UNIT - I

MATRICES.

① For a given Matrix A of order 3, $|A| = 32$. and two of its eigenvalues are 8 and 2. Find the Sum of the eigenvalues.

②. Check whether the Matrix B is orthogonal? Justify?

$$B = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

③ If the Sum of two eigenvalues and trace of a 3x3 matrix A are equal, find the Value of $|A|$.

④. Use Cayley - Hamilton theorem to find $A^4 - 4A^3 - 5A^2 + A + 2I$ when $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$.

⑤. Given $A = \begin{bmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$. Find the eigen values of A^2.

⑥ Can $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ be diagonalized? why?

⑦ If 1 and 2 are the eigenvalues of a 2x2 matrix A, what are the eigenvalues of A^2 and A^{-1}?

⑧. State Cayley - Hamilton theorem.

⑨ If -1 is an eigen Value of the Matrix $A = \begin{bmatrix} 1 & -2 \\ -3 & 2 \end{bmatrix}$, then find the eigen values of A^4 Using properties.

⑩. Use cayley - Hamilton theorem to find $A^4 - 8A^3 - 12A^2$ when $A = \begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$.

⑪ Find the values of a and b Such that the Matrix $\begin{bmatrix} a & 4 \\ 1 & b \end{bmatrix}$ has 3 and -2 its eigen values.

⑫. Find the index and Signature of the Q.F $x_1^2 + 2x_2^2 - 3x_3^2$.

⑬ The product of two eigenvalues of the Matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \end{bmatrix}$

(14). If 3 and 6 are two eigen Values of $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ write down all the eigen values of A^{-1}.

(15) Write down the Quadratic form Corresponding to the Matrix $\begin{bmatrix} 0 & 5 & -1 \\ 5 & 1 & 6 \\ -1 & 6 & 2 \end{bmatrix}$.

① Find the Characteristic equation of the Matrix A. Verify & find A^{-1}.

Given $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Hence find A^{-1} and A^4.

② Find the eigen values and eigen vectors of

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

③ Reduce the given Quadratic form Q to its Canonical form using Orthogonal transformation. $Q = x^2 + 3y^2 + 3z^2 - 2yz$.

④. Use Cayley-Hamilton theorem to find the Value of the Matrix given by. $A^8 - 5A^7 + 7A^6 - 3A^5 + 8A^4 - 5A^3 + 8A^2 - 2A + I$. if the Matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

⑤ Find the eigenvalues and eigen vectors for the Matrix

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}.$$

⑥ Reduce the quadratic form $x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 + 2x_3x_2$ to the Canonical form through an Orthogonal transformation and hence show that it is positive Semi-definite. Also give a non-zero set of values x_1, x_2, x_3 which makes this quadratic form zero.

⑦. Reduce the Quadratic form $10x_1^2 + 2x_2^2 + 5x_3^2 + 6x_2x_3 - 10x_3x_1 - 4x_1x_2$ to a Canonical form through an Orthogonal transformation and hence find rank, index, Signature, nature and also give non-zero set of values for x_1, x_2, x_3 (if they exist), that will make the Quadratic form zero.

⑧ Find the Eigenvalues and Eigen Vectors of the Matrix

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

⑨ Reduce the quadratic form $2x^2 + 5y^2 + 3z^2 + 4xy$ to Canonical form by Orthogonal reduction and State its nature.

⑩. Using Cayley-Hamilton theorem, find the

(11). Reduce the quadratic form $2x_1^2+x_2^2+x_3^2+2x_1x_2-2x_1x_3-4x_2x_3$ to Canonical form by an Orthogonal transformation. Also find the rank, index, Signature and nature of the quadratic form.

