand}} \ll \delta_{\text{shale}}$) and should not be neglected. Nevertheless, it is usually ignored in practice, with the undoubted effects of anisotropy being normalized away using log data.

One context in which anisotropy cannot be thus ignored is in wide-azimuth P-wave surveys (done on land or an OBS kit at sea). Here, the expression corresponding to equation 10 contains an azimuthal variation, which commonly leads to azimuthal variation (of the P-AVO gradient) on the order of 100% (see, e.g., Hall and Kendall, 2000).

There is another context in which small anisotropy parameters make for large effects. In the v_{sv} expression in equations 3, the anisotropic variation is governed by $(v_{P0}/v_{S0})^2(\varepsilon-\delta) \equiv \sigma$. Since (v_{P0}/v_{S0}) is of the order of 3 for unconsolidated marine sediments, the square is of the order of 9, and so σ may be rather large. This can result in some large anisotropic variation, even when the leading term in the equation is 1. For example, the SV equivalent of equation 7 is:

$$v_{SV-NMO}^2 = v_{S0}^2(1+2\sigma) \tag{11}$$

Since SV surveys are rarely performed in exploration geophysics, this is not an important result in itself. However, with the recent interest in OBS surveys, many converted-wave data sets are being acquired. The dominant upcoming S-wave arrival in these data is usually converted from the downgoing P-wave at the reflector. These C-waves image a conversion point whose location (between source and receiver) is governed by the asymptotic conversion point (ACP) (Thomsen, 1999):

$$ACP = \frac{\Gamma_{eff}}{1+\Gamma_{eff}}, \tag{12}$$

where the effective gamma is given by

$$\Gamma_{eff} \equiv \frac{(v_{P-NMO}/v_{SV-NMO})^2}{v_{P0}/v_{S0}}. \tag{13}$$

For the case of a single anisotropic layer, this reduces to

$$\Gamma_{eff} \equiv \frac{v_{P0}}{v_{S0}}\frac{1+2\delta}{1+2\sigma}, \tag{14}$$

which may be significantly smaller than the vertical velocity ratio because σ may be large by the previous argument. This can have a significant impact on imaging, on acquisition design, and hence on acquisition costs.

Small anisotropy can cause completely new effects

An important aspect of anisotropy is that the polarization of shear waves is determined by the medium, not by the source. Thus, in vertical polar anisotropy (see equation 3) we have (in each propagation direction) SV-waves and SH-waves but no waves polarized otherwise (other symmetries have corresponding restrictions). If we attempt (for example, by orienting the source excitation) to create shear waves polarized other than SV or SH, such waves will not propagate at all — not one meter. Instead, the shear strains decompose themselves tensor-wise into the components of SV and SH propagation in each direction, and those waves propagate, each with its own velocity.

Hence, measurement of shear arrivals conveys new information about the rocks, i.e., their symmetry properties, and this is expressed by completely new phenomena (e.g., shear-wave splitting). (Some information about subsurface symmetry is provided by P-waves as well, but possible conclusions are limited by restrictions on travel paths, e.g., sources and receivers on the surface.) In the case of vertical polar anisotropy, the revealed symmetry information is the direction of the symmetry axis (vertical), which we already know.

Even so, completely novel phenomena do appear in polar anisotropic media. Consider the wavefronts in Figure 6, (from

Dellinger, 1991) — a particularly clear discussion of the principles of anisotropy. The red wavefront is SH, just an oblate ellipsoid, but the green SV surface shows cusps and a triplication near 45°. This phenomenon was known already to Rudzki (1911) and was verified independently by Helbig (1958) and by Dellinger (1991). Its existence is governed by a sixth-order inequality in four elastic constants, which resists intuitive understanding. In particular, it is clear that this bizarre phenomenon occurs only for strong anisotropy, but which measure(s) of anisotropy are involved and how large they must be was not clear (see, however, the discussion by Helbig, 1994). Cusps have been observed in the field and theoretically modeled in a walkaway VSP in the Juravskoe oil field in the Caucasus Basin (Slater et al., 1993) and in a survey over the Natih oil field (Oman) by Hake et al. (1998). A schlieren photo of cusps in Pertinax (a resinated stack of paper as a model for a layered medium) was obtained by Helbig (1958).

