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ON THE DETECTION OF EXOPLANETS VIA RADIAL VELOCITY DOPPLER SPECTROMETRY

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ABSTRACT

Since the discovery of the first exoplanet occurred in the late 1980s, physicists and astronomers have confirmed the existence of over 1783 planets orbiting other stars. Over the years, several methods have been developed for detecting these celestial systems, but none has thus far been more successful than the use of Radial Velocity Doppler Spectrometry. As such, this paper attempts breaks down the barriers placed before new entries into the field by detailing the theoretical models and experimental process of this most successful technique at the senior undergraduate level.

Keywords: Exoplanet: General — Exoplanet: Individual(Kepler XXXX) — Planetary Physics: General — Doppler Spectrometry: General — Orbital Mechanics: General

1. INTRODUCTION TO EXOPLANETS

For centuries, some of humanities greatest minds have pondered over the possibility of other worlds orbiting the uncountable number of stars that exist in the visible universe.

2. OVERVIEW OF TERMINOLOGY

It is important to begin by defining exactly what it is researchers are finding with these surveys. In 2003, the Working Group on Extrasolar Planets (WGESP) of the International Astronomical Union (IAU) modified its definition of sub-stellar companions to be defined as:

> Objects with true masses below the limiting mass for thermonuclear fusion of deuterium (currently calculated to be 13 Jupiter masses for objects of solar metallicity) that orbit stars or stellar remnants are "planets" (no matter how they formed). The minimum mass/size required for an extrasolar object to be considered a planet should be the same as that used in our Solar System.

Thus, when we discuss these discoveries it is important to keep in mind the maximum size of the sub-stellar companions we consider to be exoplanets.

3. THE TWO BODY PROBLEM

A fundamental part of constructing our detection method is ability to translate the "wobble" of a star into the orbital information of its sub-stellar companion(s). For simplicity's sake, we will only consider the case of a singular companion for the entirety of this paper. For a more detailed description of multi-planet systems, the author suggests the review of the numerical methodology employed by Beaugé et al. (2012).

Consider a star, M_s, with a planet, M_p, in orbit around it. It is well known to any undergraduate in physics that by converting coordinate systems to that of the reference frame of the stationary center of mass, the two-body problem reduces to that of a single body moving within a potential well. We do this by stating:

$$\vec{r}_{cm} = \frac{M_p \vec{r}_p + M_s \vec{r}_s}{M_s + M_p} \tag{1}$$

$$\vec{r} = \vec{r}_s - \vec{r}_p \tag{2}$$

with $\vec{r}_s$ being the position vector of star from the origin, $\vec{r}_p$ being the position vector of the planet, $\vec{r}$ being the difference in position vectors, and $\vec{r}_{cm}$ being the vector pointing to the center of mass from the origin. Without loss of generality, we may set $\vec{r}_{cm} = 0$. Combining this fact with Eq. 1 & Eq. 2 yields the following relationships:

$$\vec{r}_s = \frac{M_p}{M_s + Mp} \vec{r} \tag{3}$$

$$\vec{r}_p = -\frac{M_s}{M_s + Mp} \vec{r} \tag{4}$$

Thus, if we know $\vec{r}$, which is the solution to the equivalent Lagrange one-body problem, we can solve for the individual motions of the star and the planet.

When considering a planet orbiting in a plane parallel to the XY-plane, we express $\vec{r}$ in the following manner:

$$\vec{r} = \frac{a(1-e^2)}{1 + e\cos f} \begin{pmatrix} \cos f \\ \sin f \\ 0 \end{pmatrix} \tag{5}$$

where a is the semi-major axis, e is the eccentricity, and $f = \theta - \varpi$ is the true anomaly[3]. However, this implies that the observation axis is that of the orbital plane's Z-axis. In most cases, the plane of the orbit has been rotated about each of the observation axes individually. We can represent this coordinate transformation under rotations about an axis by the following linear transforms:

$$\boldsymbol{P}_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \tag{6}$$

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[3] It should be noted that the true anomaly is being expressed here as a function of the current angular position, θ, and the longitude of periapse, ϖ. There are other methods of expression, but this is the most common, regardless of its difficulty of measurement.