(12). Verify Cayley-Hamilton theorem for the Matrix $A=\begin{bmatrix}1 & -2 & 3\\ 2 & 4 & -2\\ -1 & 1 & 2\end{bmatrix}$

(13). The eigenvectors of a 3x3 real symmetric matrix A corresponding to the eigenvalues 2, 3, 6 are $(1,0,-1)^T$, $(1,1,1)^T$ and $(1,2,-1)^T$ respectively. Find the Matrix A.

(14). Reduce the Quadratic form $2x_1x_2+2x_1x_3-2x_2x_3$ to a canonical form by an orthogonal reduction. Also find its nature.

(15) Find A^n Using Cayley-Hamilton theorem, taking $A=\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}$ Hence find A^3

(16) If λ_i for $(i=1,2,\dots n)$ are the non-zero eigenvalues of A, then prove that,

(1) $k\lambda_i$ are the eigenvalues of kA, where k being a non-zero Scalar,

(2) $1/\lambda_i$ are the eigenvalues of A^{-1}.

(17). Verify Cayley-Hamilton theorem for the Matrix

$\begin{bmatrix}2 & 0 & -1\\ 0 & 2 & 0\\ -1 & 0 & 2\end{bmatrix}$ and hence find A^{-1} and A^4.

(18). Reduce the quadratic form $x^2+y^2+z^2-2xy-2yz-2zx$ to Canonical form through an orthogonal transformation. Write down the transformation.

UNIT-IV.

FUNCTIONS OF SEVERAL VARIABLES.

1) Using Euler's theorem, given $u(x,y)$ is a homogeneous function of degree n, prove that $x^2u_{xx} + 2xy\,u_{xy} + y^2u_{yy} = n(n-1)u$.

2) Using the definition of total derivative, find the value of $\frac{du}{dt}$ given $u = y^2 - 4ax$, $\cancel{x = at^2,\ y = 2at}$.
$u = x^3y^2 + x^2y^3$.

3) If $u = 2xy$, $v = x^2 - u^2$, $x = r\cos\theta$, $y = r\sin\theta$ then compute $\frac{\partial(u,v)}{\partial(r,\theta)} = ?$.

4) Find $\frac{du}{dt}$ if $u = \sin(x/y)$ where $x = e^t$, $y = t^2$.

5) If $u = \frac{y^2}{2x}$, $v = \frac{x^2+y^2}{2x}$ find $\frac{\partial(u,v)}{\partial(x,y)}$.

6) Given $u(x,y) = x^2\tan^{-1}(y/x)$, find the value of $x^2u_{xx} + 2xy\,u_{xy} + y^2u_{yy}$.

7) Write the Sufficient Condition for $f(x,y)$ to have a Maximum Value at (a,b).

8) If $u = f(x-y,\ y-z,\ z-x)$ then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

9) If $u = \frac{x+y}{1-xy}$. and $v = \tan^{-1}x + \tan^{-1}y$. find $\frac{\partial(u,v)}{\partial(x,y)}$.

10) Find $\frac{du}{dt}$ if $u = x/y$, where $x = e^t$, $y = \log t$.

11) If $x = u^2 - v^2$, $y = 2uv$ then evaluate the Jacobian of x,y with respect to u,v.

12) If $u = x^y$. Show that $\frac{\partial^2 u}{\partial x\partial y} = \frac{\partial^2 u}{\partial y\partial x}$.

13) If $u = x/y + y/z + z/x$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$

1. If $u = x^y$, Show that $u_{xxy} = u_{xyx}$.

2. If $u = \log(x^2+y^2) + \tan^{-1}(y/x)$. Prove that $u_{xx} + u_{yy} = 0$.

3. Find the Jacobian $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$ of the transformation

$x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$ & $z = r\cos\theta$.

