New York State Testing Program

NYS MATH STUDY GUIDE FOR 6th GRADE

This study guide covers much of what will be expected of you to know for your state test. Each category explains what we have learned, often visually shows the content, and usually gives you a BLUE UNDERLINED LINK that will send you to the Internet for extra practice. If you want to look at some sample questions for this year's test you can click here.

1. GCF (Greatest Common Factor)
2. LCM (Least Common Mult.)
3. Exponents
4. Operations with Decimals + and –
5. Operations with Decimals Multiplying
6. Operations w/Decimals Division
7. Ratios
8. Unit Rates
9. Percent Problems
10. +/– numbers (Integers)
11. Plotting Integers
12. Absolute Value
13. Expressions
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15. Solving for variable in inequalities
16. Graphing an Inequality on a number line
17. Equations
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20. Area of a Rectangle or Square
21. Area of a Triangle
22. Area of Pentagons and more
23. Volume
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25. Coordinate Graphing
26. OOO (Order of Operations)

1. **GCF stands for Greatest Common Factor**. The example below shows how to find the GCF of the numbers 12 and 16. The GCF is the biggest match and equals 4. GCF with Venn Diagram GCF and LCM with Prime Factorization

2. Least Common Multiple

Also known as LCM. You can find the LCM when you skip count by the numbers you are given. The first match is the Least (smallest) Common(shared) Multiple. Video on LCM Snowball fight!

You can also find the least common multiple of 3 (or more) numbers.

Example: Find the least common multiple for 4, 6, and 8

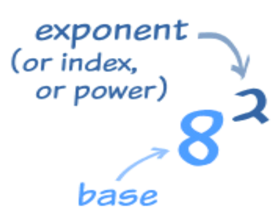
Multiples of 4 are: 4, 8, 12, 16, 20, **24**, 28, 32, 36, ...
Multiples of 6 are: 6, 12, 18, **24**, 30, 36, ...
Multiples of 8 are: 8, 16, **24**, 32, 40, ...

So, **24** is the least common multiple (I can't find a smaller one !)

3. Exponents

An exponent is a number that is multiplied by itself a certain number of times. The tall number is called the base number. The small number (above the base number) is called an exponent. The exponent tells us how many of the base number we will need to multiple by itself. Alien powers! Asteroids by mathdork.com!

Exponents



The **exponent** of a number says **how many times** to use the number in a multiplication.

In $\mathbf{8^2}$ the "2" says to use 8 twice in a multiplication,
so $\mathbf{8^2 = 8 \times 8 = 64}$

Exponents are also called Powers or Indices.

- In words: 8^2 could be called "8 to the power 2" or "8 to the second power", or simply "8 squared"

Some more examples:

Example: $\mathbf{5^3 = 5 \times 5 \times 5 = 125}$

- In words: 5^3 could be called "5 to the third power", "5 to the power 3" or simply "5 cubed"

Example: $\mathbf{2^4 = 2 \times 2 \times 2 \times 2 = 16}$

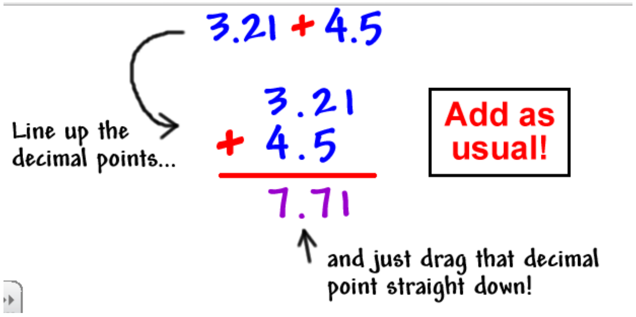
- In words: 2^4 could be called "2 to the fourth power" or "2 to the power 4" or simply "2 to the 4th"

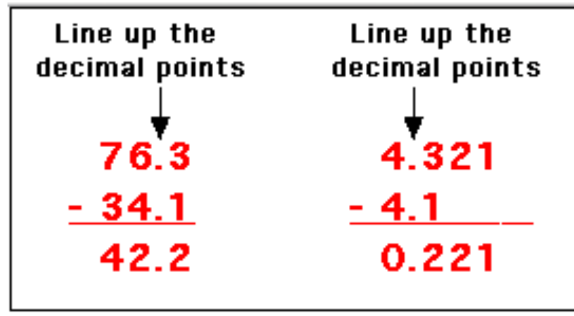
Exponents make it easier to write and use many multiplications

Example: $\mathbf{9^6}$ is easier to write and read than $\mathbf{9 \times 9 \times 9 \times 9 \times 9 \times 9}$

4. **Operations with Decimals (Adding and Subtracting)**

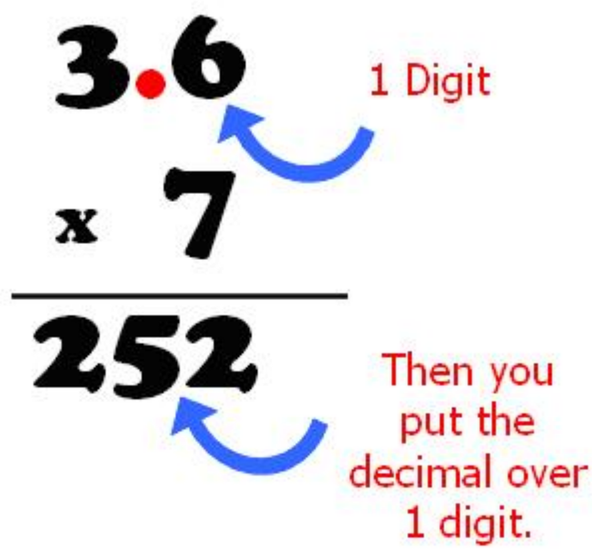
The most important thing you need to remember is to line up your decimal points for all of the numbers you are working with. If you do not you will not get the answer correct. Look at the examples below. [Practice by shooting hoops! Remember to lineup your decimal points!]