Recently, Thomsen and Dellinger (2003) discovered a strategic approximation that reduces this complexity to extreme simplicity; they found that triplications near 45° occur whenever

$$\sigma > \sigma_{critical} \equiv \tfrac{2}{3}[1 + \delta - (v_{S0}/3v_{P0})^2], \qquad (15)$$

where σ is the quantity defined earlier in the weak-anisotropy context, with the exact form for δ included. The approximation is that the trailing terms above are small (i.e., there is no assumption of weak shear anisotropy) and is probably valid for all sedimentary rocks. The implication is that rocks that exhibit P-wave anisotropy (e.g., through nonhyperbolic P-wave moveout) with

$$\eta \equiv \sigma (v_{S0}/v_{P0})^2/(1 + 2\delta) > 0.07, \quad \text{approximately}, \qquad (16)$$

also support shear-wave triplication. Such values of η are typical of many marine sediments; it follows that they would also exhibit shear-wave triplication in an appropriately designed survey. It is not yet known if this is an important result, but it illustrates the power of the parameterization in equation 5.

What is not uncertain is that the consequences of azimuthal anisotropy are pervasive and reveal themselves in phenomena completely outside the scope of isotropy. The early contributions of Gupta and the comprehensive contributions of Crampin in this arena have already been noted. After a long period of investigation by global geophysicists using earthquake sources, a breakthrough for exploration geophysics came with the realization by Alford (1986) that the control of source polarization, as in industry practice, provided a crucial advantage.

Because shear-wave polarizations are determined by the medium rather than by the source, so-called "SH-surveys" of the 1970s and 1980s (2D, with crossline sources and crossline receivers) were usually failures, yielding uninterpretable data (Willis et al., 1986). Since these surveys were generally conducted at an angle to the (unknown) axes of azimuthal anisotropy, the crossline-sourced waves decomposed into two modes (now termed simply "fast and "slow"), each with both a crossline component and an inline component. These waves propagated, each at its own velocity, and upon reflection and return to the surface, both were recorded on the crossline receivers, interfering constructively and destructively. The interference caused problems for interpreters as soon as the fast-slow delay reached a small fraction of the dominant period of the signal. Since this period is small compared to the traveltimes of interest, this criterion means that a small azimuthal shear-wave anisotropy can have large effects in the data.

Alford's solution to this was ingenious: He installed inline geophones as well and also executed inline-polarized sources (a technique unavailable to global geophysicists), recording both sources into both sets of receivers. This resulted in a 2C × 2C tensor of data, replacing the scalar data that had been the industry standard for half a century. By tensor rotation (about a vertical axis) of this data set at each time sample, one could (if the propagation were vertical) find an angle for which the off-diagonal signal was zero and the two diagonal terms contain the fast and the slow modes separately. This angle determines the azimuth of azimuthal anisotropy (i.e., in a simple

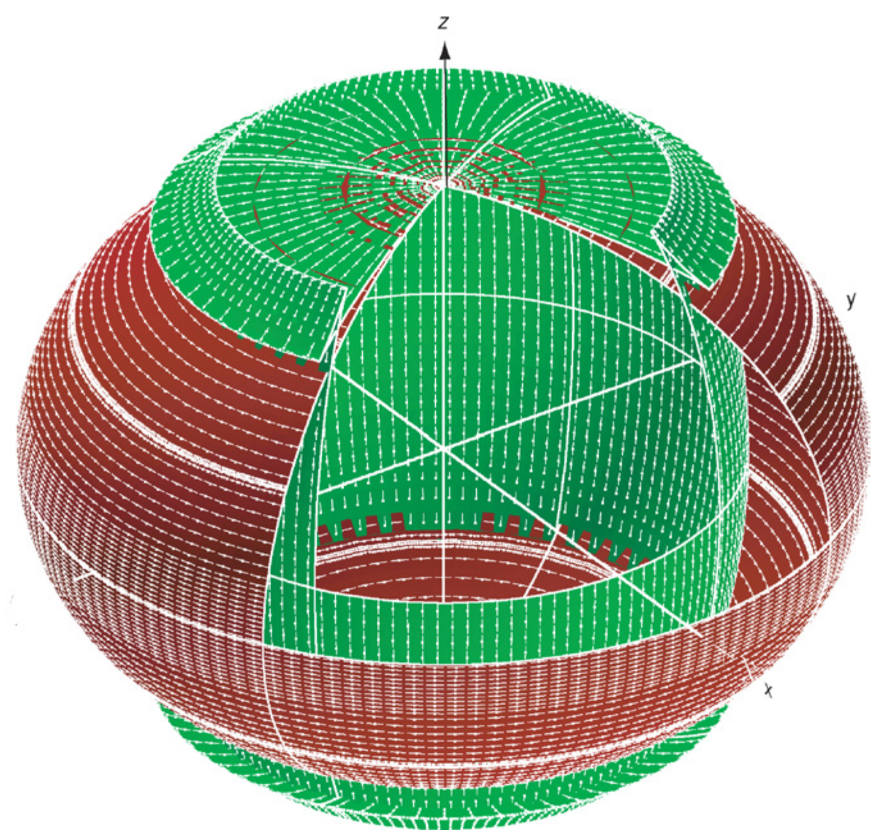

Figure 6. Wavefronts for a particular case of polar anisotropy (SH in red, SV in green). Note the SV cusps near 45°.