4. Find the Maximum Value of $x^m y^n z^p$ Subject to the Condition $x+y+z = a$.

5. Find the Taylor's Series expansion of $e^x \sin y$ at the point $(-1, \pi/4)$ upto the third degree terms.

6. If $u = f\left[x/y, y/z, z/x\right]$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$.

7. Examine the function $f(x,y) = x^3y^2(12-x-y)$ for extreme values.

8. Find the volume of the greatest rectangular parallelopiped inscribed in the ellipsoid whose equation is,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

9. If $z = f(x,y)$; where $x = u^2 - v^2$, $y = 2uv$, prove that,

$$\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} = 4(u^2+v^2)\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right)$$

10. Find the Taylor's Series expansion of $x^2y^2 + 2x^2y + 3xy^2$ in Powers of $(x+2)$ and $(y-1)$ upto 3rd degree terms. $x^3+y^3+xy^2$
$(x-1)$ & $(y-2)$. $x^2y+3y-2$ $(x-1)(y+2)$.

11. If $x+y+z = u$, $y+z = uv$, $z = uvw$, prove that,

$$\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v.$$

12. Find the extreme Values of the function,

$f(x,y) = x^3+y^3-3x-12y+20$.

(13). If $u = f(x,y)$ where $x = r\cos\theta$, $y = r\sin\theta$, prove that,

$$u_x^2 + u_y^2 = u_r^2 + \frac{1}{r^2} u_\theta^2$$

(14) Find the Maximum & Minimum Values of

$x^2 - xy + y^2 - 2x + y$. $x^4 + y^4 - 2x^2 + 4xy - 2y^2$.

(15). A rectangular box open at the top, is to have a Volume of 32 cc. Find the dimensions of the box, that requires the least material for its Construction.

(16). Find the Taylor's Series expansion of $e^x \cos y$ in the neighborhood of the point $(1, \pi/4)$ upto third degree terms.

(17). Find the Jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3 if $y_1 = \frac{x_2 x_3}{x_1}$, $y_2 = \frac{x_3 x_1}{x_2}$, $y_3 = \frac{x_1 x_2}{x_3}$.

(18). If $u = \sin^{-1} \frac{x+y}{\sqrt{x}+\sqrt{y}}$, then prove that,

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\frac{\sin u \cos 2u}{4\cos^3 u}.$$

(19). Transform the equation $z_{xx} + 2z_{xy} + z_{yy} = 0$ by changing the independent Variables using $u = x - y$, & $v = x + y$.

Unit - III

Analytic Functions:

Part - A

1. State the Cauchy-Riemann equations in polar co-ordinates satisfied by an analytic functions.

2. Find the invariant points of the transformation
$$w = \frac{2z+6}{z+7}.$$

3. State the basic difference between the limit of a function of a real variable and that of a complex variable.

4. Find the bilinear transformation has atmost two fixed points.

5. Verify whether the function $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic.

6. Find the constants a, b, c if $f(z) = x + ay + i(bx + cy)$ is analytic.

7. Verify whether $f(z) = \bar{z}$ is analytic function or not.

8. Find the map of the circle $|z| = 3$, under the transformation $w = 2z$.

9. ~~Define Singular point~~. Show that $u = 2x - x^3 + 3xy^2$ is harmonic.

(16). Find the image

(10) Show that an analytic function with constant imaginary part is constant.

(11) Find the invariant points of the transformation $w = \frac{1+z}{1-z}$.

(12) Show that the function $f(z) = \bar{z}$ is nowhere differentiable.

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PART - B

1. Find the analytic function $f(z) = P+iQ$, if $P-Q = \dfrac{\sin 2x}{\cosh 2y + \cos 2x}$

2. Find the bilinear transformation which maps the points $z = 0, -i, -1$ into $w = i, 1, 0$ respectively.

3. If $f(z)$ is a regular function of z, prove that $\left[\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right] |f(z)|^2 = 4|f'(z)|^2$.

4. Find the image of the half plane $x > c$, when $c > 0$ under the transformation $w = 1/z$. Show the regions graphically. Also find the fixed points of w.