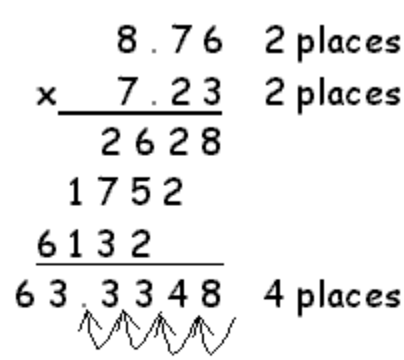


5. **Operations with Decimals (Multiplying)**

When multiplying numbers with decimals, ignore the decimal point until you multiply all of the numbers. Count the number of spaces the decimal point is over in both numbers multiplied. Move the decimal point that number of spaces in the answer. The problem to the left has one digit space of decimals in the original expression (3.6) The student multiplied and then should move the decimal point one spot at the end. The answer would be 25.2
[Multiplying decimal football]
[Video showing how to multiply decimals…best song ever.]



This problem shows a multiplication problem with 4 digits of decimals. Look at how the student determines the answer.



6. Operations with Decimals (Dividing)

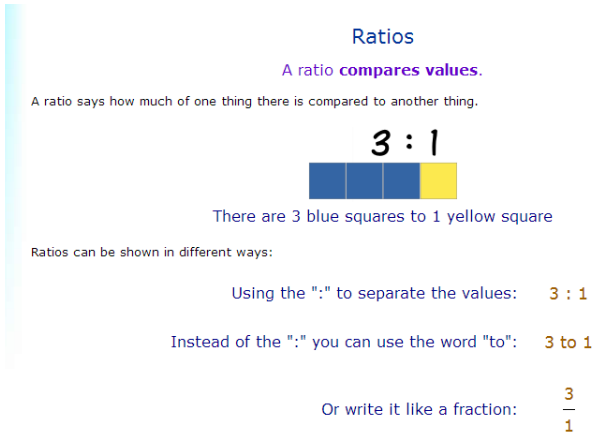
Follow this link for the step by step instructions. Or click [HERE](https://www.khanacademy.org/math/arithmetic/decimals/dividing_decimals/v/dividing-decimals).

https://www.khanacademy.org/math/arithmetic/decimals/dividing_decimals/v/dividing-decimals

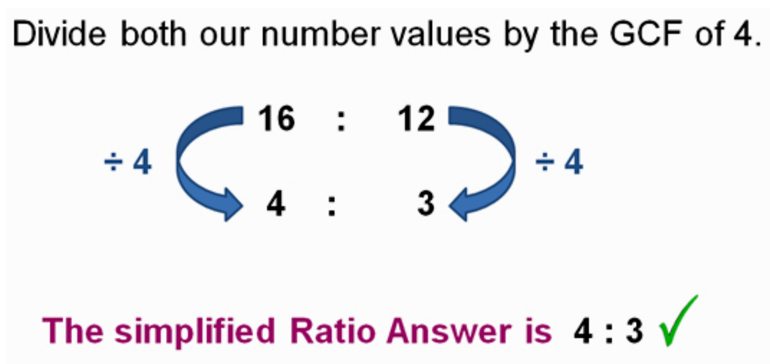
7. Ratios

For the BEST ratio and proportion practice CLICK HERE.

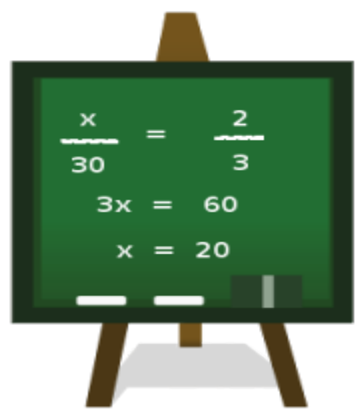
A ratio compares any two groups.



Ratios can be simplified.



Ratios can be solved using PROPORTIONS (think → CROSS MULTIPLY to find X)



Fill in Ratio Tables

Use Tables to Compare Ratios

Antonio and Maria each swam laps in a pool. Who has the best time per lap?

Antonio

Number of laps	1	2	4
Time (min)		10	20

Maria

Number of laps	1	3	6
Time (min)		6	12

8. Unit Rates

Unit Rates find the rate of one thing. We can find these in the grocery store. The example below shows a price tag for a 25.25 oz box of Cheerios. The price per ounce is listed in the Unit Rate.

Unit Rate Jeopardy

- You can buy 4 apples for $2 at the store. To find the Unit Rate we would want to find out how much we are paying for each apple. Simply divide the cost by the number of apples. Each apple would cost .50.
- Another example of unit rate is this. You can do 6 pushups every 30 seconds. How long does it take to do one? Divide the time by the pushups. The answer is 5 seconds per pushup. This is the Unit Rate.

9. Percent Problems

When you see a problem that has a percentage sign (%) in it, you should immediately write down the Percent Proportion we learned in class. The word problem will give you two pieces of important information. One piece will be unknown and we will have to find the value. We will use the variable X to show this. Below is the Percent Proportion (we call it the Magic Proportion).

$$\frac{\text{part}}{\text{whole}} = \frac{\%}{100}$$

THESE PRACTICE PROBLEMS WILL MAKE YOU A PERCENT PRO! Let plug in the numbers from this word problem to the proportion.

