Math 1401 Application Project Spring 2013

Name: ____

This project is due on Thursday May 9th. All work must be done on a separate sheet of paper. I must be able to clearly see all of your work and reasoning as you develop your answer. Convince me that you know what you are doing! **Do not skip steps**, do not approximate your answers, calculators are only allowed for arithmetic calculations and where noted.

Rancher Rob has 1000 yards of fencing and he plans to use the fencing to make 2 enclosures, one circular and one a square. How much of the 1000 yards should be used for each region if Rob wants to maximize the combined area of both regions?

Setup: Let x yards of fencing be used for the circumference of the circle and the rest $(1000 - x)$ be used for the perimeter of the square.

Task 1: Show that the area function to be maximized is:

$$A(x) = (\frac{\pi + 4}{16\pi})x^2 - 125x + 62500$$

Task 2: Show that the critical point of of $A(x)$ is $x = \frac{1000\pi}{\pi+4}$

Task 3: Since the domain of the continuous function $A(x)$ is $[0, 1000]$, we can find the value of x that maximizes the combined area $A(x)$ by using the Extreme Value Theorem. Find the values of $A(0)$, $A(\frac{1000\pi}{\pi+4})$, and $A(1000)$. The function values, in no particular order, are $\frac{250000}{\pi}$, 62500, and $\frac{250000}{\pi+4}$
This is not what Rancher Rob expected and the mathematician part of Rob is not satisfied.

Task 4: Using your calculator, sketch the graph of $A(x)$ on graph paper using the following window: $x : [0, 1000]$ with a scale of 100 and $y : [0, 90000]$ with a scale of $10,000$. Does your graph verify your results from Task 3?

Rancher Rob now wants to verify that the critical point will always be a relative minimum for all lengths of fencing. Let the length of the fence be L.

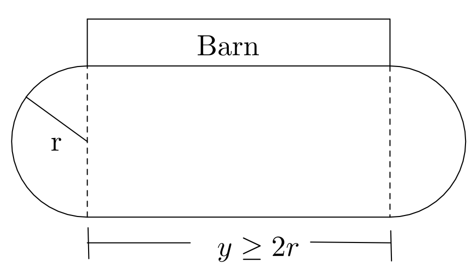
Task 5: Now let x yards of fence be used for the circumference of the circle and the rest, $(L - x)$ yards, be used for the perimeter of the square. Show that the area function to me maximized is:

$$A(x) = (\frac{\pi + 4}{16\pi})x^2 - (\frac{L}{8})x + \frac{L^2}{16}$$

Show that the critical point of $A(x)$ is $x = \frac{\pi}{\pi+4}L$. Finally, since the domain of the continuous function $A(x)$ is $[0, L]$, w can find the value of x that maximizes the combined area $A(x)$ using the Extreme Value Theorem. Find the values of $A(0)$, $A(\frac{\pi}{\pi+4}L)$, and $A(L)$. The function values in no particular order are $\frac{L^2}{4\pi}$, $\frac{L^2}{4(\pi+4)}$, and $\frac{L^2}{16}$.

Task 6: Verify that your earlier results, when $L = 1000$, fit the solution from Task 5.

Rancher Rob wants to maximize the combined areas and at the same time have a circular region to train his animals. What he hopes to do is create a rectangular region with semicircles on the sides with added restriction that the width of the rectangle must be at least 2 times the radius of the semicircles. He has also decided to save fencing by creating this area with one of the rectangular sides along his barn. Here is the new region:



Task 7: If Rancher Rob still has 1000 yards of fencing and $y \geq 2r$, find the values of r and y that maximizes the are of this new region. What is the area of the region?