

# POKHARA UNIVERSITY

Level: Bachelor Semester – Fall Year : 2005  
Programme: BE Full Marks: 100  
Course: Electromagnetic Propagation and Antenna Time : 3hrs.

*Candidates are required to give their answers in their own words as far as practicable.*

*The figures in the margin indicate full marks.*

***Attempt all the questions.***

1. Find the expression for power radiated by a current element. 15
2. a) What do you mean by the directive gain of an antenna? Show that the directivity of a half wave dipole is 2.15 dBi. 7  
b) Prove that maximum effective area of any antenna is  $1.5 \lambda^2 / 4\pi$  8
3. a) What do you mean by parasitic array. Discuss about Yagi-Uda antenna. 8  
b) Find the pattern for an eight-element array obtained by principle of multiplication of patterns. 7
4. a) For transmit-receive system, derive the expression for free space loss (FSL) in decibel (dB) and signal-to-noise ratio (S/N) for a receiving system. 8  
b) Consider a link at a frequency  $f = 14$  GHz between a TV satellite in geostationary orbit and a parabolic receiving antenna of the surface of the Earth, at a distance  $d = 36,000$  Km from the satellite. The transmit power is  $P_T = 100$  W, and the transmit antenna gain is  $G_T = 40$  dB. 7
  - i. Determine the power density  $dP/dS$  (Watt/m<sup>2</sup>) at the receive antenna.
  - ii. The TV picture quality is acceptable if the receiver antenna (assumed lossless) receives a power  $P_R$  which exceeds a threshold  $P_0 = 2 \times 10^{-11}$  W. What should the antenna area  $S_A$  and the antenna gain  $G_A$  (in dB) to achieve this?
5. a) Define Fresnel reflection coefficient ( $\Gamma$ ). Assuming earth as lossy dielectric medium, derive the expression for reflection coefficient when E-field is in plane of incidence. 7  
b) Derive the expression for refractive index of ionosphere and the maximum usable frequency (MUF). 8

6. a) Consider the case for synchronous satellite relay, where 6 GHz is used for ground to satellite link and 4 GHz is used for satellite to ground link. Consider 28 meter diameter ground antenna and 0.28 meter diameter satellite antenna assuming 67% effective area and height of the satellite is 36,000 km. Find the following: 10
- i. Basic transmission loss
  - ii. Maximum directive gain
  - iii. With ground transmitter power of 12kw, the power received at the satellite receiver.
  - iv. With satellite transmitter power of 1w, the power received at ground antenna.
- b) Draw the block diagram of optical fiber communication. Explain each block in brief. 5
7. **Write short notes on (Any Two)** 2×5
- a) Horn antenna
  - b) SID
  - c) Half power beamwidth
  - d) Antenna Temperature and signal to Noise ratio.

# Electromagnetic Propagation & antenna.

## CHAPTER-1 INTRODUCTION (15-25 marks)

- \* Review of Electromagnetic waves and equations:
  - ↳ Maxwell's eqn.
  - ↳ Electromagnetic wave & eqn of wave propagation.

### \* Plane wave and Uniform plane wave

- # A plane wave is a wave whose phase is constant over a set of plane.
- # Uniform plane wave is the one whose magnitude and phase are constant.
- # A spherical plane wave is the one which appears to be a uniform plane wave observed at far distance.
- # E.M waves originates from a point spreads out uniformly in all direction and is a form of spherical wave.

### \* Properties of plane wave

1. At every point in space, electric field  $E$  and magnetic field vector  $H$  are perpendicular to each other and to direction of propagation. Thus plane wave is transverse i.e.  $E$  &  $H$  are both  $90^\circ$  to direction of propagation.

2. velocity of wave in free space

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8$$

3.  $E$  &  $H$  oscillates in phase and ratio of magnitude of  $E$  &  $H$  is constant i.e.  $120\pi$ ,  $377 \Omega$ .

$$\text{i.e. } E/H = 120\pi$$

## \* Wave Impedance.

# The wave impedance is defined as:-

$$Z = \frac{\text{Electric field component}}{\text{magnetic field component}} \quad (\Omega)$$

# For Transverse EM wave, there exists only one component of each of the electric field and magnetic fields resulting only one wave impedance generally called intrinsic wave impedance  $\eta_0$ ,

$$\eta_0 = \frac{E_x}{H_y} \quad [\text{free space}]$$

## \* Poynting vector.

# A poynting vector  $\vec{P}$  is the cross product of  $\vec{E}$  &  $\vec{H}$ .

$$\vec{P} = \vec{E} \times \vec{H}$$

The magnitude of  $\vec{P}$  represents an instantaneous power density ( $\text{W/m}^2$ ) at a point and its direction indicates direction of power flow at that point & is perpendicular to plane containing  $\vec{E}$  and  $\vec{H}$ .

# For perfect dielectric medium

$$E_x = E_{x0} \cos(\omega t - \beta z)$$

$$H_y = \frac{E_x}{\eta} = \frac{1}{\eta} E_{x0} \cos(\omega t - \beta z)$$

&

$$P_z = E_x \cdot H_y = \frac{1}{\eta} E_{x0}^2 \cos^2(\omega t - \beta z)$$

# Average power density can be calculated as

$$P_{z \text{ avg}} = \frac{1}{T} \int_0^T \frac{1}{\eta} E_{x0}^2 \cos^2(\omega t - \beta z) dt$$
$$= \frac{1}{2\eta} E_{x0}^2 \quad (\text{W/m}^2)$$

## \* Retarded Potential

- # The scalar electric potential  $V$  at a point caused by a line charge with a linear charge density  $\rho_L$  is defined by:

$$V = \int \frac{\rho_L dL}{4\pi\epsilon_0 r} \quad (V)$$

where  $r$  is distance between  $dL$  & pt under consideration.

- # Similarly vector magnetic potential is defined as

$$\vec{A} = \int \frac{\mu I d\vec{L}}{4\pi r} \quad (Wb/m)$$

- # Here  $\rho_L$  and  $I$  don't change with time and therefore  $V$  and  $A$  at the point of interest are fixed for all the time

- # But if  $\rho_L$  and  $I$  vary with time then their values seen at the time of measurement can't be used to calculate  $V$  &  $\vec{A}$  at distant point because it takes time to reach the effect from the source to the point of interest. The values of  $\rho_L$  &  $I$  which contributed the effect has already been changed to some other values.

$$V = \int \frac{[\rho_L] dL}{4\pi\epsilon_0 r}$$

$$\vec{A} = \int \frac{\mu [I] d\vec{L}}{4\pi r}$$

The  $V$  and  $A$  are respectively termed as retarded electric scalar potential & retarded vector magnetic potential. The symbol  $[\ ]$  represents that the corresponding quantity has been retarded in time in order to encompass the time lapsed in propagating the effect from the source to the point where the quantity is being calculated.

$$[I] = I_0 \cos[\omega ct - t'] \quad t' = r/v$$

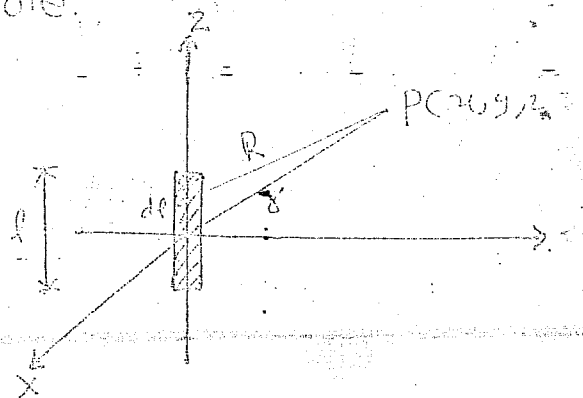
\* Electric and Magnetic Fields due to alternating current along electric dipole.

# Consider a current element or electric dipole of elementary length 'l' with variation in current as

$$I = I_0 \cos \omega (t - R/v)$$

where  $v$  = velocity

$R$  = distance between centre of small current element  $dl$  and point of consideration



# Now retarded potential can be obtained as magnet vector potential

$$\vec{A} = \oint \frac{\mu [I] d\vec{l}}{4\pi R}$$

Here the direction of  $\vec{A}$  is in the direction of current or  $d\vec{l}$  i.e. along  $z$  axis and is retarded in time by  $R/v$  sec.

# Assuming the current to be uniform throughout the length at any time  $t$  and since the length of dipole is very small.

$$d\vec{l} = dz \hat{z} \quad \& \quad R \approx r$$

$$\therefore \vec{A} = \oint \frac{\mu I_0 \cos \omega (t - r/v) dz \cdot \hat{z}}{4\pi r}$$

The integration along length will be

$$\vec{A} = \frac{\mu I_0 \cos \omega (t - r/v) l \hat{z}}{4\pi r}$$

# we know,

$$\vec{A} = A_x + A_y + A_z$$

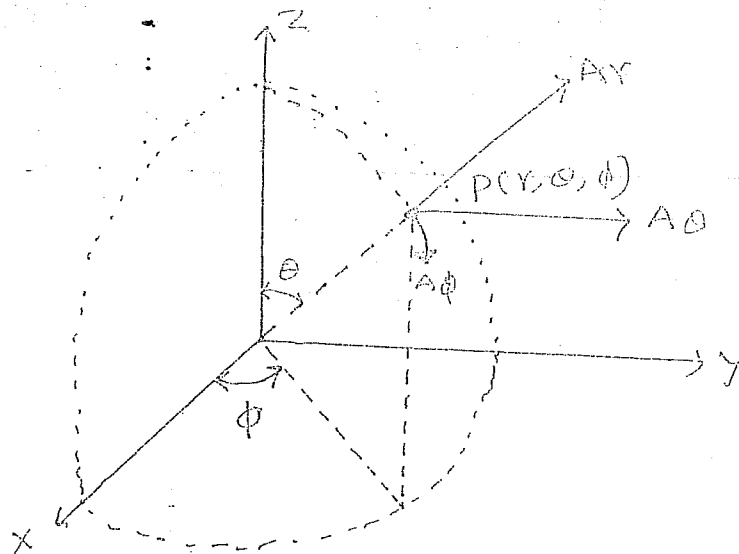
$$A_x = A_y = 0$$

$$A_z = \frac{\mu I_0 \cos \omega (t - r/v) l}{4\pi r}$$

$$A_r = A_2 \cos \theta = \frac{\mu_0 I_0 \cos \omega (t - r/v)}{4\pi r} \cos \theta$$

$$A_\theta = -A_2 \sin \theta = -\frac{\mu_0 I_0 \cos \omega (t - r/v)}{4\pi r} \sin \theta$$

$$A_\phi = 0$$



	$\hat{e}_r$	$\hat{e}_\theta$	$\hat{e}_\phi$
$\hat{e}_r$	$\sin \theta \cos \phi$	$\cos \theta \cos \phi$	$-\sin \phi$
$\hat{e}_\theta$	$\sin \theta \sin \phi$	$\cos \theta \sin \phi$	$\cos \phi$
$\hat{e}_\phi$	$-\cos \theta$	$-\sin \theta$	$0$