24 students in a class took an algebra test. If 18 students passed the test, what percent do not pass?

We are given 24 students taking the algebra test. We are told 18 passed. We are asked in the question to find the percent.

24 is the whole number of students taking the test. This number goes to the "whole" section.

The 18 tells us the PART that passed. 18 CAN'T be the whole number of students. This goes to the "part" section.

$$\frac{\text{part}}{\text{whole}} = \frac{\%}{100}$$

We need to find the "%", which means percent. We don't know this, soit becomes and X. The proportion should look like this when completed.

$$\frac{18}{24} = \frac{X}{100}$$

Your final steps are to cross multiply and divide. You could simplify the 18/24 to ¾ to make the calculations easier.

Sometimes, you are given the percent and have to find either the “part” or the ‘whole”. If the percent (%) is given to you in the problem it should go over the 100. Here is an example of a question where you are given the percent.

In a school, 25 % of the teachers teach basic math. If there are 50 basic math teachers, how many teachers are there in the school?

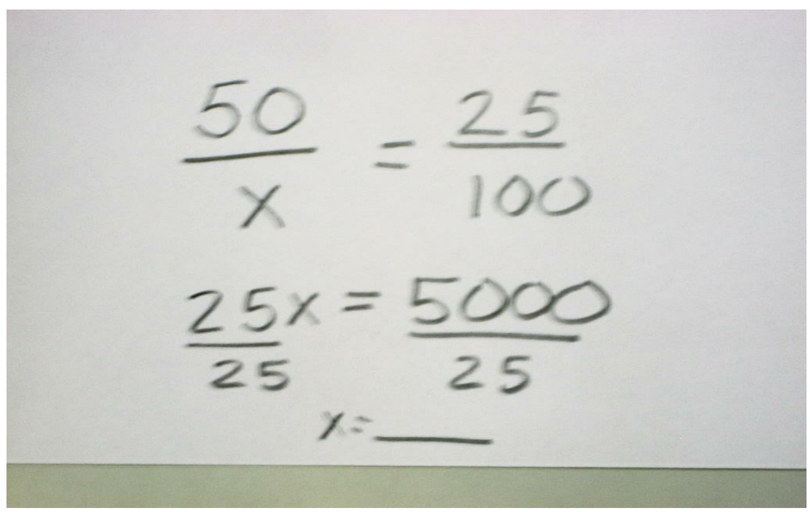
$$\frac{\text{part}}{\text{whole}} = \frac{\%}{100}$$

The “percent” is given to us. It’s going to go over the “100”.

The number that is given to us is 50. We need to determine if the 50 is the part or whole. I find that this is the part that troubles students the most. Try to make sense of what the 50 stands for. It says there are 50 basic math teachers in the problem. Is this the total teachers in the school or just the part that teach math? It’s the “part” that teach math. The 50 will go where the part is.

The question asks to find out how many teachers are in the school. This means the “whole” school. Since we are being asked to find this we do not know the answer yet. Since it’s an unknown number, we’ll use the variable X to find the answer. Below is what the proportion should look like.

I cross multiplied and got 25x=5000. One would have to divide 5,000 by 25.



Try this problem for practice.

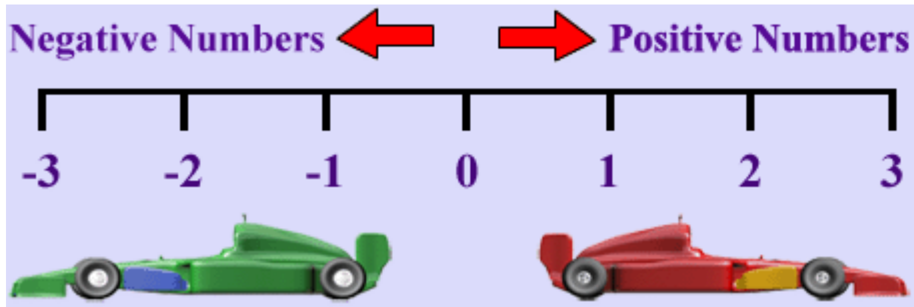
A test has 20 questions. If Peter gets 80% correct, how many questions did Peter missed?

$$\frac{\text{part}}{\text{whole}} = \frac{\%}{100}$$

10. Positive and Negative Numbers (INTEGERS)

A positive number is any number greater than 0. A negative number is the opposite of that statement. They are any number less than zero. Zero is neither negative nor positive. The number line below shows both positive and negative numbers.

Note that all negative numbers have a minus side before the digit. You need not put a + in front of a positive number. No symbol means it's positive.



We often can compare these numbers by creating Inequalities The symbols below can be used to compare any integers.

Negative numbers work in a way we are not used to.

Some examples below explain this.

Inequality Symbols

$<$ – Less Than

$>$ – Greater Than

$\leq$ – Less Than or Equal to

$\geq$ – Greater Than or Equal to

Compare the following integers using the symbols: <, >, or =.