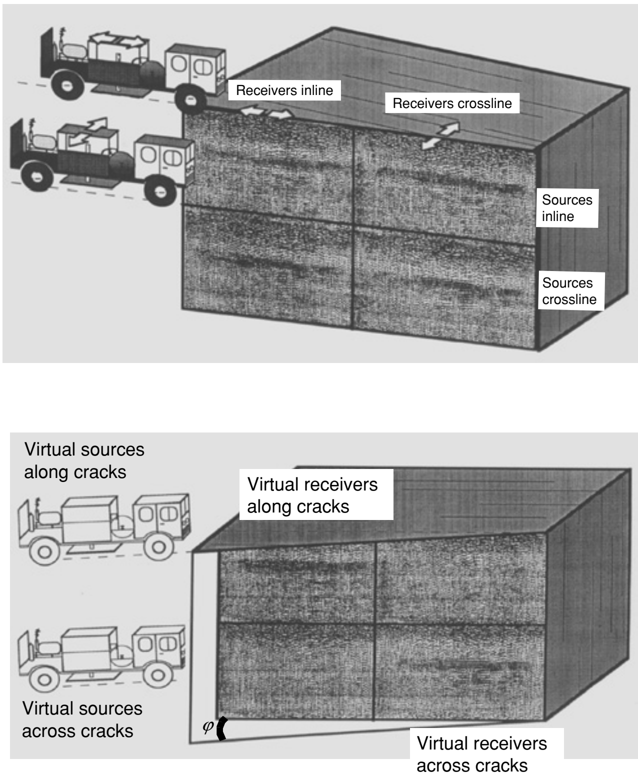

Figure 7. 2C × 2C matrix of data after preprocessing and stacking (from Beaudoin et al., 1997). A conventional SH survey is shown as the lower-right element. According to isotropic theory, in this flat-lying stratigraphy the lower-left element should be only noise, but the signal there is just as strong, although both elements show poor continuity of reflectors. The top row was never acquired prior to Alford's work. (b) 2C × 2C data from (a), rotated through the angle shown. Note that the off-diagonal sections now show only noise; the lower-right section can be shown (by crosscorrelation) to slightly lag the upper-left section. This determines that the angle shown identifies the strike of the imputed cracks (as shown cartoon-style), rather than the normal to the cracks.

case, the orientation of subsurface fractures). As applied to stacks of real data, this technique works amazingly well, given that none of the traces going into the stack precisely obeys the assumption of vertical travel (see Figure 7a, b).

In cases where reservoir production is dominated by crack permeability, such subsurface characterization can be important in exploring for patches of intense fracturing, with an implied enhancement of crack permeability. As mentioned earlier, this is a new exploration concept — not exploring for the presence of reservoir rock, or for the presence of hydrocarbons, but for the presence of (crack) permeability. Figures 8 and 9 illustrate the concept; although only two wells were studied (in the Valhall oil field in Norway), they were well-matched in all respects, aside from their anisotropy differences, and unambiguously show the connection between shear-wave splitting and production.

In the case where the principle directions of azimuthal anisotropy vary with depth, this algorithm needs to be augmented with a layer-stripping procedure; see Winterstein and Meadows (1991) for VSPs, and Thomsen et al. (1999) for surface surveys (also giving a vector algebra for analyzing such data sets).

With the introduction of OBS methods, C-wave surveys have become common in contexts where the value they provide justifies their higher cost. Such data sets offer rich possibilities for subsurface characterization. A first step is often determination of the principle axes of azimuthal anisotropy. If the survey has a wide distribution of source-receiver azimuths, this is especially important. In this context, Alford's algorithm is not possible since the P-S conversion provides only a radial (not a transverse) excitation.

The simplest method to determine the principal axes was recognized by Garotta and Granger (1988) long before the OBS era. In a wide-azimuth common-conversion-point gather, those shear waves that happen to arrive from either of the principle directions will have no energy on the transverse component. Hence, by taking the ratio of transverse to radial amplitudes averaged over an appropriate time window, the principal directions become obvious (Figure 10).