5. Verify that the families of curves $u = c_1$ and $v = c_2$ cut orthogonally, when $u+iv = z^3$.

6. Find the analytic function $u+iv$, if $u = (x-y)(x^2+4xy+y^2)$. Also find the conjugate harmonic function v.

7. Find the image of the circle $|z-1| = 1$ in the complex plane under the mapping $w = 1/z$.

8. When the function $f(z) = u+iv$ is analytic, prove that the curves, $u =$ Constant and $v =$ Constant are orthogonal.

9. Prove that every analytic function $w = u+iv$ can be expressed as a function of z alone, not as a function of $\bar{z}$.

10. Find the bilinear transformation which maps the points $z = 0, 1, \infty$ into $w = i, 1, -i$ respectively.

(11) Find the image of the hyperbola $x^2 - y^2 = 1$, under the transformation $w = 1/z$ is the lemniscate $r^2 = \cos 2\phi$.

(12) Prove that the transformation $w = \frac{z}{1-z}$ maps the upper half of z-plane onto the upper half of w-plane. what is the image of $|z| = 1$ under this transformation.

(13) Prove that $u = e^x [x \cos y - y \sin y]$ is harmonic & hence find the analytic function $f(z) = u + iv$.

(14) Find the bilinear transformation that transforms $1, i$ and -1 of the z-plane onto $0, 1$ and ∞ of the w-plane. Also show that the transformation maps interior of the unit cube of the z-plane on to upper half of the w-plane.

(15) Prove that $u = x^2 - y^2$ and $v = \frac{-y}{x^2+y^2}$ are harmonic but $u+iv$ is not regular.

(16) If $w = f(z)$ is analytic, prove that $\frac{dw}{dz} = \frac{\partial w}{\partial x} = -i \frac{\partial w}{\partial y}$.

(17) Show that $u = \frac{1}{2} \log (x^2 + y^2)$ is harmonic. Determine its analytic function. Find also its conjugate.

(18) Find the image of $|z| = 2$ under the mapping

(i) $w = z + 3 + 2i$, (ii) $w = 3z$.

(19) Find the analytic function $w = u + iv$ when $v = e^{-2y} (y \cos 2x + x \sin 2x)$ and find u.

UNIT-III.

PART-B.

(20). Show that the map $w = 1/z$ maps the totality of circles and straight lines as Circles or Straight lines.

(21) If $u(x,y)$ and $v(x,y)$ are harmonic functions in a region R, prove that the function

$$\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + i\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$ is an analytic function of $z = x+iy$.

(22) If $f(z)$ is an analytic function of z, prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \cdot \log |f(z)| = 0.$$

(23) Find the bilinear transformation that maps the Points $z = \infty, i, 0$ onto $w = 0, i, \infty$ respectively.

(24) Determine the analytic function whose real part is $\dfrac{\sin 2x}{\cosh 2y - \cos 2x}$.

UNIT - IV

COMPLEX INTEGRATION:

PART - A

(1) Evaluate $\int_C \tan z \, dz$ where C is $|z| = 2$.

(2) Find the Taylor Series for $f(z) = \sin z$ about $z = \pi/4$.

(3) Identify the type of Singularities of the following function $f(z) = e^{(1/(z-1))}$.

(4) Calculate the residue of $f(z) = \dfrac{e^{2z}}{(z+1)^2}$ at its pole.

(5) What is the value of the integral $\int_C \dfrac{3z^2+7z+1}{z+1} \cdot dz$ where C is $|z| = 1/2$?

(6) If $f(z) = \dfrac{-1}{z-1} - 2\left[1+(z-1)+(z-1)^2+\cdots\right]$ find the residue of $f(z)$ at $z=1$.

(7) Evaluate $\int_C \dfrac{e^z}{z-1} \cdot dz$, if C is $|z| = 2$.

(8) Expand $f(z) = \sin z$ in a Taylor series about Origin.

(9) Define Singular point.

(10) Evaluate $\int_C \dfrac{z\,dz}{(z-1)(z-2)}$ where C is the circle $|z| = 1/2$.