1. 9 ____ -12

2. -13 ____ -5

3. -8 _____ -18

Solutions

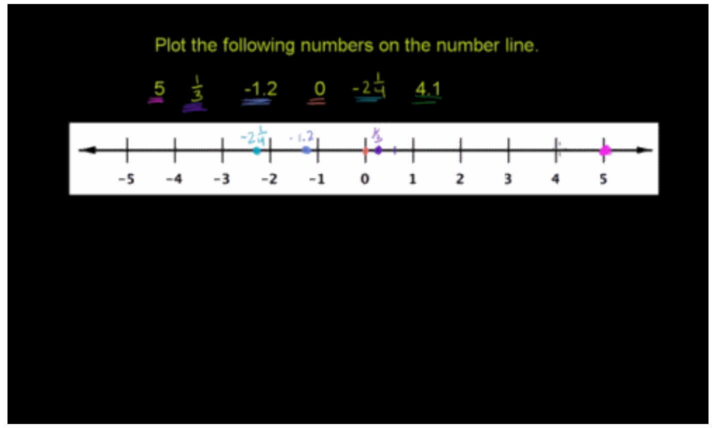
1. 9 __>__ -12 - This is read as **"9 is greater than -12**."
   A positive number is ALWAYS greater than a negative number.

2. -13 __<__ -5 - This is read as **"-13 is less than -5."**
   -13 is farther to the left of 0, therefore it is less than -5.

3. -8 __>__ -18 - This is read as "**-8 is greater than -18"**.
   -8 is closer to 0, therefore it is greater than -18.

11. Plotting Integers on a Number Line

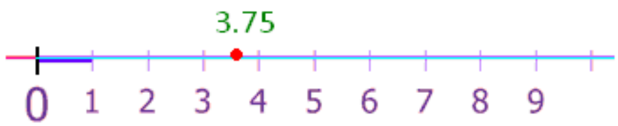
Sometimes you will be asked to put positive or negative numbers on a number line. You will have to "graph a point", which simply means put a point, on a number line. Here is an example that uses whole numbers and mixed numbers.



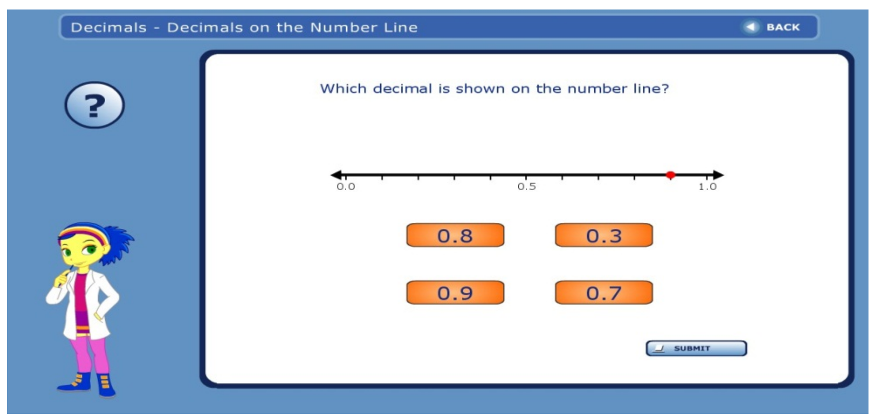
Compare decimals on a number line

Move the red flag to the integer

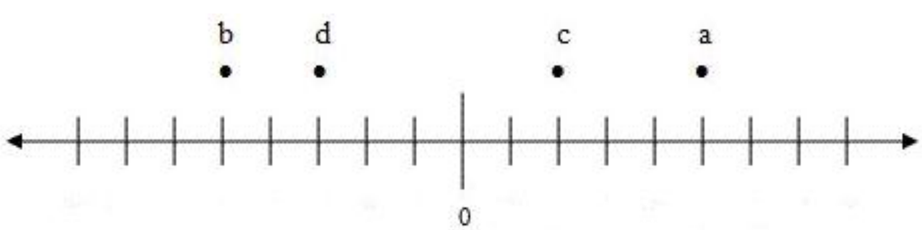
You may be asked to graph points that have a decimal as part of the value. Here is an example of that.



Try this problem.



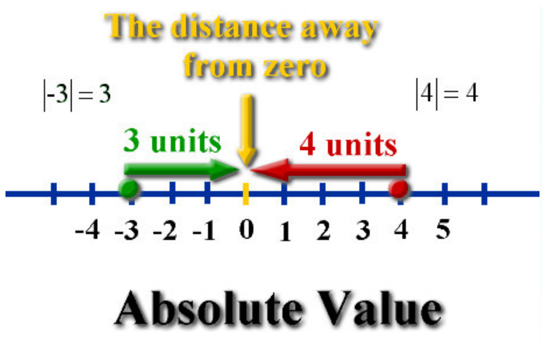
Fill in the points.



Estimate the value of the points below.

12. Absolute Value

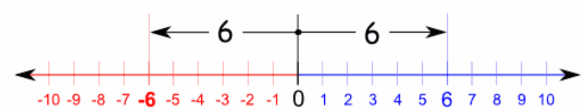
Absolute Value tells you what an integer's distance is from zero. The answer, or absolute value, is NEVER negative. Here are some examples below. [Click the balls in ascending (smaller to bigger) order in terms of absolute value!] [Or play this.]



Absolute Value

Absolute Value means ...

... only **how far** a number is from zero:



"6" is 6 away from zero,
and "-6" is **also** 6 away from zero.

So the absolute value of 6 is **6**,
and the absolute value of -6 is also **6**

More Examples:

- The absolute value of -9 is **9**
- The absolute value of 3 is **3**
- The absolute value of 0 is **0**
- The absolute value of -156 is **156**

No Negatives!

So in practice "absolute value" means to remove any negative sign in front of a number, and to think of all numbers as positive (or zero).