Dellinger et al. (2002) show more formally how to (1) generalize Alford's logic to treat the case of upcoming shear waves from a wide-azimuth converted-wave survey and (2) replace the stacking step with any migration, i.e., not limited by the zero-reflector-dip assumption of stacking.

Application of such principles normally results in much better C-wave images, as the principal-component images have only one arrival per reflector (see, e.g. Granger et al., 2000). The differences between the two split-shear arrivals sometimes carry remarkable new information concerning the subsurface (e.g., as in Figure 10).

Of course, deformation of the subsurface like this is often accompanied by microseismic activity. Caley et al. (2001) noticed a time-dependence in overburden anisotropy (Figure 11), the first time such time dependence, corresponding to oil-field activity, has been seen.

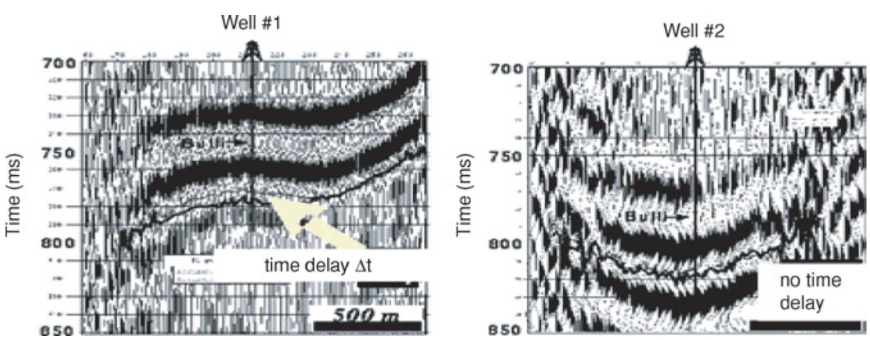

Figure 8. An Alford analysis of a 2C × 2C survey in a coal-bed methane province was used to make a 2D section from the fast polarization (Beaudoin et al., 1997). (Horizontal compression of the section makes the layers appear to dip strongly, but they are nearly flat.) Also shown (thin line) is the trough just below the Bulli coal beds and the corresponding pick (thick line) from the slow-polarization section. At the site of well #1, there is a delay between the two picks, indicating substantial anisotropy there, with an inferred intensity of cleating in the coal beds. At well #2, there is no separation, implying no cleating.

It has been shown by Crampin (1999) and Crampin and Peacock (2005) that the stress-aligned, fluid-saturated microcracks in almost all rocks, including most hydrocarbon reservoirs, are so closely spaced that the rock-crack system is close to fracture criticality and fracturing. This makes reservoirs (and the rock mass) highly compliant to small changes in conditions and thus explains the sensitivity of the Valhall field to changing conditions (Figure 12). This novel aspect might have implications for earthquake prediction.

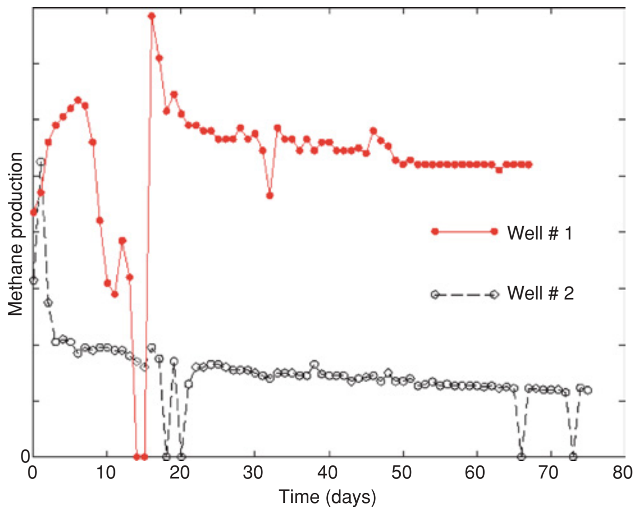

Figure 9. Two wells in the Valhall field (Norway) completed identically, showing that the fractured well had four times the production of the unfractured well.