(11) Find the residue of the function $f(z) = \frac{4}{z^2(z-2)}$ at a simple pole.

PART-B

① Evaluate $\int_C \frac{z\,dz}{(z-1)(z-2)^2}$ where c is the circle $|z-2| = \frac{1}{2}$ using Cauchy's Integral formula.

② Evaluate $\int_0^{2\pi} \frac{d\theta}{1-2x\sin\theta+x^2}$ $(0<x<1)$, using contour integration.

③ Find the Laurent's Series of $f(z) = \frac{z^2-1}{z^2+5z+6}$ valid in the region $2<|z|<3$.

④ Evaluate $\int_{-\infty}^{\infty} \frac{x^2\,dx}{(x^2+a^2)(x^2+b^2)}$, using Contour Integration.

⑤ Evaluate ~~$\int_C \frac{z\,dz}{\ }$~~ $f(z) = \frac{1}{(z+1)(z+3)}$ is Laurent Series valid for the regions $|z|>3$ and $1<|z|<3$.

⑥ Evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$ using Contour integration.

⑦ Evaluate $\int_0^{\infty} \frac{dx}{(x^2+a^2)^3}$ $a>0$, using contour integration.

⑧ Find the Laurent's Series of $f(z) = \frac{7z-2}{z(z+1)(z+2)}$ in $1<|z+1|<3$.

⑨ Using Cauchy's Integral formula, evaluate $\int_C \frac{4-3z}{z(z-1)(z-2)}\,dz$, where c is the circle $|z| = \frac{3}{2}$

⑩ Evaluate $\int_{-\infty}^{\infty} \frac{x^2-x+2}{x^4+10x^2+9}\,dx$ Using Contour integration.

11. Evaluate $\int_C \frac{z+4}{z^2+2z+5}\,dz$, where C is the circle $|z+1+i| = 2$, Using Cauchy's integral formula.

12. Find the residues of $f(z) = \frac{z^2}{(z-1)^2(z+2)^2}$ at its isolated Singularities Using Laurent's Series expansions. Also State the valid region.

13. Evaluate $\int_{-\infty}^{\infty} \frac{x^2\,dx}{(x^2+1)(x^2+4)}$ Using contour integration.

14. Evaluate $\int_C \frac{(z+1)}{(z^2+2z+4)^2}\,dz$ where c is $|z+1+i| = 2$ using cauchy's integral formula.

15. Evaluate Using Cauchy's residue theorem, $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)}\,dz$, where C is $|z| = 3$.

16. Evaluate Using Contour integration $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)^2}\,dx$.

17. Evaluate $\int_C \frac{z-1}{(z+1)^2(z-2)}\,dz$, where C is the circle $|z-i| = 2$ using Cauchy's residue theorem.

18. Evaluate $\int_0^{\infty} \frac{\cos mx}{x^2+a^2}\,dx$, Using Contour integration.

19. ~~Apply Convolution theorem to evaluate~~ $\cancel{L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]}$. Evaluate $\int_0^{2\pi} \frac{\sin^2\theta}{a+b\cos\theta}\,d\theta$, $a>b>0$.

Unit - V

Laplace Transforms

Part - A

1. Find the Laplace transform of $\frac{1-\cos t}{t}$.

2. Find the inverse Laplace transform of $\cot^{-1}(k/s)$.

3. Find the Laplace transform of $t \cos at$.

4. Verify initial value theorem for $f(t) = 1+e^{-t}[\sin t+\cos t]$.

5. Find the Laplace transform of unit step function.

6. Find $L^{-1}[\cot^{-1}(s)]$.

7. Find Laplace transform of $t \sin 2t$.

8. Find $L^{-1}\left[\frac{1}{s^2+4s+4}\right]$.

9. State the conditions under which Laplace transform of $f(t)$ exists.

10. Find $L[e^{-3t} \sin t \cos t]$.

11. Find inverse Laplace Transform of $\frac{e^{-as}}{s}$.

12. State the first shifting theorem on Laplace transforms.

13. Is the linearity property applicable to $L\left[\frac{1-\cos t}{t}\right]$. Reason out.

(14) Find the inverse Laplace transform of

$$\frac{1}{(S+1)(S+2)}.$$

PART - B

(23) Find $L^{-1}\left[\frac{s^2}{(s^2+4)^2}\right]$ using convolution theorem.