Absolute Value Symbol

To show that you want the absolute value of something, you put "|" marks either side (they are called "bars" and are found on the right side of your keyboard), like these examples:

$|-5| = 5$ $\quad\quad$ $|7| = 7$

Sometimes absolute value is also written as "abs()", so **abs(-1) = 1** is the same as **|-1| = 1**

Subtract Either Way Around

And it doesn't matter which way around you do a subtraction, the absolute value will always be the same:

$|8-3| = 5$ (8-3 = 5) $\quad\quad$ $|3-8| = 5$ (3-8 = -5, and **|-5| = 5**)

More Examples

Here are some more examples of how to handle absolute values:

$|-3\times6| = 18$ (-3×6 = -18, and **|-18| = 18**)

$-|-12| = -12$ (**|-12| = 12** and then the first minus gets you **-12**)

Use the Absolute Value symbols to show the absolute value of each number.

PHRASE

the absolute value of -9
the absolute value of 18
the absolute value of -2
the absolute value of 630
the absolute value of -210

13. Expressions

Expressions are mathematical phrases that have no answer. You are not asked to solve the phrase. Often there are variables involved although an expression. Play Algebraic Expressions Millionaire for practice!

Phrase	Expression
the sum of nine and eight	9 + 8
the sum of nine and a number x	9 + x

The expression **9 + 8** represents a single number (17). This expression is a numerical expression, (also called an arithmetic expression). The expression **9 + x** represents a value that can change. If x is 2, then the expression **9 + x** has a value of 11. If x is 6, then the expression has a value of 15. So **9 + x** is an algebraic expression. In the next few examples, we will be working solely with algebraic expressions.

Example 2: Write each phrase as an algebraic expression.

Phrase	Expression
nine increased by a number x	$9 + x$
fourteen decreased by a number p	$14 - p$
seven less than a number t	$t - 7$
the product of 9 and a number n	$9 \cdot n$ or $9n$
thirty-two divided by a number y	$32 \div y$ or $\frac{32}{y}$

In Example 2, each algebraic expression consisted of one number, one operation and one variable. Let's look at an example in which the expression consists of more than one number and/or operation.

Example 3: Write each phrase as an algebraic expression using the variable n.

Phrase	Expression
five more than twice a number	$2n + 5$
the product of a number and 6	$6n$
seven divided by twice a number	$7 \div 2n$ or $\frac{7}{2n}$

When creating an expression you must pay attention to the words in the problem.

Click here for video on this.

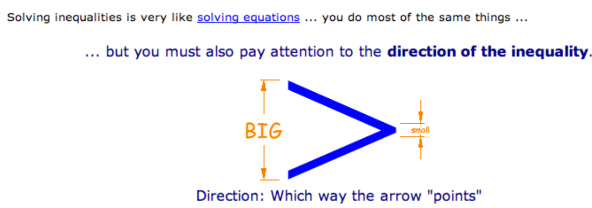
Addition	Subtraction
add(ed) to all together both combined in all increase by more than perimeter plus sum total	decreased by difference fewer than how many more left less less than minus remaining take away
Multiplication	**Division**
a area multiplied by of per product of rate times triple twice	divided half how many each out of percent quarter quotient of percent

14. Inequalities (Symbols)

Inequalities compare two groups that are not equal. Sometimes there is a variable involved and you may need to find the value of the variable. There are four main symbols used when making inequalities.

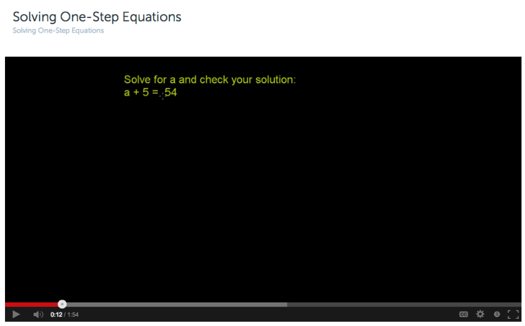
Inequality Symbols

- $<$ – Less Than
- $>$ – Greater Than
- $\leq$ – Less Than or Equal to
- $\geq$ – Greater Than or Equal to



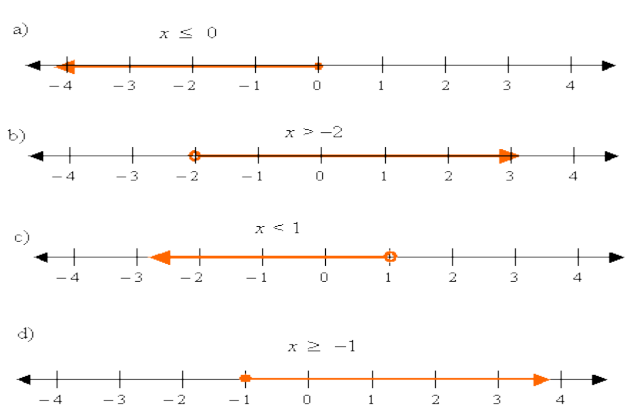
15. **Solving for a Variable in Inequalities**

How do we solve for inequalities? Well, the first thing we need to do is get the VARIABLE all by itself on one side of the inequality. We do that using the INVERSE PROPERTY! Watch THIS VIDEO to review how it works!



16. Graphing an Inequality on a Number Line

Once you have an inequality that has a variable on one side and a constant on the other you can graph the answers (or solution set) on a number line. Some examples are below. Pay attention to the symbols and whether you need to fill in your circle. The table below shows when to do this. Practice here! Or here, shooter!



Symbol	Meaning	Closed or Open Circle
<	Less Than	Open ○
>	Greater Than	Open ○
≤	Less Than or Equal to	Closed ●
≥	Greater Than or Equal to	Closed ●

17.Equations

Equations are the opposite of inequalities. Equations compare two groups that are equal. If variables are used we often will call them algebraic expressions. One of the first equations you ever learned was 1+1=2. The two sides of the equation are separated by the equal sign. Both sides have a value of 2.