$$\boldsymbol{P}_z(\phi) = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{7}$$

Allowing our orientation angles to be i, Ω, & ω, we can summarize the coordinate transformation from the orbital plane (x, y, z) to the observer's plane (x_{ob}, y_{ob}, z_{ob}) as the product of three rotations:

1. The rotation about the z-axis through the angle ω to align the periapse with the ascending node.
2. The rotation about the x'-axis through the angle i to allow the orbital plane and the observer's plane to be parallel.
3. The rotation about the z''-axis through the angle Ω to align the periapse with the x_{ob}-axis.

Thus, our transform is of the form:

$$\vec{r}_{ob} = \boldsymbol{P}_z(\Omega)\boldsymbol{P}_x(i)\boldsymbol{P}_z(\omega)\vec{r} \tag{8}$$

And we arrive with:

$$\vec{r}_{ob} = \frac{a(1-e^2)}{1+e\cos f}\begin{pmatrix} \cos\Omega\cos\psi - \sin\Omega\sin\psi\cos i \\ \sin\Omega\cos\psi + \cos\Omega\sin\psi\cos i \\ \sin\psi\sin i \end{pmatrix} \tag{9}$$

with $\psi = \omega + f$. This is useful because we can now accurately describe any planetary system's orientation towards our line of sight and the sky's reference frame.

With the ability to describe an orbit about the center of mass of either body, we can now continue towards deriving the radial velocity equation. This will allow us to relate our observations to the parameters of the sub-stellar companion's orbit. The radial velocity of the star can be expressed as:

$$v_r = \dot{\vec{r}}_s \cdot \hat{z}_{ob} = V_{cm,z} + \frac{M_p}{M_s + M_p}\dot{r}_{ob,z} \tag{10}$$

where $V_{cm,z}$ is the proper motion of the barycenter and $\dot{r}_{ob,z} = \dot{\vec{r}}_{ob} \cdot \hat{z}_{ob}$ is the velocity of $\vec{r}_{ob}$ projected onto the z_{ob}-axis. We now obtain that:

$$\dot{r}_{ob,z} = \dot{r}\sin\psi\sin i + r\dot{f}\cos\psi\sin i \tag{11}$$

Next, we obtain $\dot{r}$ by taking the derivative with respect to time:

$$\dot{r} = \frac{a(1-e^2)}{1+e\cos f} \cdot \frac{\dot{f}e\sin f}{1+e\cos f} = \frac{r\dot{f}e\sin f}{1+e\cos f} \tag{12}$$

Noting Kepler's Second and Third Laws, we know that:

$$L = \frac{2\pi}{T}a^2\sqrt{1-e^2} = r^2\dot{f} \tag{13}$$

After some algebra is done, we obtain:

$$r\dot{f} = \frac{2\pi}{T}\frac{a}{\sqrt{1-e^2}}(1+e\cos f) \tag{14}$$

Making use of Eq. 12 & 14, we are left with:

$$\dot{r}_{ob,z} = \frac{2\pi}{T}\frac{a\sin i}{\sqrt{1-e^2}}(\cos\psi + e\cos\omega) \tag{15}$$

We can now finally write our final form of the radial velocity equation as:

$$v_r = V_{cm,z} + K(\cos\psi + e\cos\omega) \tag{16}$$

where:

$$K = \frac{2\pi}{T}\frac{M_p\sin i}{M_s + M_p}\frac{a}{\sqrt{1-e^2}} \tag{17}$$

Thus, we now have a model which relates the measurable radial velocity and mass of the star to the parameters of its sub-stellar companion's orbit: the period T; the minimum mass $M_p\sin i$; the semi-major axis a; and the eccentricity e.

4. THE RELATIVISTIC DOPPLER SHIFT

It is a great tryumph of physics to be able to utilize the radial velocity of a star to detect the presence of sub-stellar companions. However, to have the precision needed to measure the effects on the parent star, we need a technique stronger than that of astrometry. The way we retrieve this radial velocity is through none-other than the relativistic Doppler effect.