# From the definition of  $\vec{A}$  we may derive the magnetic field  $\vec{H}$  as

$$\vec{H} = \frac{1}{\mu} (\nabla \times \vec{A})$$

$$\nabla \times \vec{A} = \frac{1}{r^2 \sin \theta} \begin{vmatrix} \hat{e}_r & r \hat{e}_\theta & r \sin \theta \hat{e}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & r A_\theta & r \sin \theta A_\phi \end{vmatrix}$$

$$H_r = \frac{1}{\mu} \left[ \frac{1}{r^2 \sin \theta} \left( \frac{\partial}{\partial \theta} (r \sin \theta A_\phi) - \frac{\partial}{\partial \phi} (r A_\theta) \right) \right]$$

$$H_r = 0$$

$$H_\theta = \frac{1}{\mu} \left[ \frac{r}{r^2 \sin \theta} \left( -\frac{\partial}{\partial r} (r \sin \theta A_\phi) + \frac{\partial}{\partial \phi} (A_r) \right) \right]$$

$$H_\theta = 0$$

$$H_\phi = \frac{1}{\mu} \left[ \frac{r \sin \theta}{r^2 \sin \theta} \left\{ \frac{\partial}{\partial r} (r A_\theta) - \frac{\partial}{\partial \theta} (A_r) \right\} \right]$$

$$H_\phi = \frac{1}{\mu} \left[ \frac{1}{r} \left\{ \frac{\partial}{\partial r} (r A_\theta) - \frac{\partial}{\partial \theta} (A_r) \right\} \right]$$

$$\begin{aligned}
 \frac{\partial}{\partial r}(rA_\theta) &= \frac{\partial}{\partial r} \left( r \cdot \frac{-\mu l I_0 \cos \omega(t-r/v) \sin \theta}{4\pi r} \right) \\
 &= \frac{-\mu l I_0 \sin \theta}{4\pi} - \sin \omega(t-r/v) \cdot (-\omega/v) \\
 &= \frac{-\mu l I_0 \omega \sin \theta \sin \omega t'}{4\pi v} \quad [\because t' = t - r/v]
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial}{\partial \theta}(Ar) &= \frac{\partial}{\partial \theta} \left[ \frac{\mu l I_0 \cos \omega(t-r/v) \cos \theta}{4\pi r} \right] \\
 &= -\frac{\mu l I_0 \cos \omega t' \sin \theta}{4\pi r}
 \end{aligned}$$

Then,

$$H_\phi = \frac{1}{\mu} \left[ \frac{-\mu l I_0 \omega \sin \theta \sin \omega t'}{4\pi r v} + \frac{\mu l I_0 \cos \omega t' \sin \theta}{4\pi r^2} \right]$$

$$H_\phi = \frac{l I_0 \sin \theta}{4\pi} \left[ \frac{\cos \omega t'}{r^2} - \frac{\omega \sin \omega t'}{v r} \right]$$

Next,

Electric field can be obtained as

$$\vec{E} = \frac{1}{\epsilon} \int \nabla \times \vec{H} \, d\ell$$

Now,

$$E_r = \frac{1}{\epsilon} \int \frac{1}{r^2 \sin \theta} \left[ \frac{\partial}{\partial \theta} (r \sin \theta H_\phi) - \frac{\partial}{\partial \phi} (r H_\theta) \right] d\ell$$

on differentiation

$$E_r = \frac{I_0 l \cos \theta}{2\pi \epsilon r} \left( \frac{\cos \omega t'}{r^2} - \frac{\omega \sin \omega t'}{r v} \right)$$

$$E_r = \frac{I_0 l \cos \theta}{2\pi \epsilon} \left[ \frac{\sin \omega t'}{r^3 \omega} + \frac{\cos \omega t'}{r^2 v} \right]$$



$$E_{\theta} = \frac{1}{\epsilon} \left[ \frac{r}{r^2 \sin \theta} \left[ -\frac{\partial}{\partial r} (r \sin \theta H_{\phi}) \right] + \frac{\partial}{\partial \phi} (H_r) \right]$$

$$E_{\theta} = -\frac{1}{\epsilon} \left[ \frac{1}{r} \frac{\partial}{\partial r} (r H_{\phi}) \right] dt$$

Solving we get (5-6 steps)

$$E_{\theta} = \frac{I_0 l \sin \theta}{4\pi \epsilon} \left[ \frac{\cos \omega t'}{r^2 v} + \frac{\sin \omega t'}{\omega r^3} - \frac{\omega \sin \omega t'}{v^2 r} \right]$$

$$E_{\phi} = \frac{1}{\epsilon} \left[ \frac{r \sin \theta}{r^2 \sin \theta} \left\{ \frac{\partial}{\partial r} (r H_{\theta}) - \frac{\partial}{\partial \theta} (H_r) \right\} \right] dt$$

$$E_{\phi} = 0$$

Ques Derive electric field and magnetic field components and explain the significance of different terms in the derived eqn.

$$H_{\phi} = \frac{I_0 l \sin \theta}{4\pi} \left[ \frac{\cos \omega t'}{r^2} - \frac{\omega \sin \omega t'}{v r} \right]$$

$$E_r = \frac{I_0 l \cos \theta}{2\pi \epsilon} \left[ \frac{\sin \omega t'}{r^3 \omega} + \frac{\cos \omega t'}{r^2 v} \right]$$

$$E_{\theta} = \frac{I_0 l \sin \theta}{4\pi \epsilon} \left[ \frac{\sin \omega t'}{r^3 \omega} + \frac{\cos \omega t'}{r^2 v} - \frac{\omega \sin \omega t'}{v^2 r} \right]$$

# The term which varies inversely as distance ( $1/r$ ) is k/a Far field or Radiation Field which accounts for radiation of EM waves.

# The term which varies inversely as square of distance ( $1/r^2$ ) is k/a near field or Induction Field. This field will predominate at points close to current carrying elements where  $r$  is small. In distance where  $r$  is large the effect becomes negligible compared to radiation field.

field and exists only at the close periphery of current carrying elements.

\* Power Radiated by Alternating current element.

Power radiated by a.c element is given by Poynting vector.

$$\vec{P} = \vec{E} \times \vec{H} = \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{\phi} \\ E_r & E_{\theta} & E_{\phi} \\ H_r & H_{\theta} & H_{\phi} \end{vmatrix}$$

$$P_r = E_{\theta} H_{\phi} - \cancel{H_{\theta} E_{\phi}}^{\nearrow 0} \quad [\because H_{\theta} = 0, E_{\phi} = 0]$$

$$= \frac{I_0 l \sin \theta}{4\pi \epsilon} \left[ \frac{\sin \omega t'}{r^3 \omega} + \frac{\cos \omega t'}{r^2 v} - \frac{\omega \sin \omega t'}{v^2 r} \right] \times$$

$$\frac{I_0 l \sin \theta}{4\pi} \left[ \frac{\cos \omega t'}{r^2} - \frac{\omega \sin \omega t'}{v r} \right]$$

(3 step) ↓

$$P_r = \frac{I_0^2 l^2 \sin^2 \theta}{16\pi^2 \epsilon} \left[ \frac{\cos 2\omega t'}{r^4 v} - \frac{\omega \sin 2\omega t'}{r^3 v^2} + \frac{\sin 2\omega t'}{2\omega r^5} + \frac{\omega^2 (1 - \cos 2\omega t')}{2 r^2 v^3} \right]$$

$$P_{\theta} = -E_r H_{\phi} + \cancel{E_{\phi} H_r}^{\nearrow 0}$$

$$= - \frac{I_0 l \cos \theta}{2\pi \epsilon} \left[ \frac{\sin \omega t'}{r^3 \omega} + \frac{\cos \omega t'}{r^2 v} \right] \cdot \left[ \frac{I_0 l \sin \theta}{4\pi} \left[ \frac{\cos \omega t'}{r^2} - \frac{\omega \sin \omega t'}{v r} \right] \right]$$

$$\frac{I_0 l \sin \theta}{4\pi} \left[ \frac{\cos \omega t'}{r^2} - \frac{\omega \sin \omega t'}{v r} \right]$$

$$P_{\theta} = \frac{I_0^2 l^2 \sin 2\theta}{16\pi^2 \epsilon} \left[ \frac{\omega \sin \omega t'}{2 r^3 v^2} - \frac{\cos 2\omega t'}{r^4 v} - \frac{\sin 2\omega t'}{2\omega r^5} \right]$$

$$P_{\phi} = \cancel{E_r H_{\theta}}^{\nearrow 0} - \cancel{E_{\theta} H_r}^{\nearrow 0}$$

$$P_{\phi} = 0$$

average power contributed by  $\sin 2\omega$  &  $\cos 2\omega$  is zero. So the only term contributing to power is

$$\frac{I_0^2 \ell^2 \sin^2 \theta}{16 \pi^2 \epsilon} \left[ \frac{\omega^2}{2 r^2 v^3} \right] \text{ of } P_r$$

we get

$$P_r = \frac{I_0^2 \ell^2 \sin^2 \theta}{16 \pi^2 \epsilon} \frac{\omega^2}{2 r^2 v^3}$$

$$P_\theta = 0$$

$$P_\phi = 0$$

# Thus the avg power is only in r direction

$$\therefore P_{avg} = \left( \frac{I_0 \ell \sin \theta \omega}{4 \pi r v} \right)^2 \cdot \frac{1}{2 \epsilon v}$$

# For Free space  $\epsilon = \epsilon_0$  &  $v = v_0$

$$\epsilon_0 v_0 = \epsilon_0 \cdot \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \sqrt{\frac{\epsilon_0}{\mu_0}} = \frac{1}{\eta_0} \quad \eta_0 = \text{Intrinsic impedance}$$

$$\therefore P_{avg} = \frac{\eta_0}{2} \left( \frac{I_0 \ell \sin \theta \omega}{4 \pi r v} \right)^2 \text{ watt/m}^2$$

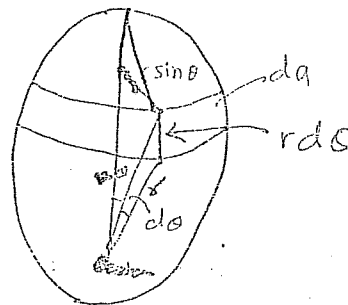
# Total power radiated can be obtained by integrating average power.

$$P_{total} = \oint P_{avg} \cdot da$$

$$= \pi \int_0^\pi \frac{\eta_0}{2} \left( \frac{I_0 \ell \sin \theta \omega}{4 \pi r v} \right)^2 2 \pi r^2 \sin \theta d\theta$$

$$= \frac{\eta_0}{2} 2 \pi r^2 \left( \frac{I_0 \ell \omega}{4 \pi r v} \right)^2 \int_0^\pi \sin^3 \theta d\theta$$

$$P_{total} = \frac{\eta_0 \omega^2 I_0^2 \ell^2}{12 \pi v^2}$$



$$da = 2 \pi r \sin \theta \cdot r d\theta = 2 \pi r^2 \sin \theta d\theta$$

$$da = 2 \pi r^2 \sin \theta d\theta$$

$$\text{Again } \left( \frac{\omega}{v} \right)^2 = \left( \frac{2 \pi f}{\lambda} \right)^2 = \frac{4 \pi^2}{\lambda^2}$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120 \pi$$

$$P_{\text{total}} = \frac{80\pi^2 I_0^2}{d^2}$$

current

In terms of effective or rms value.

$$I_0 = I_{\text{eff}} \sqrt{2}$$

$$P_{\text{total}} = \frac{80\pi^2 \rho^2}{d^2} (I_{\text{eff}})^2$$

$$(I_{\text{eff}})^2 R_{\text{rad}} = \frac{80\pi^2 \rho^2}{d^2} (I_{\text{eff}})^2$$

$$\therefore R_{\text{rad}} = \frac{80\pi^2 \rho^2}{d^2}$$

where  $R_{\text{rad}}$  is Radiation Resistance.