Below there is an illustration about equations. What would the X have to be worth to make this an equation?

What is an Equation?

An equation says that two things are equal. It will have an equals sign "=" like this:

$$x + 2 = 6$$

That equations says: **what is on the left (x + 2) is equal to what is on the right (6)**

So an equation is like a **statement** "*this* equals *that*"

Solving Equations

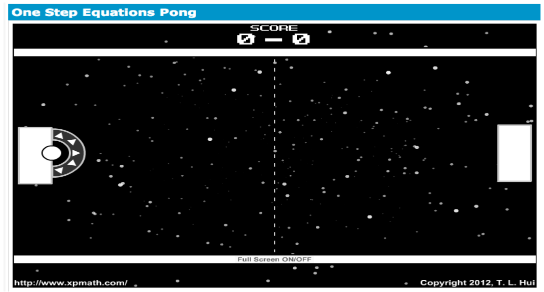
$$\begin{array}{l} x + 5 = 7 \\ -5 \qquad -5 \\ x = 7 - 5 \\ x = 2 \end{array} \qquad \begin{array}{l} x - 5 = 7 \\ +5 \qquad +5 \\ x = 7 + 5 \\ x = 12 \end{array}$$

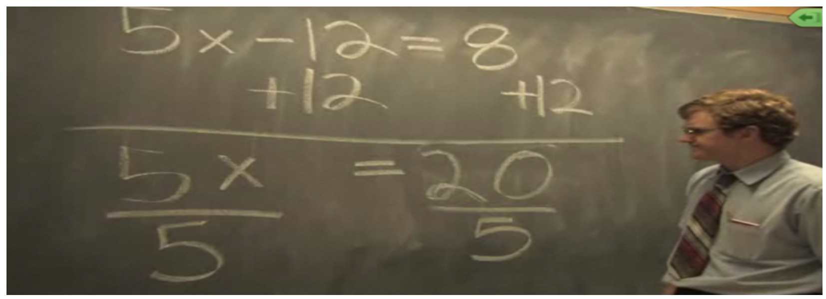
Remember, when trying to solve for a variable, you must try to get the variable alone. Here is an example of an addition problem where you have to find the value of X. Practice here to shoot some hoops and get better at this!

Sometimes there are coefficients involved. Coefficients are numbers that are attached to a variable. Like 3x, 12z, or 18r. If there is a coefficient you must divide both sides of the equation by the coefficient in order to get the variable by itself. CLICK HERE for extra help with this. Fun game!

$$\frac{\cancel{2}x}{\cancel{2}} = \frac{10}{2}$$

$$x = 5$$





Try some of these practice problems. Make sure you try to get the variable alone!

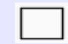
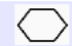
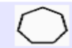
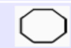
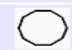
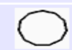
Solve the equations below and write the algebraic steps.
The first one is done for you.

5n - 5 = 15 5n - 5 + 5 = 15 + 5 5n = 20 5n ÷ 5 = 20 ÷ 5 **n = 4**	**2n + 6 = 18**	**5n + 4 = 24**
2a - 1 = 7	**4n + 3 = 11**	**8n - 7 = 33**
10n + 9 = 89	**5n + 11 = 46**	**6a - 7= 53**
9b + 15 = 60	**5d - 6 = 24**	**9n + 12 = 30**

18. Polygon Names

[Click here for a fun song about this]

Names of Polygons

		If it is a
Name	**Sides**	**Shape**
Triangle *(or Trigon)*	3	
Quadrilateral *(or Tetragon)*	4	
Pentagon	5	
Hexagon	6	
Heptagon *(or Septagon)*	7	
Octagon	8	
Nonagon *(or Enneagon)*	9	
Decagon	10	
Hendecagon *(or Undecagon)*	11	
Dodecagon	12	

19. Perimeter

Perimeter measures the distance around any type of polygon. Click here for more.

Perimeter of a Square

Taking the following square with side length 6 inches, calculate the perimeter.

In order to calculate perimeter, you need to add together the lengths of all four sides of the square. You are given the length of one side. Remember, all sides of a square are equal, so really you already have the measures of each side.

Then, you add them together, so 6 + 6 + 6 + 6 = 24 inches. Thus, 24 inches is your final answer.

Perimeter of a Rectangle

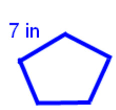
Taking the following rectangle with length 8 inches and width 4 inches, calculate the perimeter.

In order to calculate perimeter, you need to add together the lengths of all four sides of the rectangle. You are given the length of one side and the width of one side. Remember, opposite sides of a rectangle are equal, so really you already have the measures of each side.

Then, you add them together, so 8 + 8 + 4 + 4 = 24 inches. Thus, 24 inches is your final answer.

Perimeter of a Polygon

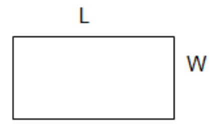
The perimeter of a polygon is calculated using the same method of adding together each side. Remember that if all the sides are equal, you only need to know one side of the polygon. If the sides are unequal, however, you do need to know the length of each different side. Taking the following pentagon with side length 7, calculate the perimeter.