We know that the general Lorentz transforms for a frame S' moving at a velocity $\vec{V}$ with respect to the frame S is given by:

$$\vec{R}' = \vec{R} + (\gamma - 1)\frac{(\vec{R}\cdot\vec{\beta})\vec{\beta}}{||\beta||^2} - \gamma\vec{\beta}ct \tag{18}$$

$$ct' = \gamma(ct - \vec{\beta}\cdot\vec{R}) \tag{19}$$

with $\vec{\beta} = \vec{V}c^{-1}$ and $\gamma = \sqrt{1 - ||\vec{\beta}||^2}^{-1}$, as per the usual convention.

Let us now imagine a plane wave of light is emitted in the S rest frame, denoted as $e^{i(\vec{k}\cdot\vec{R} - 2\pi\nu t)}$, at the position $\vec{R}$. Now, allow an observer in the moving S' frame to be looking at the light along the $\hat{k}'$ direction in his rest frame. The observer would measure the emitted wave as having the form $e^{i(\vec{k}'\cdot\vec{R}' - 2\pi\nu' t)}$. Thus, we can substitute the Lorentz transforms into the phase equation of the light and retrieve:

$$\vec{k}' \cdot \left[\vec{R} + (\gamma - 1)\frac{(\vec{R}\cdot\vec{\beta})\vec{\beta}}{||\vec{\beta}||^2} - \gamma\vec{\beta}ct\right] - \frac{2\pi\nu'}{c}\left[\gamma(ct - \vec{\beta}\cdot\vec{R})\right] \tag{20}$$

Note that when the plane wave is viewed in the source's S frame, it has the form $e^{i(\vec{k}\cdot\vec{R} - 2\pi\nu t)}$. Thus, we can separate the spacial and temporal parts of the equation to find:

$$\vec{R} \cdot \underbrace{\left[\vec{k}' + \gamma(\vec{k}'\cdot\vec{\beta})\frac{\vec{\beta}}{||\vec{\beta}||^2} + \gamma||\vec{k}'||\vec{\beta}\right]}_{\vec{k}} \tag{21}$$

for the spacial part and:

$$-t\underbrace{\left[2\pi\nu'\gamma((\hat{k}'\cdot\vec{\beta}) + 1)\right]}_{2\pi\nu} \tag{22}$$

for the temporal part. Therefor, an observer in the S' frame, which sees the light source in the S frame moving at a relative velocity of $\vec{V}$, will measure the Doppler shift to be:

$$\frac{\nu}{\nu'} = \frac{\lambda'}{\lambda} = \gamma(1 + \hat{k}'\cdot\vec{\beta}) \tag{23}$$

For the case related to exoplanets, we have the ability to approximate $\hat{k}' \cdot \vec{\beta}$ as equal to v_r. This is because if the distance to the source's barycenter is on the order of light years and the orbit's semi-major axis is on the order of AU, then $\vec{k}'_{z'} >> \vec{k}'_{x'}, \vec{k}'_{y'}$ and thus $\hat{k}' \approx \hat{z}_{ob}$. In addition, if one is interested in low-amplitute variations in v_r of more than 0.1 m s^-1, the relativistic term can be dropped. This allows us to neglect the need to measure the transverse velocity of the star, which would require direct observation methods. Thus, we find that the radial velocity of a star is related to the Doppler shift of its received spectrum by:

$$v_r = c\frac{\lambda' - \lambda}{\lambda} = c(z - 1) \tag{24}$$

where $z = \frac{\lambda'}{\lambda}$ is the Doppler parameter.

5. INSTRUMENTATION

5.1. *The Echelle Grating*

The center piece of the echelle spectrometer is the echelle grating: a type of blazed reflection grating. A diagram of this type of grating can be found in Figure **??**. Since knowing the point-spread function (PSF) of the instrumentation will become vital to precisely measuring the Doppler parameter, it is worth deriving the contribution of the echelle grating itself. Not only will this showcase why the grating is used in the spectrograph, but also allow us to understand exactly how it works.