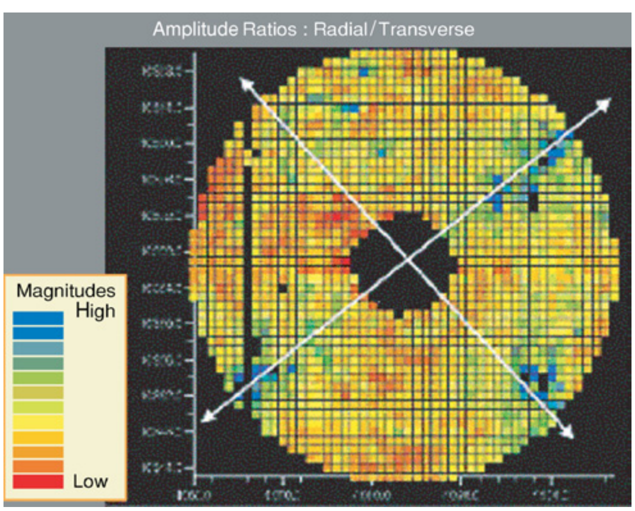

Figure 10. Colors show the ratio of radial/transverse energy for traces coming from sources and receivers separated in distance and azimuth according to the position on the plot. The large values for the ratio (in blue) indicate the two orthogonal principal directions of azimuthal anisotropy. (Garotta and Granger, 1988).

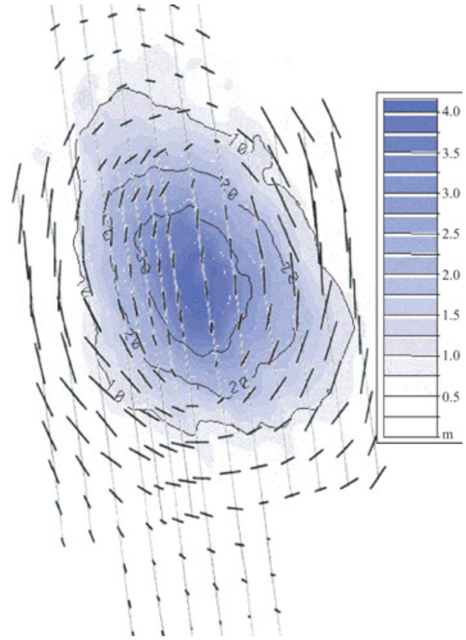

Figure 11. At the Valhall field (Norway), production causes collapse of the reservoir formation, deformation of the overburden, and subsidence of the sea floor. The figure (from Olofsson et al., 2002) shows contours of the subsidence, and arrows indicate azimuthal anisotropy from C-wave analysis; the patterns are convincing evidence that the anisotropy is causally related to the deformation.

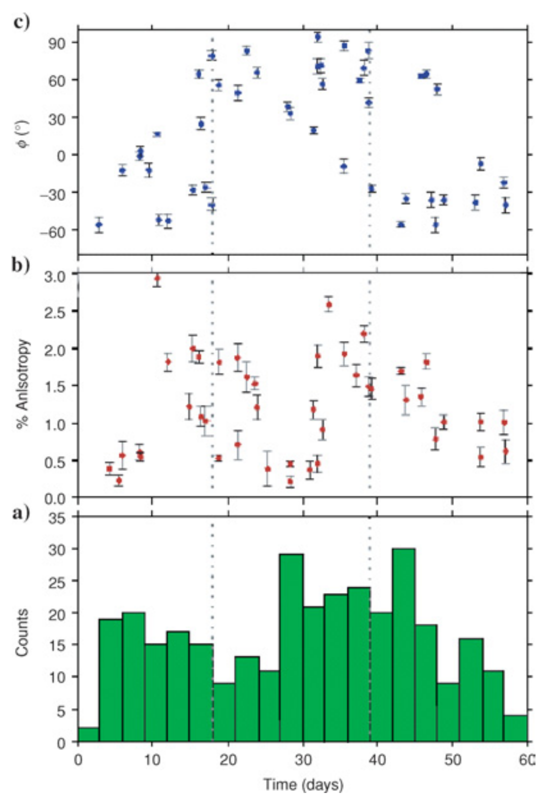

Figure 12. 4D azimuthal anisotropy at Valhall field. Note the shift (by about 90°) in the azimuth of the fast polarization during the three weeks in the middle of the eight-week period.

SUMMARY

We have reviewed how elastic anisotropy has ancient roots (stretching back 167 years), how seismic anisotropy has very old roots (stretching back 107 years), yet how the beginnings of anisotropy in exploration seismics are a matter of living memory. We have seen how anisotropy has evolved from a nuisance, rarely noticed, to a ubiquitous characteristic of our data and how it can now be an important avenue for subsurface characterization. We reviewed the conceptual and operational difficulties that delayed the realization of its importance. We showed how (usually small) anisotropy creates some effects that are correspondingly small (2nd order), some effects that are, nevertheless, very noticeable (1st order), and some effects that are new altogether (0th order). Because of these historical developments, anisotropy today is a mainstream issue in exploration and reservoir geophysics, one whose principles every working geophysicist should understand.