A pentagon has five sides, and all of these sides are equal, therefore you can perform the following calculation:

7 + 7 + 7 + 7 + 7 = 35

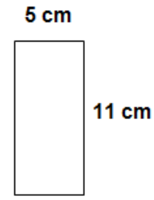
20. **Area of a Rectangle or Square**



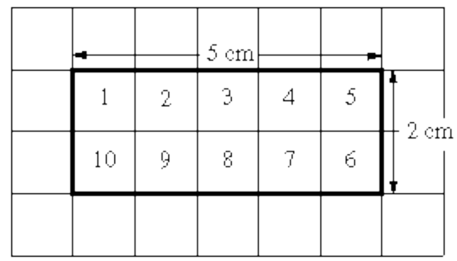
Area = L x W
Area = Length x Width

Area counts number of square units that can fit inside of a shape. You can find the area of any 2-D (two dimensional—length and width or base and height) object. To find the area of a rectangle or square simply count the number of squares that is shown inside or take the length measurement and width measurement and multiply. This is the area formula and can be shown as A=lw. For practice with this CLICK HERE.

Example:

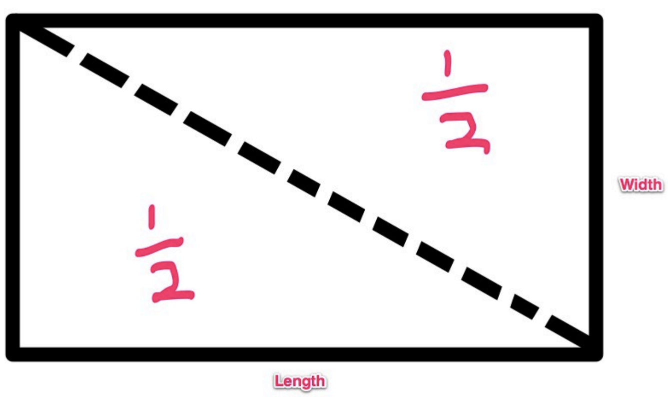


Area = L x W
Area = 5 cm x 11 cm
Area = 55 cm^2



21. **Area of a Triangle**

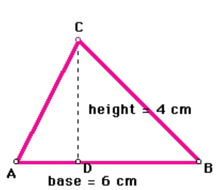
We learned this year that a triangle can be shown as half of a rectangle.



This could help us understand the formula for area of a triangle better. Click here for practice.

$$A = \frac{1}{2} bh$$

The area of a triangle equals one half times the base times the height.



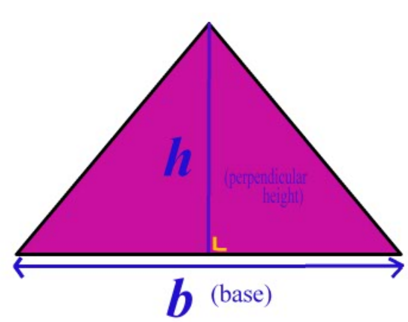
Here is how the math would look:

$$A = \frac{1}{2} bh$$

$$A = \frac{1}{2} \times 6 \times 4$$

$$A = \frac{1}{2} \times 24$$

A= 12 square cm



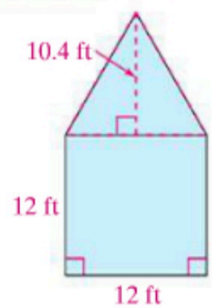
$$\text{Area} = \frac{bh}{2}$$

22. Area of Pentagons and More

If you have to find the area of a pentagon you can do so by DECOMPOSING it, or breaking it into rectanlges/squares and/or triangles.

The pentagon (looks like a house) below has been decomposed into a triangle and square. Once the areas are found of each you can add them together to find the total area. The process is shown below. Click here for more practice on decomposing and finding area.

EXAMPLE **Making Simpler Shapes**



1 **Multiple Choice** Karl drew this diagram of a deck he's going to build. What is the area of the deck?

Ⓐ 134.4 square feet Ⓒ 268.8 square feet
Ⓑ 206.4 square feet Ⓓ 748.8 square feet

The deck is in the shape of a pentagon. You can think of the pentagon as a triangle and a square that share an edge.

To find the area of the deck, find the sum of the areas of the square and the triangle

Square

$$\begin{aligned} A &= b \times h \\ &= 12 \times 12 \\ &= 144 \end{aligned}$$

Triangle

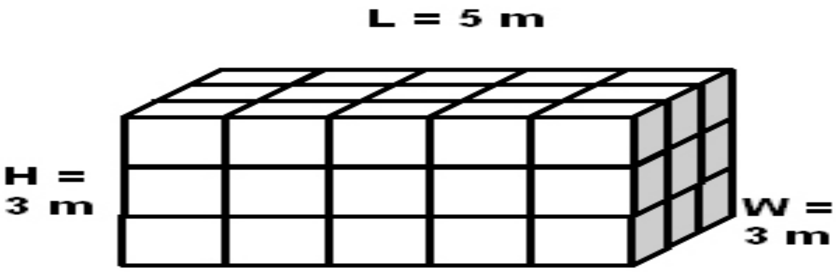
$$\begin{aligned} A &= \frac{1}{2} \times b \times h \\ &= \frac{1}{2} \times 10.4 \times 12 \\ &= 62.4 \end{aligned}$$

Sum of the areas: $144 + 62.4 = 206.4$

The total area is 206.4 square feet. The correct answer is choice B.