(24) Find the Laplace transform of the Half wave rectifier $f(t) = \begin{cases} \sin\omega t & 0 < t < \pi/\omega \\ 0 & , \pi/\omega < t < 2\pi/\omega \end{cases}$ and

$f(t+2\pi/\omega) = f(t)$.

PART - B.

1. Find the Laplace transform of $te^{-2t}\cos 3t$.

2. Find the inverse Laplace transform of $\frac{1}{(s+1)(s^2+4)}$.

3. Solve the equation $y''+9y = \cos 2t$, $y(0)=1$ and $y(\pi/2) = -1$ using Laplace transform.

4. Find the Laplace transform of $f(t) = \begin{cases} t, & \text{in } 0\le t\le a \\ 2a-t, & \text{in } a\le t\le 2a. \end{cases}$ and $f(t+2a) = f(t)$. Instead of 'a' 'π' is given.

5. Find the Laplace transform of $e^{-4t}\int_0^t t\sin 3t\,dt$.

6. Using Convolution theorem find the inverse Laplace transform of $\frac{1}{(s^2+1)(s+1)}$.

~~7. Find laplace transform of $f(t) = \{ t, 0<t<a$~~

7. Find the Laplace transform of square wave function defined by $f(t) = \begin{cases} 1 & \text{in } 0<t<a \\ -1 & \text{in } a<t<2a \end{cases}$ with period $2a$.

8. Solve the differential equation $\frac{d^2y}{dt^2}+y = \sin 2t$, $y(0)=0$, $y'(0)=0$ by Using Laplace transform method.

9. Apply Convolution theorem to evaluate $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$.

10. Verify initial & final value theorem for the function $f(t) = 1+e^{-t}(\sin t+\cos t)$.

11. Using Laplace transform solve the differential equation $y''-3y'-4y = 2e^{-t}$ with $y(0)=1=y'(0)$.

(12) Find the laplace transform of

$$f(t) = \begin{cases} \epsilon & 0 \le t \le a \\ -\epsilon, & a \le t \le 2a \end{cases} \quad \& \quad f(t+2a) = f(t) \text{ for all } t.$$

(13) Find the inverse Laplace transform of $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$. Using convolution theorem.

(14) Using convolution theorem find $L^{-1}\left[\frac{1}{(s+a)(s+b)}\right]$.

(15) Find $L\left[\frac{\cos at - \cos bt}{t}\right]$.

(16) Solve $\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 2$, given $x=0$, & $\frac{dx}{dt} = 5$, for $t=0$, Using Laplace transform method.

(17) Find $L^{-1}\left[\frac{1}{s(s^2+4)}\right]$ using convolution theorem.

(18) Find the Laplace transform of a square wave

(19) function given by $f(t) = \begin{cases} E & \text{for } 0 \le t \le a/2 \\ -E & \text{for } a/2 \le t \le a \end{cases}$ and $f(t+a) = f(t)$.

(19) Evaluate $\int_0^\infty t e^{-2t} \cos t \, dt$ Using Laplace transform.

(20) Solve $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = \sin t$, if $\frac{dy}{dt} = 0$ and $y=2$ when $t=0$ using Laplace transforms.

(21) Find the Laplace transform of $\frac{e^{at} - e^{-bt}}{t}$.

(22) Solve the differential equation $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = e^{t}$ with $y(0)=1$ & $y'(0)=0$, Using Laplace transforms.