It is defined as the fictitious resistance which when inserted in series with antenna will consume same amount of power as is actually radiated.

- 3 Dipole  $< \lambda/50$  infinitesimal dipole (Hertzian dipole)
- $\approx \lambda/50$  to  $\lambda/10$  small dipole
- $> \lambda/10$  Large dipole.

### \* Input Impedance of Long Antenna / Long dipole

2010 Fall Q Derive the expn for power radiated by half wave dipole.

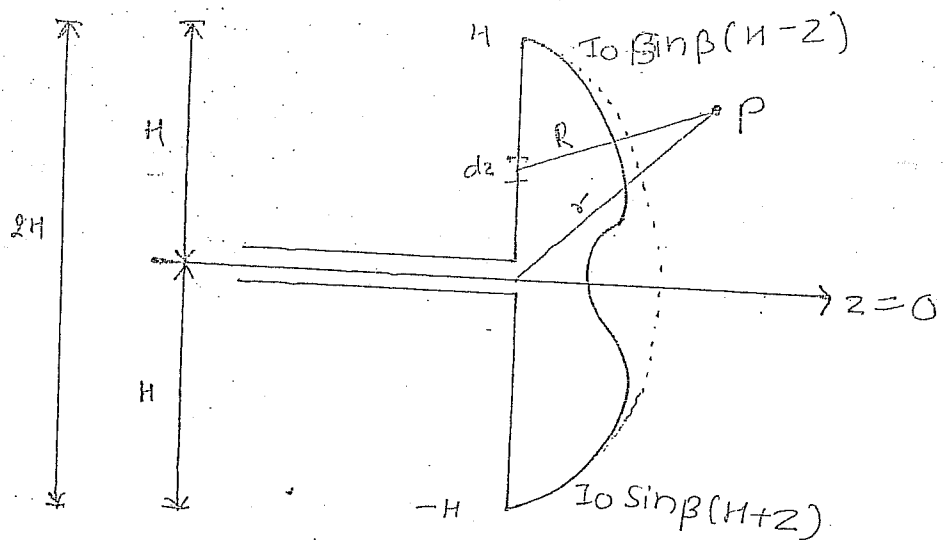


fig: center fed dipole with sinusoidal current distribution.

# Long Antenna is the antenna which

i) has length  $> \lambda/10$

ii) carries current distribution as

$$I = \begin{cases} I_0 \sin \beta(H-z) & \text{for } z > 0 \\ I_0 \sin \beta(H+z) & \text{for } z < 0 \end{cases}$$

where

$H =$  half of length of antenna

$z =$  point on antenna wrt which effect is considered

$\beta =$  phase constant  $= \frac{2\pi}{\lambda}$

$$A_z = \oint \frac{\mu I \cos \omega t'}{4\pi R} dz$$

here  $\omega t' = 2\pi f t' = 2\pi \frac{v}{\lambda} t' = \beta R$   $\left[ \begin{array}{l} \beta = 2\pi/\lambda \\ v \times t = R \end{array} \right]$

with assumption that we take only real part of  $e^{-j\beta R}$  we have,

$$A_z = \oint \frac{\mu I e^{-j\beta R}}{4\pi R} dz$$

# Total vector potential at P due to all current element

$$\begin{aligned} A_z &= \frac{\mu}{4\pi} \left[ \int_{-h}^0 \frac{I e^{-j\beta R}}{R} dz + \int_0^h \frac{I e^{-j\beta R}}{R} dz \right] \\ &= \frac{\mu}{4\pi} \left[ \int_{-h}^0 \frac{I_0 \sin \beta(h+z) e^{-j\beta R}}{R} dz + \int_0^h \frac{I_0 \sin \beta(h-z) e^{-j\beta R}}{R} dz \right] \end{aligned}$$

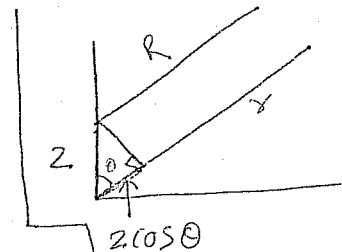
# Since P is at larger distance we can assume  $R \approx r$  in case of denominator

But in case of R in the phase factor lines R & r will be essential

$$\therefore R = r - z \cos \theta$$

# So we will have

$$A_z = \frac{\mu I_0}{4\pi r} \left[ \int_{-h}^0 \sin \beta(h+z) e^{-j\beta(r-z \cos \theta)} dz + \int_0^h \sin \beta(h-z) e^{-j\beta(r-z \cos \theta)} dz \right]$$



$$\therefore r = R + z \cos \theta$$

$$\begin{aligned} &= \frac{\mu I_0 e^{-j\beta r}}{4\pi r} \left[ \int_{-h}^0 \sin \beta(h+z) e^{j\beta z \cos \theta} dz + \int_0^h \sin \beta(h-z) e^{j\beta z \cos \theta} dz \right] \end{aligned}$$

# For  $\lambda/2$  Antenna, (half wave antenna)

$$L = 2h = \lambda/2$$

$$\therefore h = \lambda/4$$

$$\sin \beta(h+z) = \sin \frac{2\pi}{\lambda} \left( \frac{\lambda}{4} + z \right) = \sin \left( \frac{\pi}{2} + \beta z \right) = \cos \beta z$$

Similarly

$$\sin \beta(h-z) = \cos \beta z$$

$$A_z = \frac{\mu I_0 e^{-j\beta r}}{4\pi r} \left[ \int_{-h}^0 \cos \beta z e^{j\beta z \cos \theta} dz + \int_0^h \cos \beta z e^{j\beta z \cos \theta} dz \right]$$

On Solving, (9 steps)

$$A_z = \frac{\mu I_0 e^{-j\beta r}}{2\pi r \beta} \left[ \frac{\cos \left( \frac{\pi}{2} \cos \theta \right)}{\sin^2 \theta} \right]$$

we know that radiated power is only in  $H_\phi$  direction  
using maxwell's equation

$$H_\phi = \frac{1}{\mu} \left[ -\frac{d}{dr} (A_z \sin \theta) \right] \quad [\because H_\phi = \frac{1}{\mu} \left[ \frac{1}{r} \frac{\partial}{\partial r} (A_\theta) - \frac{\partial}{\partial \theta} (A_r) \right]]$$

$$= \frac{1}{\mu} \left( \frac{-\mu I_0}{2\pi \beta} \right) \left[ \frac{\cos \left( \frac{\pi}{2} \cos \theta \right)}{\sin \theta} \right] \frac{d}{dr} \left( \frac{e^{-j\beta r}}{r} \right)$$

$$= -\frac{I_0}{2\pi \beta} \left[ \frac{\cos \left( \frac{\pi}{2} \cos \theta \right)}{\sin \theta} \right] \frac{e^{-j\beta r} \cdot (-j\beta)}{r^2}$$

$$H_\phi = \frac{j I_0 e^{-j\beta r}}{2\pi r} \left[ \frac{\cos \left( \frac{\pi}{2} \cos \theta \right)}{\sin \theta} \right]$$

Now,

$$E_\theta = \eta H_\phi$$

$$\therefore E_\theta = \frac{j \eta I_0 e^{-j\beta r}}{2\pi r} \left[ \frac{\cos \left( \frac{\pi}{2} \cos \theta \right)}{\sin \theta} \right]$$

$$|H_\phi|_{\text{peak}} = \frac{I_0}{2\pi r} \left[ \frac{\cos(\pi/2 \cos \theta)}{\sin \theta} \right]$$

$$|E_\theta|_{\text{peak}} = \frac{\eta I_0}{2\pi r} \left[ \frac{\cos(\pi/2 \cos \theta)}{\sin \theta} \right]$$

# Peak value of Poynting vector is the product of peak values of  $E_\theta$  &  $H_\phi$  :

$$P_{\text{peak}} = |E_\theta| |H_\phi|$$

# Thus avg value of power

$$P_{\text{avg}} = \frac{|E_\theta|}{\sqrt{2}} \cdot \frac{|H_\phi|}{\sqrt{2}} = \frac{1}{2} |E_\theta| |H_\phi|$$

$$= \frac{1}{2} \frac{\eta I_0^2}{4\pi^2 r^2} \left[ \frac{\cos(\pi/2 \cos \theta)}{\sin \theta} \right]^2$$

$$= \frac{\eta I_0^2}{8\pi^2 r^2} \left[ \frac{\cos(\pi/2 \cos \theta)}{\sin \theta} \right]^2$$

# Total power radiated is

$$P_{\text{total}} = \int P_{\text{avg}} \cdot d\Omega$$

$$= \frac{\eta I_0^2}{8\pi^2 r^2} \int_0^\pi \frac{\cos^2(\pi/2 \cos \theta)}{\sin^2 \theta} 2\pi r^2 \sin \theta d\theta$$

$$= \frac{\eta I_0^2}{4\pi} \int_0^\pi \frac{\cos^2(\pi/2 \cos \theta)}{\sin \theta} d\theta$$

$$= \frac{\eta I_0^2}{4\pi} \times 1.219$$

here  $\eta = 120\pi$  &  $I_0 = \sqrt{2} I_{\text{eff}}$

$$\therefore P_{\text{total}} = 60 I_{\text{eff}}^2 \times 1.219$$

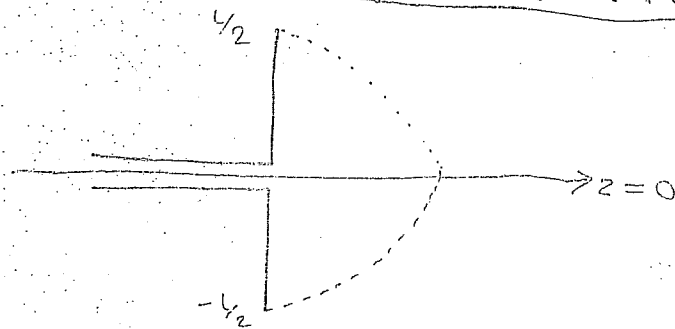
$$\therefore P_{\text{total}} = 73.14 I_{\text{eff}}^2$$

$$R_{\text{rad}} \cdot I_{\text{eff}}^2 = 73.14 I_{\text{eff}}^2$$

$$R_{\text{rad}} = 73.14 \Omega$$



## Input Impedance of Short Antenna



2024 Define sw

# Short Antenna is the one which

- i) has length  $\lambda/50 < l < \lambda/10$
- ii) carries current distribution as