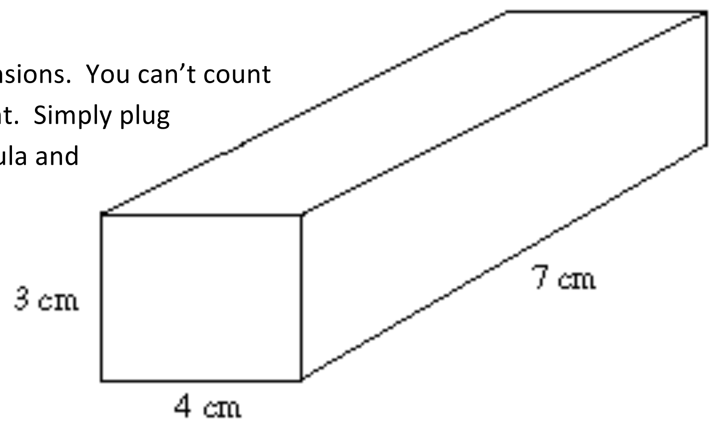
23. Volume

Volume measures the mass, or size, of a 3-D object. 3-D stands for 3-Dimensional and has three measurement types: A length, a width, and a height. Counting all of the cubes will give you the volume in cubic units, which can also be shown in units^3. You can try counting all of the cubes but sometimes that isn't possible. We've learned that if you multiply all of the three measurements (length, width, height) together you can find the volume. There are some examples on the next page.

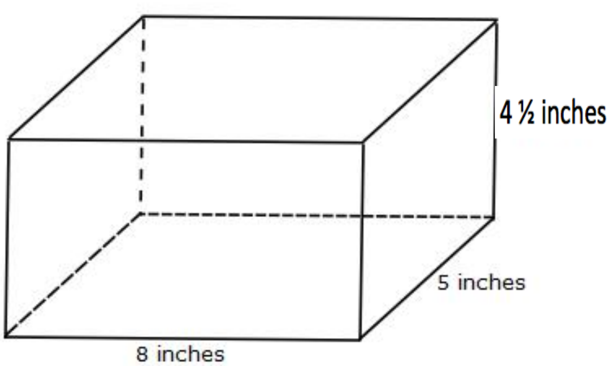


Multiplty the L (5), W (3) and H(3) to find the volume. 5x3x3=45 cubic m (meters) or 45m ^{3}.

This rectangular prism only shows it's dimensions. You can't count the cubes because they aren't there to count. Simply plug the three dimensions into our volume formula and you will get it. V= lwh. V= 3x4x7=84 cm ^{2}.



There will be times where all of your dimensions won't be perfect whole numbers. If they are Mixed Numbers, or Fractional Numbers you will have to convert all parts to fractions.



Here you would multiply 8x5x4 ½. Since 4 ½ is a mixed number you would have to switch it to an improper fraction. Remember, multiply the denominator (2) by the whole number(4) and then add the numerator (1). This will get you a total of 9. The denominator of the improper fraction will remain what it was (2). So the improper fraction is $\frac{9}{2}$. The v=lwh looks like $\frac{8}{1} \times \frac{5}{1} \times \frac{9}{2.}$ The product will be $\frac{360}{2}$ *which means* $360 \div 2$. *The answer is* 180 *cubic inches.*

Click here for more practice.

24. Surface Area

Surface Area finds the area of each face of a 3-D object. When we did Surface Area we looked at food boxes and many other types of boxes. The boxes were broke down and you drew a NET, which is a drawing of what the 3-D shape looked like before it was put together.

To find the surface area you must find the area of each FACE, or side.

Explore 3-D shapes with Nets, Vertices, and Edges. Go from Net to 3-D

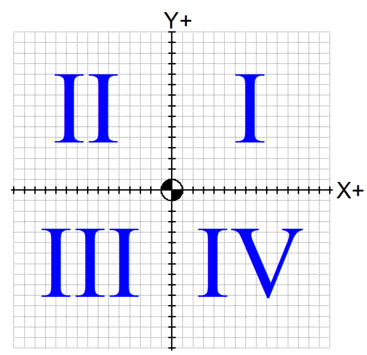
Explore more 3-D shapes

Video on how to find Surface Area

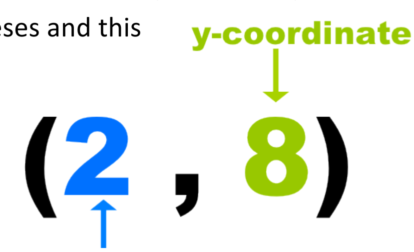
25. Coordinate Graphing

We've done quite a bit with coordinate graphing this year. This is what a coordinate plane looks like.

The Roman Numerals (I, II, III, IV) represent the QUADRANTS. There are four QUADRANTS, or sections. Notice that the X-axis is labeled as is the Y-axis.

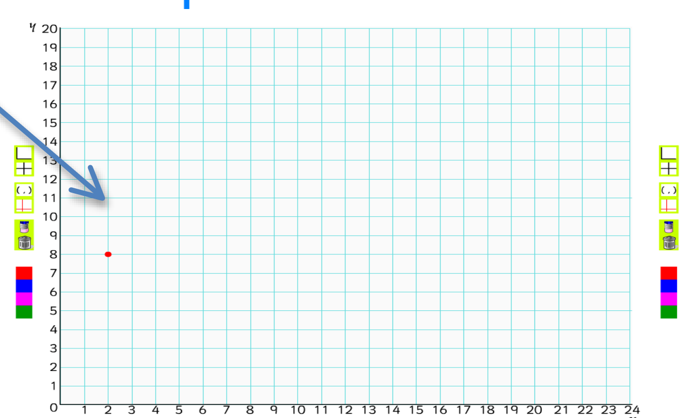


We are often asked to graph, or plot, a point or points on to the coordinate plane. The point has an address that we must follow. The address is inside of parentheses and this is called an ORDERED PAIR. They look like this



The first number will be found on the X-axis. This would be 2 in this situation. The second number shows you how high to climb the Y-Axis.

FIND THE ALIENS!



26. Order of Operations

Look at the example to help remind you. [Click here for practice.]