ACKNOWLEDGMENTS

The authors acknowledge prompt, constructive, and expert reviews by Ilya Tsvankin and Stuart Crampin, themselves major contributors to the substance of the recent work reviewed herein.

APPENDIX A

LAYER-INDUCED ANISOTROPY

The concept of fine layering as a source of anisotropy is as old as the concept of seismic anisotropy (Rudzki, 1898). Today we know that fine layering is not the most important cause of seismic anisotropy, nor is it the model we use to determine reservoir properties. However, it is the best-studied case; it can be solved with elementary means; it gives a straightforward and unambiguous answer, and it provides a case history in the development of scientific thought that is worth reading.

When Rudzki mentioned layering as the most likely cause of anisotropy, he was talking qualitatively. He referred to experiments on rock samples, but again only to show that rocks had different properties in different directions. The problem finding of the long-wave equivalent to a stack of isotropic layers is not very difficult. However, nobody seems to have attacked the problem in the first third of the 20th century. Riznichenko (1949), Postma (1955) and Helbig (1956, 1958) all used the same approach: subject a cube that is representative of the compound to different stresses and strains in a series of thought experiments and connect its overall reaction to the elastic properties of the constituents. For instance, to obtain c_{33}, the stiffness describing how normal stress on a horizontal surface element depends on compressive strain in vertical direction, we use a thought experiment of vertical compression without lateral expansion (Figure A-1). The compound stiffness is the thickness-weighted harmonic average of the constituent stiffnesses. A thought experiment with shear strain in the vertical plane gives a similar expression for c_{55} (Figure A-2). For c_{66} (shear in the horizontal plane) the result is different (Figure A-3): the stiffness is the thickness-weighted arithmetic average of the corresponding constituent stiffnesses. The expressions for the compound stiffnesses c_{11} and c_{13} turn out to be more complicated than those for c_{33}, c_{55}, and c_{66} (see Figure A-4).

All three (independent) papers gave the same result, so everything seemed to be in order. However, Riznichenko referred to a paper (Bruggeman, 1937) that, according to him, could not be correct because of a faulty method. In fact, Bruggeman had solved the problem — and a bit more — elegantly with much less space and effort (and without error!).

Bruggeman used the following idea: there are three stress-components (σ_{yz}, σ_{xz}, σ_{zz}) and three strain-components (ε_{xx}, ε_{xy}, ε_{yy}) that are continuous at the interfaces. Therefore, each of these six components has a constant value throughout

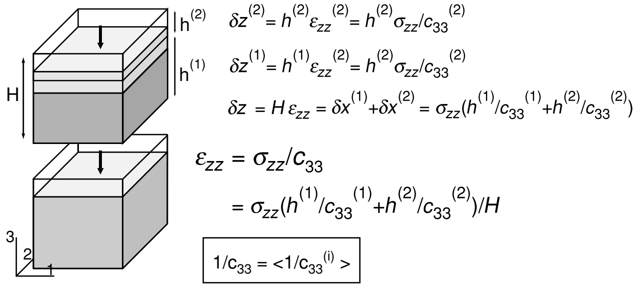

Figure A-1. Determination of the stiffness component c_{33}, where the brackets indicate the average over the constituent layers.

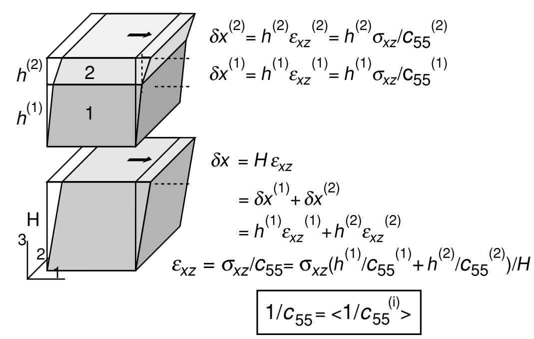

Figure A-2. Determination of the stiffness component c_{55}.