$$I(z) = I_0 \left(1 - \frac{2|z|}{L}\right)$$

# The current distribution is triangular as shown in fig

# For short Antenna,  $I_{eff} = I_{eff}/2$  as that for Infinitesimal small antenna.

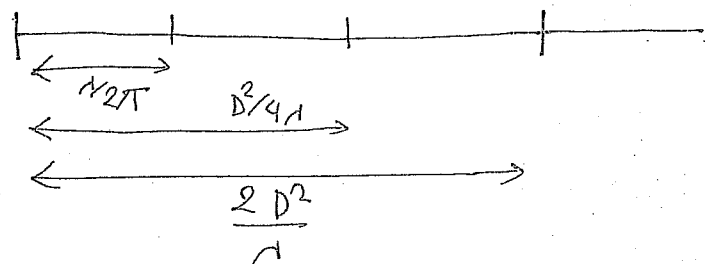
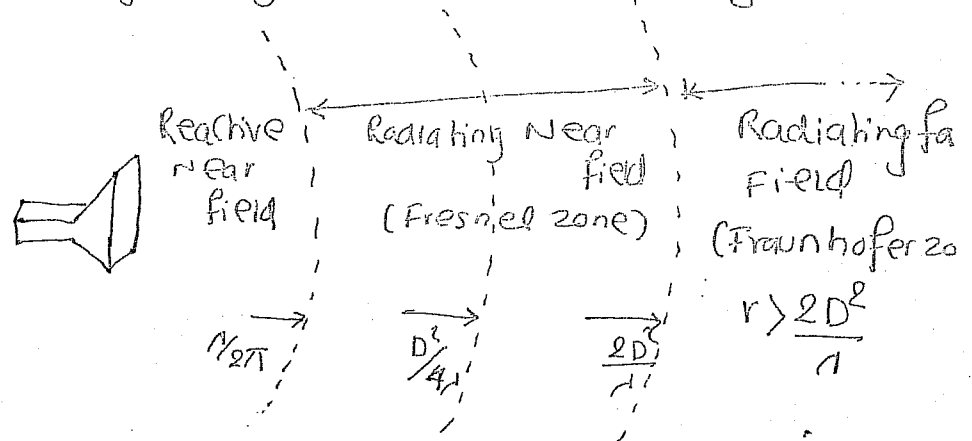
$$P_{total} = \frac{80\pi^2 l^2}{\lambda^2} \cdot (I_{eff})^2 \quad \text{for infinitesimal antenna}$$

$$= \frac{80\pi^2 l^2}{\lambda^2} \left(\frac{I_{eff}}{2}\right)^2$$

$$P_{total} = \frac{20\pi^2 l^2}{\lambda^2} (I_{eff})^2$$

$$\therefore R_{rad} = \frac{20\pi^2 l^2}{\lambda^2}$$

- # The radiation field from the transmitting antenna is characterized by the complex Poynting vector  $E \times H^*$  in which  $E$  is the electric field and  $H$  is magnetic field.
- # Close to the antenna the Poynting vector is imaginary (reactive) and  $E, H$  decays more rapidly than  $1/r$  whereas further away it is real (radiating) &  $E, H$  decays as  $1/r$ .
- # Based on these characteristics of Poynting we can identify three major regions as shown in fig.



### (1) Reactive Field

- # This is the region in space immediately surrounding antenna.
- # This region extends from  $0 < r < 1/2\lambda$
- # In this space the Poynting vector is predominantly reactive (non radiating) [has all three components in spherical coordinates  $(r, \theta, \phi)$ ] & decays more rapidly than  $1/r$ .

### ② Radiating Near Field

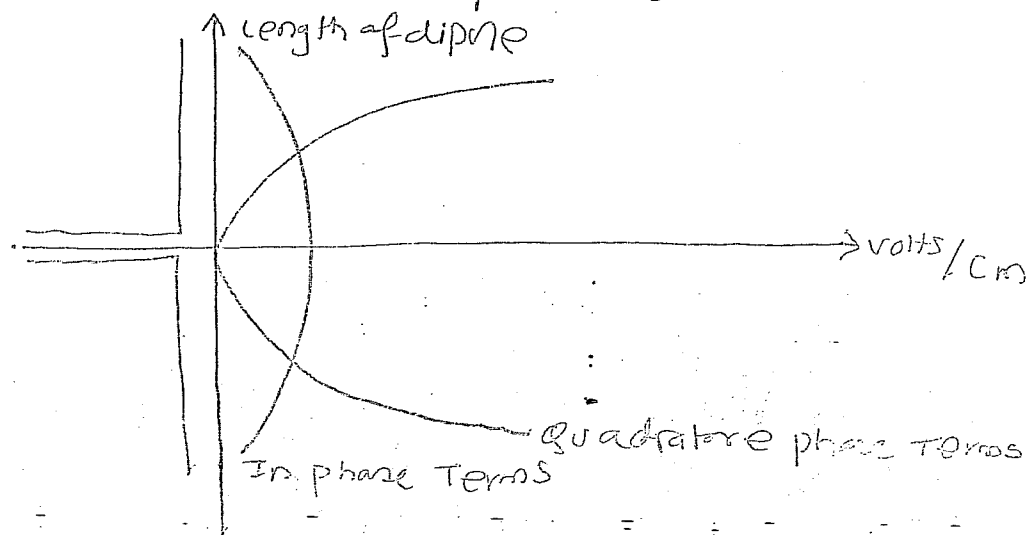
- # After the reactive field radiating field begins to dominate
- # This region extends from  $\sqrt{2\pi} < r < 2D^2/\lambda$  where  $D$  is largest dimension of the antenna.
- # This region is often referred to as Fresnel zone
- # This region is divided into two subregions.
  - i) For  $\sqrt{2\pi} < r < D^2/\lambda \Rightarrow$  The field decay more rapidly than  $1/r$  & radiation pattern is dependent on  $r$
  - ii) For  $D^2/\lambda < r < 2D^2/\lambda \Rightarrow$  The field decay as  $1/r$  but radiation pattern is dependent on  $r$

### ③ Radiating Far Field

- # Beyond the radiating near field is  $r > 2D^2/\lambda$  the Poynting vector is real (only radiating field)
- # The field decay as  $1/r$  and the radiation pattern is independent of  $r$ .
- # This region is often referred as Fraunhofer zone

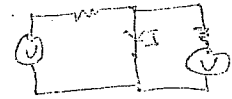
### # In phase & Quadrature phase Terms:

- # quadrature terms  $90^\circ$  phase diff



## ① Superposition Theorem

# In a n/w of generator & impedances, the current flowing at any point is the sum of currents that would flow if each generators were considered separately, all the other generators being replaced at the time by impedances equal to their internal impedance.



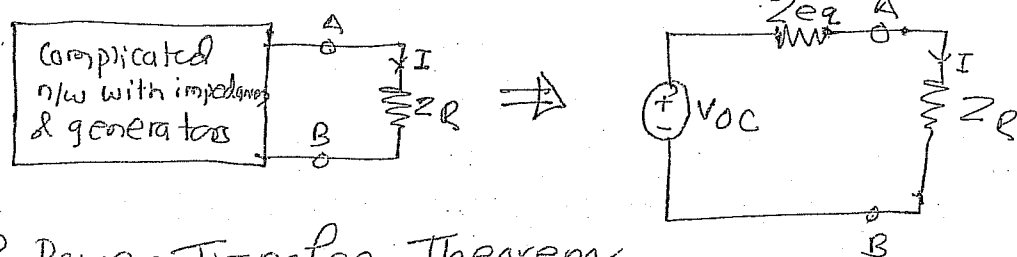
## ② Thevenin's Theorem

# In a n/w consisting of one or more generator & impedances, the current flowing through load impedance  $Z_L$  is same as obtained by replacing original network across  $Z_L$  with an eq. n/w consisting of single voltage source  $V_{oc}$  (open ckt voltage) and impedance  $Z_{eq}$  in series

where

$V_{oc}$  is open ckt voltage measured across  $Z_L$

$Z_{eq}$  is impedance measured at same open terminal looking back into the n/w replacing all the energy source by their respective internal impedance.



## ③ Max<sup>m</sup> Power Transfer Theorem

# In any network, the max<sup>m</sup> power is transferred to the Load  $Z_L$  by the generator if the Load impedance  $Z_L$  is complex conjugate of the equivalent impedance of the n/w  $Z_{eq}$  measured looking back into the n/w from the  $Z_L$  terminal.

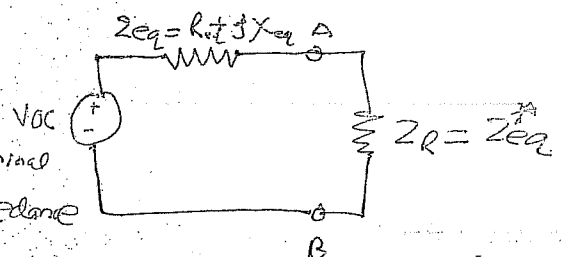
# The max<sup>m</sup> power transferred is

$$P_{max} = \frac{V_{oc}^2}{4R}$$

$V_{oc}$  = open ckt voltage from Load Terminal



$R$  = Resistive component of eq Impedance seen from Load terminal.

$$\text{If } Z_{eq} = R + jX \\ Z_L = R - jX$$



#### ④ Compensation Theorem

# Any impedance in a n/w may be replaced by a generator of zero internal impedance such that generated voltage at every instant is equal to instantaneous p.d that existed across the impedance because of current flowing through it.

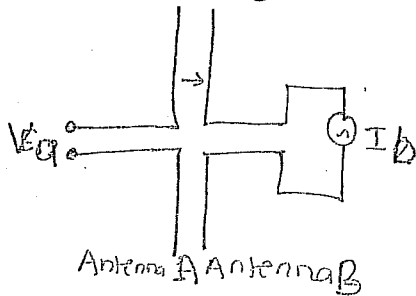
$V = IR$ 

 $\approx$ 


## Reciprocity Theorem

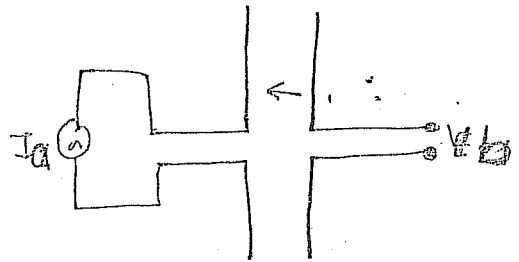
2006 2007 2009 2009

# This theorem establishes the equivalency of Transmitting and receiving Antenna. i.e. same antenna can be used as transmitter & receiver antenna (eg RADAR antenna)

# Reciprocity Theorem states that If an emf is applied to the terminal of antenna A and current measured at terminal of antenna B then an equal current both in amplitude and phase will be obtained at terminal of antenna A if same emf is applied to terminal of antenna B



Hg ⑨



Antenna A    Antenna B

If  $V_a = V_b \Rightarrow I_a = I_b$   
The eqn can be drawn as.

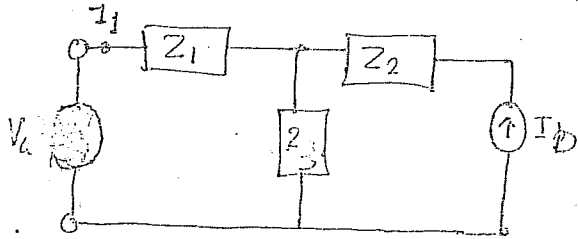


Fig (c) eq T n/w of fig a

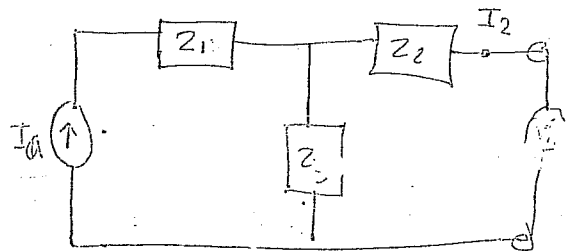


fig (d) eq 7: n/w of fig 10

#  $Z_{11}$  &  $Z_{22}$  are self impedance of Antenna 1 & 2  
and  $Z_{12}$  is mutual impedance bet<sup>n</sup> two antenna

$$I_2 Z_{22} + Z_m (I_2 - I_1) = 0$$

$$I_2 (Z_m + Z_{22}) = Z_m I_1$$

$$\boxed{I_2 = \left( \frac{Z_m}{Z_m + Z_{22}} \right) I_1} \quad \text{--- (1)}$$

From Loop 1

# From fig c

$$I_b = I_1 \left( \frac{Z_3}{Z_2 + Z_3} \right) \quad \text{--- (1) from current division}$$

where

$$I_1 = \frac{V_a}{Z_1 + [Z_2 Z_3 / (Z_2 + Z_3)]} = \frac{V_a (Z_2 + Z_3)}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3} \quad \text{--- (2)}$$

From (1) & (2)

$$\boxed{I_b = \frac{V_a Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}} \quad \text{--- (3)}$$

Similarly from n/w of fig d.

$$\boxed{I_a = \frac{V_b Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}} \quad \text{--- (4)}$$

comparing (3) & (4)

If  $V_a = V_b$  Then  $I_a = I_b$  which proves reciprocity theorem.