Let us imagine shining monochromatic light emanating from a point-source at the echelle grating. Allow the light rays to hit the grating at the center of each face and at the angle $\theta_i \approx \gamma$. Then the problem of finding the intensity pattern of the reflected light (in this case, the PSF) breaks down into two parts: the interference of the light waves and the diffraction of the light waves around the blazed edges of the grating faces.

First, we shall attack the interference problem. For an echelle grating, the derivation is that of an n-slit transmission grating of "slit width" of d. The total electric field at a screen in the far field is:

$$E = E_0 \sum_{j=1}^{N} e^{i(2j\alpha_I)} = E_0 \frac{\sin(N\alpha_I)}{\sin\alpha_I} \tag{25}$$

where $\alpha_I = \frac{\pi}{\lambda} d(\sin\theta_i + \sin\theta_o)$ is the phase difference due to different path lengths. Thus, the normalized intensity due to interference is:

$$I_I(\lambda, \theta_o) = \frac{\sin^2[\frac{\pi}{\lambda} L(\sin\theta_i + \sin\theta_o)]}{\sin^2[\frac{\pi}{\lambda} d(\sin\theta_i + \sin\theta_o)]} \tag{26}$$

Now we will tackle the problem of diffraction of the light rays around the blazed edges of the grating. As before we will only be allowing the angle $\theta_i \approx \gamma$. Given this condition, the electric field at a screen in the far field is:

$$E_B = E_0 \frac{\sin\alpha_B}{\alpha_B} \tag{27}$$

where $\alpha_B = \frac{\pi}{\lambda}(d\cos\gamma)(\sin\theta'_i + \sin\theta'_o)$ is the phase difference, $\theta'_i = \theta_i - \gamma$ and $\theta'_o = \theta_o - \gamma$. Therefor, the normalized intensity due to diffraction is:

$$I_B(\lambda, \theta_o) = \frac{\sin^2[\frac{\pi}{\lambda}(d\cos\gamma)(\sin\theta'_i + \sin\theta'_o)]}{[\frac{\pi}{\lambda}(d\cos\gamma)(\sin\theta'_i + \sin\theta'_o)]^2} \tag{28}$$

Equation 28 is referred to as the "blaze function" in most literature and allows for the central maximum to be shifted to a higher order based on the blaze angle, γ.

We now combine the two products through an algebra trick involving the diffraction grating equation. The goal is to find a relationship $\frac{d}{\lambda}$ in the interference case and substitute it into α_B to find the total intensity. We do this by:

$$\begin{aligned} n\lambda &= d(\sin\theta_i + \sin\theta_o) \\ &= d(\sin(\theta'_i + \gamma) + \sin(\theta'_o + \gamma)) \\ &= d(2\sin\gamma\cos\theta'_i) \quad \text{when } \theta'_i = -\theta'_o \end{aligned}$$

$$\frac{d}{\lambda} = \frac{n}{2\sin\gamma\cos\theta'_i} \tag{29}$$

Substituting Eq. 29 into our relationship for α_B, we arrive at final form for the PSF due to an echelle grating:

$$I(n, \theta'_o) = \text{sinc}^2\left[\frac{n\pi(\sin\theta'_i + \sin\theta'_o)}{2\tan\gamma\cos\theta'_o}\right] \tag{30}$$

This final form allows us to see that the echelle grating's primary purpose. For each wavelength, the grating shifts the central maximum out from the first order and into to another higher order. The order to which it then resides is dependent on the physical characteristics of the grating and how it is placed into the spectrograph.

6. EXPERIMENTAL PROCEDURE

6.1. *Measuring the Spectrum of the Gas Cell*

6.2. *Calculating the Instrumental PSF*

6.3. *Modeling the Observations*

6.4. *Correcting for the Relative Motion of the Observer*

6.5. *Finding Exoplanets*

6.5.1. *HD 72659b*

7. RESULTS OF THE KECK & LICK OBSERVATIONAL SURVEYS

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8. CONCLUDING THOUGHTS

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APPENDIX

APPENDIX MATERIAL

REFERENCES

Aurière, M. 1982, A&A, 109, 301

Figure 1. Derived spectra for 3C138 (see **?**). Plots for all sources are available in the electronic edition of *The Astrophysical Journal*.