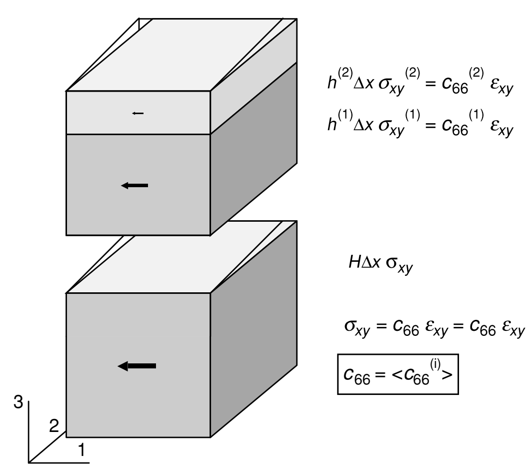

Figure A-3. Determination of the stiffness component c_{66}.

the entire medium. Any stress-strain function that can be expressed in these components must have the same form in the constituents and in the compound medium. The function

$$\phi \equiv 2E - \sigma_{zz}\varepsilon_{zz} - \sigma_{yz}\varepsilon_{yz} - \sigma_{xz}\varepsilon_{xz}, \qquad \text{(A-1)}$$

(where E is the strain energy) is expressed in these constant components as

$$\phi = -\frac{\sigma_{xz}^2}{c_{33}} - \frac{\left(\sigma_{xz}^2 + \sigma_{yz}^2\right)}{c_{44}} + (\varepsilon_{xx} + \varepsilon_{yy})^2 \left(c_{11} - \frac{c_{13}^2}{c_{33}}\right) + \left(\tfrac{1}{2}\varepsilon_{xy}^2 - 2\varepsilon_{xx}\varepsilon_{yy}\right)c_{66} + 2\sigma_{zz}(\varepsilon_{xx} + \varepsilon_{yy})\frac{c_{13}}{c_{33}}. \qquad \text{(A-2)}$$

This leads to

$$c_{33} = \left\langle c_{33}^{-1}\right\rangle^{-1} \qquad c_{44} = \left\langle c_{44}^{-1}\right\rangle^{-1} \qquad c_{66} = \langle c_{66}\rangle \qquad \text{(A-3)}$$
$$c_{13}/c_{33} = \langle c_{13}/c_{33}\rangle \qquad c_{11} - c_{13}^2/c_{33} = \left\langle c_{11} - c_{13}^2/c_{33}\right\rangle.$$

This equation holds for any number of isotropic or transversely isotropic layers.

For isotropic constituents expressed as velocities, this reduces to

$$c_{33} = \frac{1}{\left\langle 1/\rho v_P^2\right\rangle} \qquad c_{55} = \frac{1}{\left\langle 1/\rho v_S^2\right\rangle} \qquad c_{66} = \left\langle \rho v_S^2\right\rangle$$
$$c_{11} = \frac{\left(1 - 2\left\langle v_S^2/v_P^2\right\rangle\right)^2}{\left\langle 1/\rho v_P^2\right\rangle} - 4\left\langle \frac{\rho v_S^2\left(v_P^2 - v_S^2\right)}{v_P^2}\right\rangle$$
$$c_{13} = \frac{1 - 2\left\langle v_S^2/v_P^2\right\rangle}{\left\langle 1/\rho v_P^2\right\rangle}. \qquad \text{(A-4)}$$

Note: within the averaging brackets, the order of the layers and their thicknesses are irrelevant as long as the thin-bed requirement (and stationary statistics) is met. There is no requirement for periodic layering; hence, any restriction to periodic thin layering (PTL) is unnecessary, as well as geophysically unrealistic.)

Backus (1962) had essentially the same idea as Bruggeman. Today, the equations are generally called the Backus equations, although for forward modeling, his results did not go beyond those of Bruggeman (1937), Backus had done a major step for the inverse problem: He showed that there are transversely isotropic media with vertical axis that cannot be regarded as the long-wave equivalent of a sequence of stable isotropic layers, and those that can be considered as such, require at most three individual constituents.

The concept of stability enters the argument in the following way: a medium is called stable if any deformation requires work (if a deformation would instead yield energy, then one could design a perpetual motion machine, which would violate the first law of thermodynamics). Stability is equivalent to the statement that all principal minors of the elastic matrix are positive (A principal minor is the determinant of a submatrix that is symmetric to the main diagonal; see the examples in Figure A-5).

Matrix theory shows that one does not have to test all principal minors (there are many); it is sufficient to test the leading principal minors, i.e., those that are contiguous and contain the first element of the first row. If one applies this rule to the (sparse) elasticity matrix of an isotropic medium, one finds that the stability requirements are:

$$\rho v_S^2 = \mu > 0 \qquad 0 < \left(v_S^2/v_P^2\right) < \tfrac{3}{4}. \qquad \text{(A-5)}$$