## \* Application of N/w Theorem to antenna

### (1) Equality of Directional Pattern

→ The directional pattern of receiving antenna is identical with the directional pattern as a transmitting antenna.

### (2) Equivalence of Transmitting and Receiving Antenna Impedances

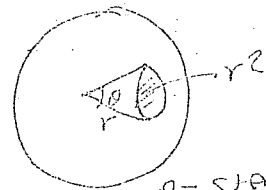
### (3) Equivalence of effective lengths

ANTENNA FUNDAMENTALS

By:

Rejan  
Sharma  
Nec

- # The measure of a solid angle is steradian.  
Since the area of a sphere of radius  $r$  is  $4\pi r^2$ .  
There are  $4\pi$  steradian in a closed sphere.

Radiation Power Density

- # It is the radiation power per unit area ( $\text{W}/\text{A}$ )  
# The quantity used to describe the power associated with EM wave is Poynting vector which gives the power density.

$$\vec{P} = \vec{E} \times \vec{H}$$

Radiation Intensity

- # Radiation Intensity is defined as the power radiated from antenna per unit solid angle.

- # It is denoted by  $U$  and mathematically,

$$U = r^2 \cdot P$$

$$\left( U = \frac{W}{4\pi}, P = \frac{W}{4\pi r^2}, W = P \cdot 4\pi r^2 \right)$$

where  $P$  = power density

for isotropic.

Isotropic Radiator

- # An isotropic radiator is an ideal source that radiate equally in all direction.  
# Although it doesn't exists in practice, it provides a convenient isotropic reference with which to compare all antennas.  
# Total power radiated.

$$W = \oint \vec{P} \cdot d\vec{s} = 4\pi r^2 P_0$$

& power density is  $P_0 = \frac{W}{4\pi r^2}$

$\therefore$  Radiation Intensity of isotropic radiator

$$U_0 = r^2 \cdot P_0 = \frac{W}{4\pi}$$



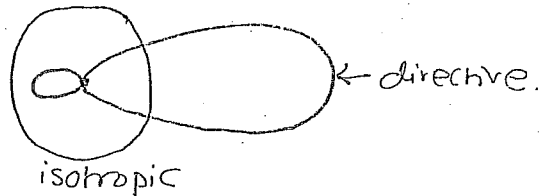
## Directivity

- # Directivity is the ability of an antenna to focus energy in a particular direction.
- # All the practical antennas concentrate their radiated energy in particular direction.
- # The degree to which a practical antenna concentrates the radiated energy relative to that of isotropic antenna is termed as directivity.
- # mathematically Directivity is the ratio of max<sup>m</sup> radiation intensity of an antenna to the radiation intensity of isotropic antenna.

$$D = \frac{U_{\max}}{U_0}$$

$$D = \frac{4\pi U_{\max}}{W}$$

- # Directivity is the max<sup>m</sup> directive gain of an antenna.



## \* Antenna Gain

- # The hypothetical isotropic antenna radiates equally in all direction. Any real antenna will radiate more energy in some directions than in others.
- # The gain of an antenna in a given direction is the amount of energy radiated in that direction compared to the energy an isotropic antenna would radiate in the same direction when driven with same input power.

## 2008 \* Directive Gain

- # Directive gain may be defined as the ratio of radiation intensity in a given direction from an antenna to the radiation intensity of isotropic antenna.

$$G_d = \frac{U}{U_0} = \frac{4\pi U}{W}$$

$$\therefore U_0 = \frac{W}{4\pi}$$

- # max<sup>m</sup> directive gain of an antenna is its directivity.

# power gain is the ratio of a test antenna to the radiated power density of an isotropic antenna with same input power to both.

$$G_p = \frac{P}{P_0}$$

Also

$$G_p = \eta G_d$$

$\eta$  = efficiency of antenna  
 $G_p$  = Power gain  
 $G_d$  = Directive gain

Define  $\eta$   
 2009

### Antenna Efficiency (Radiation Efficiency)

# The efficiency of an antenna relates the power delivered to the antenna and the power radiated from antenna

# It is a parameter which indicates losses at the input terminal and within the str. of an antenna.

# The losses associated with antenna are

1) reflection because of mismatch bet<sup>n</sup> Tx line and antenna.

2)  $I^2 R$  losses (which is due to conduction & dielectric)

$$\eta = \frac{P_{\text{radiated}}}{P_{\text{input}}}$$

$$\eta = \eta_r \eta_{cd}$$

where  $\eta$  = overall efficiency

$\eta_r$  = reflection mismatch  $\eta$

$$\eta_r = (1 - |\Gamma|^2)$$

where

$\Gamma$  = voltage reflection coeff

$$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

$Z_{in}$  = Antenna i/p impedance

$Z_0$  = characteristics impedance of transmission line

$\eta_{cd}$  = conduction dielectric efficiency

$$\eta_{cd} = \frac{R_r}{R_r + R_L}$$

where

$R_r$  = Radiation resistance

$R_L$  = Loss

For numerical

$$\eta = \frac{P_{\text{radiated}}}{P_{\text{input}}} = \frac{I^2 R_r}{I^2 (R_r + R_L)} = \frac{R_r}{R_r + R_L}$$

gain 20 & directivity 22. calculate its radiation res.  
Soln

$$\text{Loss resistance } (R_L) = 10 \Omega$$

$$\text{Directivity } (G_d) = 22$$

$$\text{Power gain } (G_p) = 20$$

$$\text{Radiation Resistance } (R_r) = ?$$

we have

$$G_p = \eta G_d$$

$$20 = \eta \cdot 22$$

$$\boxed{\therefore \eta = 0.9091}$$

$$\text{Also } \eta = \frac{R_r}{R_r + R_L}$$

$$0.9091 = \frac{R_r}{R_r + 10}$$

$$R_r (1 - 0.9091) = 10$$

$$\boxed{\therefore R_r = 100.0110 \Omega}$$

Q An Antenna has a radiation resistance of  $72 \Omega$ ,  
Loss resistance of  $8 \Omega$  & Power gain of 12 dB.  
Determine Directivity & Efficiency.

Soln

$$R_r = 72 \Omega$$

$$R_L = 8 \Omega$$

$$\text{Power gain } (G_p) = 12 \text{ dB}$$

$$\text{Efficiency } (\eta) = \frac{R_r}{R_r + R_L} = \frac{72}{72 + 8}$$

$$\therefore \eta = 0.9$$

we have,

$$G_p = \eta G_d$$

$$G_d = \frac{G_p}{\eta} = \frac{12}{0.9} = 13.3333$$

Ans

is 15 ohms antenna efficiency.

Soln,

$$R_L = 15 \Omega$$

Since dipole is  $\lambda/15$  it is short antenna

For short antenna,

$$R_r = \frac{20\pi^2 l^2}{\lambda^2}$$

$$= \frac{20\pi^2 (\lambda/15)^2}{\lambda^2}$$

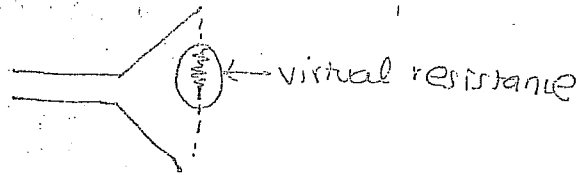
$$= 0.8764$$

$$\eta = \frac{R_r}{R_r + R_L} = \frac{0.8764}{15 + 0.8764} = 0.055$$

$$\eta = 5.5\%$$

## Radiation Resistance

# Radiation Resistance is defined as the fictitious resistance which when inserted in series with antenna will consume same amount of power as is actually radiated.



# For Infinitesimal short antenna

$$R_{rad} = \frac{80\pi^2 \ell^2}{\lambda^2}$$

For short antenna,  $R_{rad} = \frac{20\pi^2 \ell^2}{\lambda^2}$

For long antenna,  $R_{rad} = 73.14 \Omega$

# Actually Antenna will have measured ~~ohmic~~ resistance as,

$$R_t = R_r + R_f$$

$R_t$  = Total Antenna resistance

$R_r$  = Radiation resistance

$R_f$  = ohmic resistance

# Since ohmic resistance gives rise to power loss, For efficient radiation purpose, the radiation resistance must be very much higher than ohmic resistance.

## \* Bandwidth

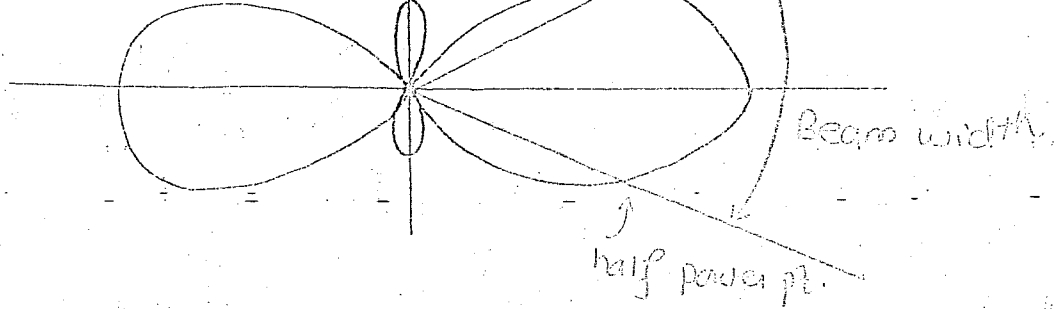
# The Bandwidth of an antenna refers to the range of frequencies over which the antenna can operate correctly.

## 0008 \* Antenna Beamwidth

# The Beamwidth is a measure of directivity of an antenna. The antenna beamwidth is defined as the angular separation between two half power points on radiation pattern of an antenna.

# Also k/a half power beamwidth or 3 dB beamwidth.

# Thus beamwidth is angular separation bet<sup>n</sup> two 3 dB points.

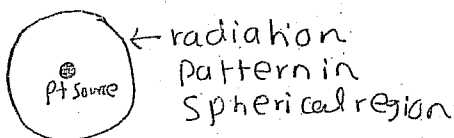


- # The beamwidth of antenna is a very important factor & it is often used as a trade off between beamwidth and side lobe level. i.e. As beam width decreases side lobe increases and vice versa.

V.V.V. 100%

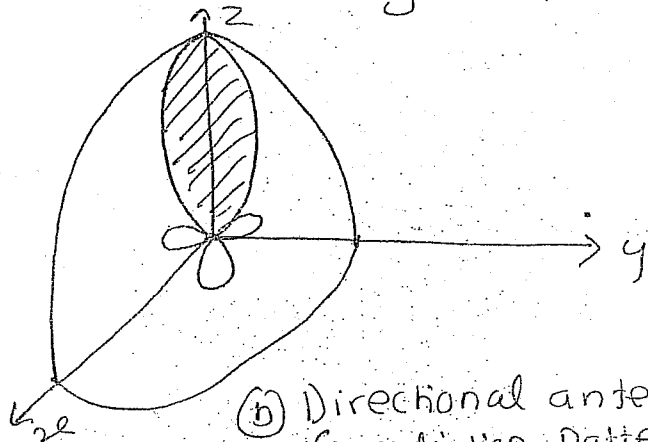
## \* RADIATION PATTERN

- # The radiation pattern of an antenna is a graphical representation of the radiation of the antenna as a function of direction.
- # The radiation pattern may be the field strength pattern or Power pattern. When the radiation pattern is expressed as Field strength  $E$ , the radiation pattern is expressed as Field strength pattern and when it is expressed in terms of power per unit solid angle the resulting pattern is power pattern.
- # ~~the~~ radiation pattern of different types of antenna is shown in fig.



### (a) Isotropic Antenna

↳ Radiates uniformly in all direction



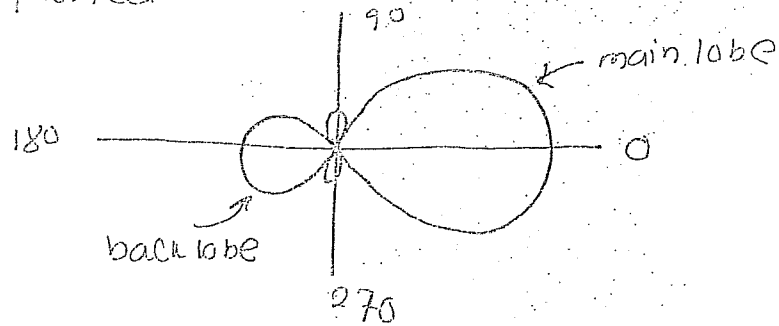
### (b) Directional antenna

↳ radiation pattern is oriented in certain direction

the radiation pattern can be plotted in two different ways.

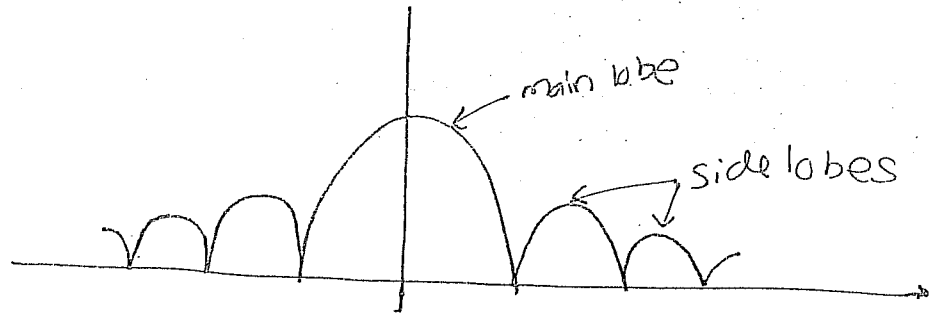
① polar plot

Here angle  $\theta$  & magnitude of power in radius are plotted



② Linear Plot

In linear plot, the angle is plotted in one of the axis and magnitude of power in another.



2007  
2009

Prove directivity of current element is 1.76 dB (7/15)

Consider a current element of Length  $L$ . Taking the magnitude of max<sup>m</sup> value of distant electric field in the direction of max<sup>m</sup> radiation, we have

$$E = \frac{I l \omega}{4\pi \epsilon_0 r^2}$$

$$= \frac{I l \omega}{4\pi \gamma_0} \times \eta$$

$$\therefore \eta = \frac{1}{\epsilon_0} = \text{Intrinsic Impedance}$$

$$= \frac{I l \cancel{2\pi f}}{4\pi \cancel{r} \cancel{1f}} \times 120\pi$$

$$\therefore \begin{aligned} v &= \lambda f \\ \omega &= 2\pi f \end{aligned}$$

$$E = \frac{60\pi I}{r} \left( \frac{l}{d} \right) \quad \text{--- (1)}$$

we have,

$$\text{radiated power, } W_{\text{rad}} = I^2 R_{\text{rad}} \Rightarrow I = \sqrt{\frac{W_{\text{rad}}}{R_{\text{rad}}}}$$

Let radiated power be 1 watt

$$I = \frac{1}{\sqrt{R_{\text{rad}}}} \quad \& \text{ For current element } R_{\text{rad}} = \frac{80\pi^2 l^2}{\lambda^2}$$

$$I = \frac{1}{\sqrt{80\pi} \cdot (l/\lambda)} \quad \text{--- 2}$$

From (1) & (2)

$$E = \frac{60\pi (l/\lambda)}{r} \cdot \frac{1}{\sqrt{80\pi} (l/\lambda)} = \frac{60}{r\sqrt{80}}$$

Now, max<sup>m</sup> Radiation Intensity is given by

$$\begin{aligned} U_{\text{max}} &= \gamma^2 \cdot P = \gamma^2 \cdot \frac{E^2}{\eta} \quad \left[ \because P = E \times \eta = E \times \frac{E}{\eta} \right] \\ &= \gamma^2 \cdot \left( \frac{60}{r\sqrt{80}} \right)^2 \cdot \frac{1}{120\pi} \end{aligned}$$

$$U_{\text{max}} = \frac{3}{8\pi}$$



$$D = \frac{4\pi U_{\max}}{W_{\text{rad}}} = 4\pi \times \frac{3}{8\pi} \quad [W_{\text{rad}} = 1]$$

$$\therefore \text{Directivity} = 1.5$$

or  
 $10 \log 1.5 = 1.76 \text{ dB}$

$$\text{Directivity} = 1.76 \text{ dB}$$

\* Directivity of Half wave Dipole

2009 Show Directivity of Half wave Dipole is 2.16 dB;

For Half wave Dipole, Electric field in max dir is

$$E = \frac{2I}{2\pi r} = \frac{60I}{r}$$

$$\eta = 120\pi$$

Let radiated power be 1 watt.

$$W_{\text{rad}} = I^2 R_{\text{rad}}$$

$$\therefore I = \frac{1}{\sqrt{R_{\text{rad}}}} = \frac{1}{\sqrt{73}}$$

$$\therefore \text{for half wave dipole } R_{\text{rad}} = 73\Omega$$

$$\therefore E = \frac{60}{\sqrt{73}r}$$

Next

max<sup>m</sup> radiation intensity is given by

$$U_{\max} = r^2 \cdot p = r^2 \frac{E^2}{\eta} = r^2 \cdot \left( \frac{60}{\sqrt{73}r} \right)^2 \cdot \frac{1}{120\pi}$$

$$\therefore U_{\max} = 0.131$$

Then

$$\text{Directivity} = \frac{4\pi U_{\max}}{W_{\text{rad}}} = 4\pi \times 0.131 = 1.646$$

$$\therefore \text{Directivity} = 10 \log 1.646$$

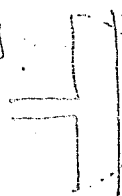
$$\text{Directivity} = 2.16 \text{ dB}$$

# The effective length of an antenna may be the ratio of induced voltage 'V' to the incident field 'E'

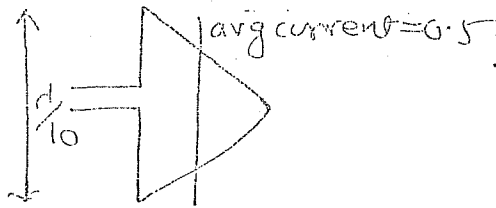
$$h_{eff} = \frac{V}{E} \text{ (m)}$$

# Effective height indicates how much of the antenna is involved in radiating (or receiving).

# It is defined as the length of antenna with a uniform and in phase current distribution along its entire length.

# Eg.  avg current = 0.64

height  $h = l/2$   
 $h_{eff} = 0.64 h$



height  $h = l/6$   
 $h_{eff} = 0.5 l$

# The effective length of an antenna is also given by the integral of normalized (avg) current distribution over the length of antenna

$$h_{eff} = \frac{1}{I_0} \int_0^h I(z) dz$$

$$h_{eff} = \frac{I_{avg}}{I_0} h_p$$

$h_p$  = physical height

$h_{eff}$  = effective "

$I_{avg}$  = avg current

$I_0$  = peak antenna current

# For an antenna of radiation resistance  $R_r$ , the power delivered to load is equal to

$$W = \frac{V^2}{4R_r} = \frac{h_e^2 E^2}{4R_r} \text{ (watt)} \quad \text{--- (1)}$$

# In terms of effective aperture same power is given by

$$W = P \cdot A_e \quad [ \because A_e = W/P ] \quad P = \text{Poynting vector of incident wave}$$

$$W = \frac{E^2}{2} A_e \quad [ \because P = \frac{E^2}{2} ] \quad \text{--- (2)}$$

From (1) & (2)

$$\frac{h_e^2 E^2}{4R_r} = \frac{E^2}{2} A_e$$

$$\therefore h_e = 2 \sqrt{\frac{R_r A_e}{2}} \text{ (m)}$$

$$\therefore A_e = \frac{h_e^2 2}{4R_r}$$

# Effective Area of an Antenna (Effective Aperture)

2009

2007

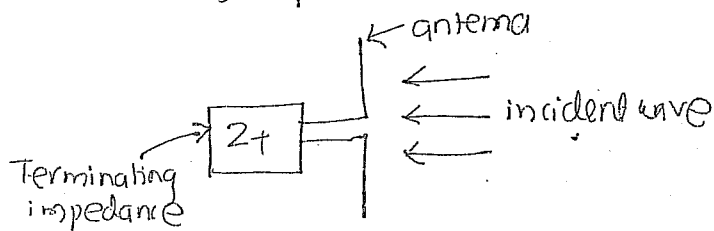
Define effective area

- # Also  $k_{\text{eff}}$  effective aperture or capture area
- # A transmitting antenna transmits E.M energy in space & receiving antenna receives a fraction of this EM energy.
- # The effective area of an antenna can be defined as the ratio of power received at the antenna load terminal to the Poynting vector (or power density) in watt/m<sup>2</sup> of incident wave

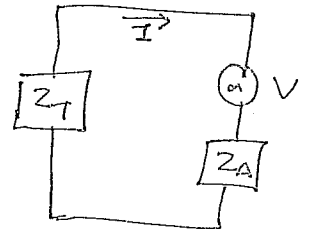
$$\therefore \text{Effective Area} = \frac{\text{Power Received}}{\text{Poynting vector of incident wave}}$$

$$\therefore A_e = \frac{W}{P} \quad \text{--- (1)}$$

- # consider a dipole receiving antenna situated in a field of an EM waves as shown in figa.



fig(a) Dipole antenna terminated with impedance  $Z_T$



fig(b) Eq circuit with antenna replaced by Thevenin's generator having eq voltage  $V$  & internal antenna impedance  $Z_A$

- # The voltage  $V$  is induced by passing E.M wave & produces current  $I$ , through Terminating impedance

- # From fig (b)

$$I = \frac{V}{Z_A + Z_T} \quad \text{--- (2)} \quad \text{where } V \text{ \& } I \text{ are effective or rms values.}$$

- # In general Terminating & Antenna impedance are complex.

$$\therefore Z_T = R_T + jX_T$$

$$Z_A = R_A + jX_A$$

- # Antenna resistance may be divided as radiation resistance  $R_r$  & non radiative or loss resistance  $R_L$

$$\therefore R_A = R_r + R_L$$

$$I = \frac{V}{\sqrt{(R_r + R_L + R_T)^2 + (X_A + X_T)^2}}$$

From ①

$A_e = \frac{W}{P}$  where  $W$  is power received i.e. power delivered by antenna to terminating impedance

$$W = I^2 R_T = \frac{V^2 R_T}{(R_r + R_L + R_T)^2 + (X_A + X_T)^2}$$

Thus

$$A_e = \frac{V^2 R_T}{P (R_r + R_L + R_T)^2 + (X_A + X_T)^2}$$

For max<sup>m</sup> power Transfer

$$X_T = -X_A$$

$$R_T = R_r + R_L$$

Then effective Area

$$A_e = \frac{V^2 (R_r + R_L)}{4P (R_r + R_L)^2}$$

If antenna is lossless then

$$A_{em} = \frac{V^2}{4PR_r}$$

$$= \frac{E^2 h_{eff}^2}{4 \frac{E^2}{\eta} R_r} \Rightarrow A_{em} = \frac{h_{eff}^2 \eta}{4 R_r} \quad \because V = E \cdot h$$

$A_{em}$  represents the area over which power is extracted from incident wave and delivered to load.