(This second inequality is equivalent to the well-known constraint on Poisson's ratio $v : -1 < v < \frac{1}{2}$.) These two inequalities A-5 are translated by the averaging rules (equation A-4) into constraints on the media that are long-wave equivalent to a stack of isotropic layers. On the other hand, a transversely isotropic medium is stable (by the same arguments) if

$$c_{33} > 0, \qquad \frac{c_{66}}{c_{33}} > 0, \qquad \frac{c_{44}}{c_{33}} > 0,$$
$$\frac{c_{11} - c_{66}}{c_{33}} > 0, \qquad \frac{c_{11} - c_{66}}{c_{33}} - \frac{c_{13}^2}{c_{33}^2} > 0. \qquad \text{(A-6)}$$

The equations (A-4) and the constraints (A-5) describe an open volume (i.e., a volume without its bounding surface) in the four-dimensional space spanned by the normalized stiffnesses. This volume lies completely inside the corresponding volume delimited by constraints (A-6); thus, there are TI media that cannot be modeled by stacking of isotropic layers.

The complete proof is somewhat involved, but a partial proof (which already makes the point) is simple. According

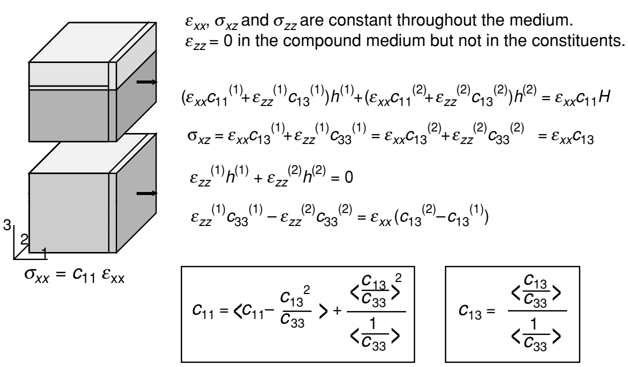

Figure A-4. Determination of the stiffness component c_{11} and c_{33}.

$$\begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{pmatrix}$$

$$\begin{matrix} C_{11} & C_{12} & C_{13} \\ C_{12} & C_{22} & C_{23} \\ C_{13} & C_{23} & C_{33} \end{matrix} \qquad \underline{\begin{matrix} C_{11} & C_{12} & C_{16} \\ C_{12} & C_{22} & C_{26} \\ C_{16} & C_{26} & C_{66} \end{matrix}} \qquad \begin{matrix} C_{11} & C_{14} & C_{16} \\ C_{14} & C_{44} & C_{46} \\ C_{16} & C_{46} & C_{66} \end{matrix} \qquad \begin{matrix} C_{44} & C_{45} & C_{46} \\ C_{45} & C_{55} & C_{56} \\ C_{46} & C_{56} & C_{66} \end{matrix}$$

Assume an arbitrary strain, e.g., $\varepsilon = (\varepsilon_1, \varepsilon_2, 0, 0, 0, \varepsilon_6)T$. (Twice) the strain energy is $2E = \varepsilon \cdot \sigma = \varepsilon_1\sigma_1 + \varepsilon_2\sigma_2 + \varepsilon_6\sigma_6$.
The stress components are $\sigma_1 = c_{11}\varepsilon_1 + c_{12}\varepsilon_2 + c_{16}\varepsilon_6$, $\sigma_2 = c_{12}\varepsilon_1 + c_{22}\varepsilon_2 + c_{26}\varepsilon_6$, $\sigma_6 = c_{16}\varepsilon_1 + c_{26}\varepsilon_2 + c_{66}\varepsilon_6$,
i.e., the stiffnesses involved are those of the second (underlined) principal minor in the examples above.
The strain energy is a quadratic form in the three strain components:
$2E = c_{11}\varepsilon_1^2 + c_{22}\varepsilon_2^2 + c_{66}\varepsilon_6 2 + 2c_{12}\varepsilon_1\varepsilon_2 + 2c_{16}\varepsilon_1\varepsilon_6 + 2c_{26}\varepsilon_3\varepsilon_6$; the strain energy is *positive* > 0 if and only if the determinant of the matrix stiffnesses involved is positive (see any book on Linear Algebra, e.g., Strang, 1986).

Figure A-5. The elastic matrix (left) and four of its third-order principal minors. The text indicates the proof that positive principal minors are necessary and sufficient for stability.

to equation A-4,

$$c_{44}^{-1} = \langle \mu^{-1} \rangle, \qquad c_{66}^{1} = \langle \mu \rangle; \tag{A-7}$$

hence, $c_{44} \le c_{66}$. The constraints A-3 for the general TI medium do not contain such restriction. Hence, a TI medium with $c_{44} > c_{66}$ can be stable, but its transverse isotropy cannot be a result of layering of stable isotropic media.

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