### \* Effective Area of Short Dipole

2005 Show that the max<sup>m</sup> effective area of any antenna is  $\frac{1.5}{4\pi} \eta^2$

The max<sup>m</sup> effective aperture of an antenna is,

$$A_{em} = \frac{V^2}{4PR_r}$$

$$\text{and } V = EL, P = \frac{E^2}{\eta}$$

$$\text{length } L = \frac{V}{E}$$

$$\therefore A_{em} = \frac{E^2 L^2}{4 \frac{E^2}{\eta} \cdot \frac{80\pi^2 L^2}{\eta^2}}$$

$$\therefore \text{For short dipole } R_r = \frac{80\pi^2 L^2}{\eta^2}$$

$$= \frac{120\pi}{320\pi^2} \eta^2$$

$$\therefore \eta = 120\pi$$

$$= \frac{3}{8\pi} \eta^2$$

$$\therefore A_{em} = 1.5 \frac{\eta^2}{4\pi}$$

For half wave dipole ,

$$V = \frac{E d}{\pi}$$

$$R_r = 73 \Omega, P = \frac{e^2}{\eta}$$

$$\begin{aligned} \therefore A_{em} &= \frac{V^2}{4 P R_r} \\ &= \frac{E^2 d^2}{\pi^2 \cdot 4 \frac{e^2}{\eta} \cdot 73} \\ &= \frac{120 \pi \cdot E^2 d^2}{4 \pi^2 e^2 \cdot 73} \end{aligned}$$

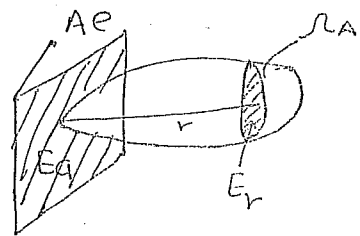
$$A_{em} = 0.13 d^2$$

\* Relation between <sup>(directivity)</sup> gain and effective area of an antenna

2001 Derive relation bet<sup>n</sup> gain & effective area.

# If the electric field in the aperture  $A_e$  be  $E_a$  then the power radiated is

$$P_{\text{radiated}} = \frac{|E_a|^2}{\eta} A_e \text{ (watts)} \quad (1)$$



# This can also be expressed in terms of received electric field  $E_r$  within the <sup>beam</sup> solid angle of antenna  $\Omega_A$  at distance

$$P_{\text{radiated}} = \frac{|E_r|^2}{\eta} r^2 \Omega_A \text{ (watts)} \quad (2)$$

# From (1) & (2)

$$P_{\text{radiated}} = \frac{|E_a|^2}{\eta} A_e = \frac{|E_r|^2}{\eta} r^2 \Omega_A \quad (3)$$

# For a constant field in the aperture, the relation bet<sup>n</sup> aperture field & field within beam area is given by

$$|E_r| = \frac{|E_a| A_e}{r^2 \Omega_A} \text{ (volts/meter)} \quad (4)$$

From (3) & (4)

$$P_{\text{radiated}} = \frac{|E_a|^2 A_e}{\eta} = \frac{|E_a|^2 A_e^2}{r^2 \Omega_A \eta} \Rightarrow \Omega_A = \frac{A_e}{r^2}$$

$$\text{Directivity} = \frac{4\pi}{\Omega_A}$$

$$\therefore D = \frac{4\pi A_{em}}{\Omega^2}$$

$A_{em}$  is max<sup>m</sup> effective aperture

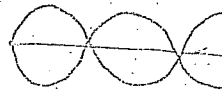
Also Gain =  $KD$  where  $K$  is constant

$$G = \frac{4\pi A_e}{\Omega^2}$$

$$K = \frac{A_e}{A_{em}}$$

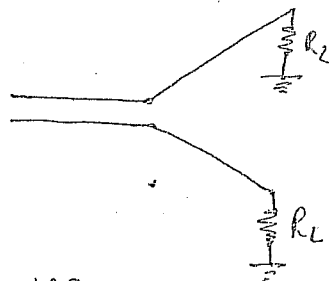
## Travelling wave antenna & Standing wave antenna

- # Standing wave antennas have standing wave of current & voltage on them.
- # In a transmitting Antenna of this type a progressive or travelling wave is supplied from the power source. When wave reaches the open end due to impedance mismatch & frequency change it is reflected.
- # The combination of the two waves sets up standing wave
- # Eg half wave dipole Antenna

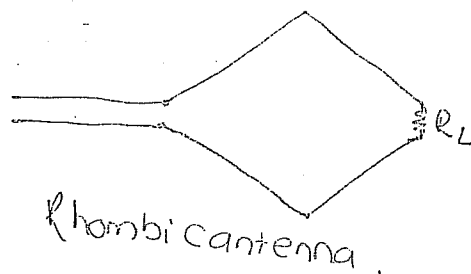
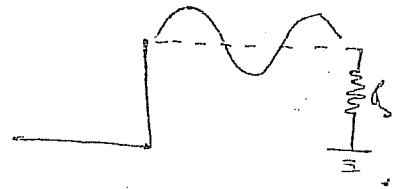


## Travelling Wave Antenna

- # Travelling wave Antennas has no standing waves.
- # This is accomplished by terminating the antenna in its characteristic impedance so that no reflections occurs
- # Eg. vee antennas, Rhombic antennas



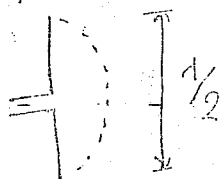
vee antenna



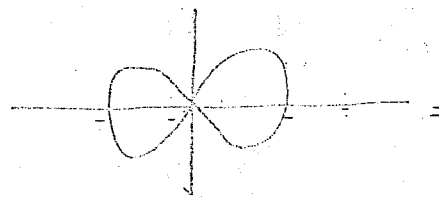
Rhombic antenna

→ Increase in length of antenna changes the shape of radiation pattern

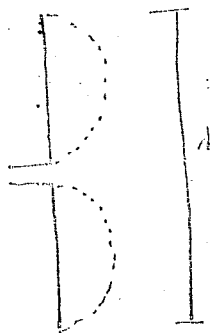
(a)



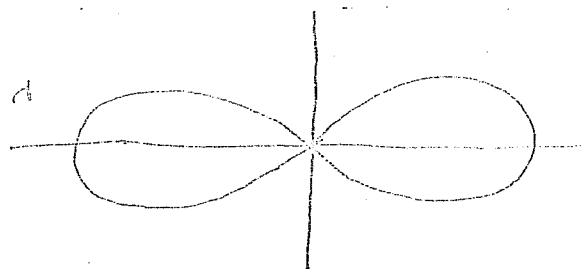
$$L = l/2$$



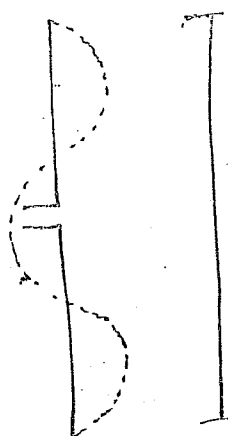
(b)



$$L = l$$

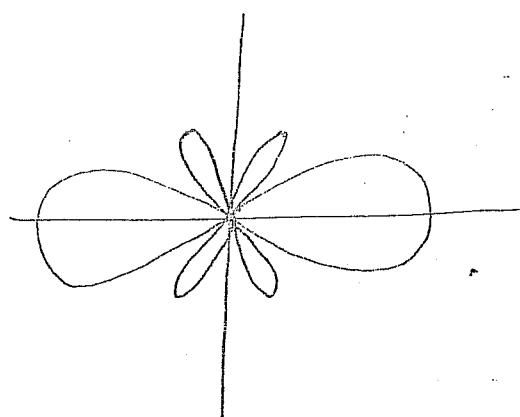


(c)

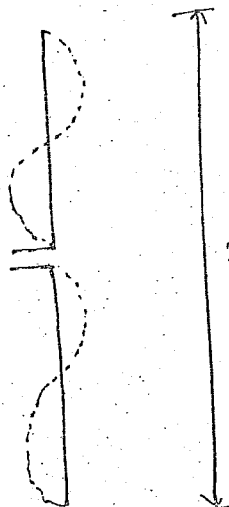


$$L = 3l/2$$

$$L = 3l/2$$

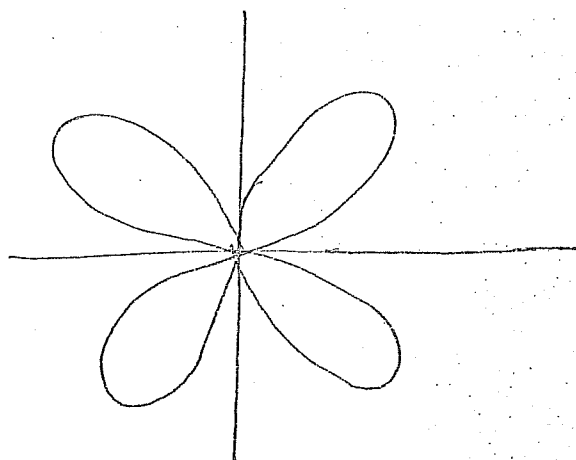


(d)



$$L = 2l$$

$$L = 2l$$



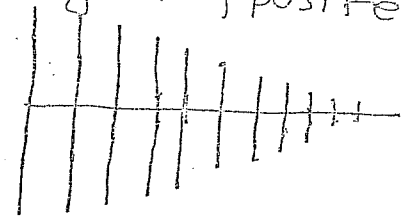


Ques What is antenna array. What are its importance

- # Antenna array is an assembly of radiating elements an electrical and geometrical configuration.
- # Total field of an array is the vector addition of field radiating from individual elements. It assumes current in each element is same as an isolated element.
- # The elements of array is arranged in such geometrical configuration & separation between elements so that field from element interfere constructively in desired direction and interferes destructively in opposite direction.

#### \* Advantages

- ① They provide the better directivity
- ② They provide high gain
- ③ Can generate different array pattern without changing its physical dimension (by exciting its elements with different currents)

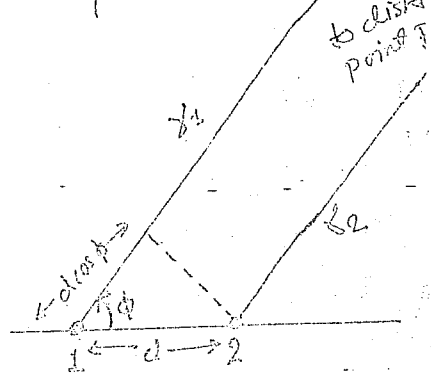


11 array antenna.

#### \* Various Forms of Antenna Array

- ① Broadside array (Radiation of array is normal to axis)
- ② End fire array (Radiation of array is along axis of array)
- ③ Collinear array (Antenna arranged in line to line ---)
- ④ Parasitic array (Feeding in only one element)

# consider two element array 1 & 2 (point sources) separated by distance  $d$ . Pt P is sufficiently far from antenna system. Then waves from antenna 1 reaches pt P later than waves from antenna 2 because of path difference  $d \cos \phi$ .



# In terms of wavelength  
path difference =  $\frac{d \cos \phi}{\lambda}$

# The total phase difference ( $\psi$ ) at pt P is

$$\psi = 2\pi \cdot \text{path difference} + \alpha, \quad \alpha = \text{phase difference due to current.}$$

$$= 2\pi \frac{d \cos \phi}{\lambda} + \alpha$$

$$\boxed{\psi = \beta d \cos \phi + \alpha}$$

Case 1: Array of two point sources of same amplitude & phase

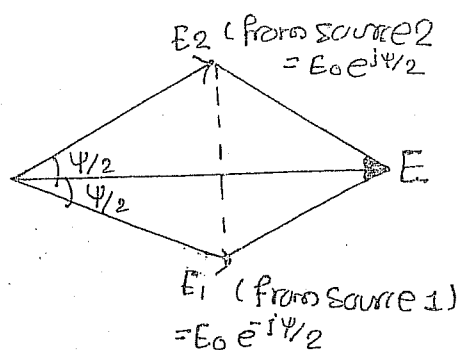
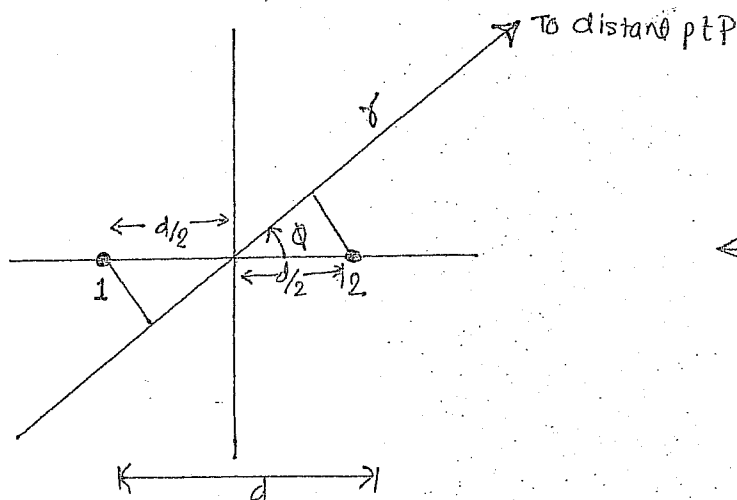
2004: Find exp<sup>n</sup> for total electric field of two element array of non directional radiator.

2006 Plot radiation pattern for two element array

2009 separated by width  $d$  &  $d = 0$ .

Find maxima, minima & half power pt.

#



# Let's consider two point sources (isotropic) separated by distance  $d$  having same amplitude and oscillating in same phase as shown in fig.

# The two point sources are located symmetrically wrt orig<sup>n</sup> of coordinates system.

angle  $\psi$  is measured anticlockwise from +ve x axis  
 # At the distant point, field from source 1 lags by  $e^{-j\psi/2}$  & from source 2 leads by  $e^{j\psi/2}$ .

$$\therefore \text{Total field } E = E_0 e^{-j\psi/2} + E_0 e^{j\psi/2}$$

$$E = E_0 (e^{-j\psi/2} + e^{j\psi/2}) \cdot 2$$

$$\boxed{E = 2 E_0 \cos \psi/2}$$

we have  $\psi = \beta d \cos \phi + \alpha$  &  $\alpha = 0 \therefore$  same phase

$$\therefore E = 2 E_0 \cos \left[ \frac{\beta d \cos \phi}{2} \right]$$

# To draw field pattern, we need maxima minima & half power points

Let's consider  $\boxed{d = \lambda/2}$

Then

$$E = 2 E_0 \cos \left[ \frac{2\pi/\lambda \cdot \lambda/2 \cos \phi}{2} \right]$$

$$E = 2 E_0 \cos \left[ \pi/2 \cos \phi \right]$$

# On normalization [make its maximum value unity, set  $2E_0 = 1$  i.e. divide by max value]

$$\boxed{E_n = \cos [\pi/2 \cos \phi]}$$

Maxima

# maxima will occur when  $\cos [\pi/2 \cos \phi] = \pm 1$

$$\text{or } \pi/2 \cos \phi = \pm n\pi$$

$$\text{For } n=0, \pi/2 \cos \phi = 0$$

$$\text{or } \cos \phi = 0$$

$$\therefore \text{maxima occur at } \boxed{\phi = 90^\circ \text{ \& } 270^\circ}$$

minima

# minima will occur when  $\cos [\pi/2 \cos \phi] = 0$

$$\text{or } \pi/2 \cos \phi = \pm (2n+1)\pi/2$$

$$\text{For } n=0, \pi/2 \cos \phi = \pm \pi/2$$

$$\cos \phi = \pm 1$$

$$\therefore \text{maxima occur at } \boxed{\phi = 0^\circ \text{ \& } 180^\circ}$$

$$\cos(\sqrt{2} \cos \phi) = \pm \frac{1}{\sqrt{2}}$$

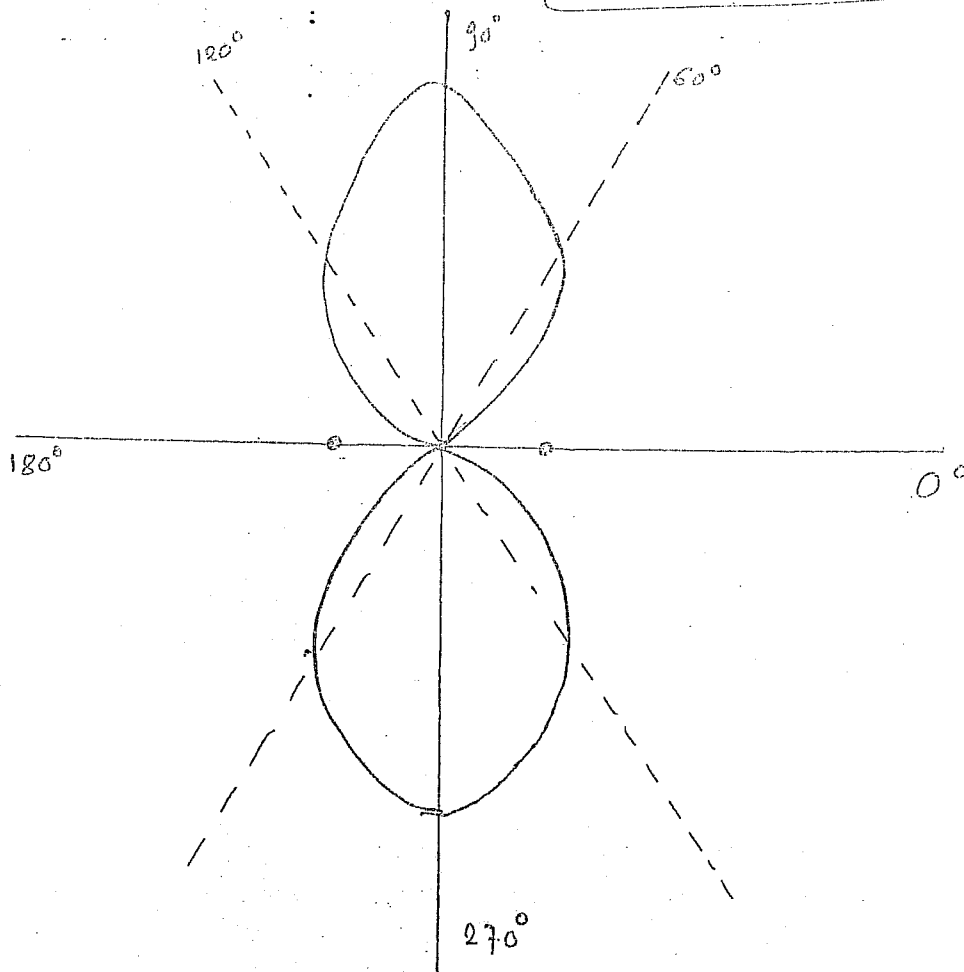
$$\text{or } \pi/2 \cos \phi = \pm (2n+1)\pi/4$$

For  $n=0$ ,

$$\pi/2 \cos \phi = \pm \pi/4$$

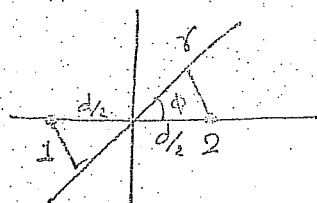
$$\cos \phi = \pm \frac{1}{2}$$

$\therefore$  half power point occurs  $\therefore \phi = 60^\circ \text{ \& } 120^\circ$



and opposite phase ( $\alpha = 180^\circ$ )

# consider two point sources separated by distance  $d$  have same amplitude and opposite phase.



$$E_2 = E_0 e^{j\psi/2}$$

$$E_1 = E_0 e^{-j\psi/2}$$

# since the two sources are opposite in phase

$$E = E_0 e^{j\psi/2} - E_0 e^{-j\psi/2}$$

$$E = E_0 \left[ \frac{e^{j\psi/2} - e^{-j\psi/2}}{2j} \right] \times 2j$$

$$= 2j E_0 \sin \psi/2$$

$$E = 2j E_0 \sin \left( \frac{\beta d \cos \phi}{2} \right)$$

$$[\because \psi = \beta d \cos \phi]$$

[ $\alpha$  considered already]

The  $j$  operator indicates phase reversal in one of the source results in  $90^\circ$  phase shift of total field.

# Let's consider  $d = \lambda/2$

$$\therefore E = 2j E_0 \sin \left( \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} \cos \phi \right)$$

$$E = 2j E_0 \sin (\pi \cos \phi)$$

[ $\alpha = 180^\circ$  considered already]

Normalizing [ $2j E_0 = 1$ ]

$$E = \sin (\pi \cos \phi)$$

maxima

$$\sin (\pi \cos \phi) = \pm 1$$

$$\text{or } \pi \cos \phi = \pm (2n+1)\pi/2$$

$$\text{For } n=0, \pi/2 \cos \phi = \pm \pi/2$$

$$\cos \phi = \pm 1$$

$$\therefore \phi = 0^\circ \text{ \& } 180^\circ$$

minima

$$\sin (\pi \cos \phi) = 0$$

$$\pi \cos \phi = \pm n\pi$$

$$\text{for } n=0, \pi \cos \phi = 0$$

$$\cos \phi = 0$$

$$\therefore \phi = 90^\circ \text{ \& } 270^\circ$$

Half power pt

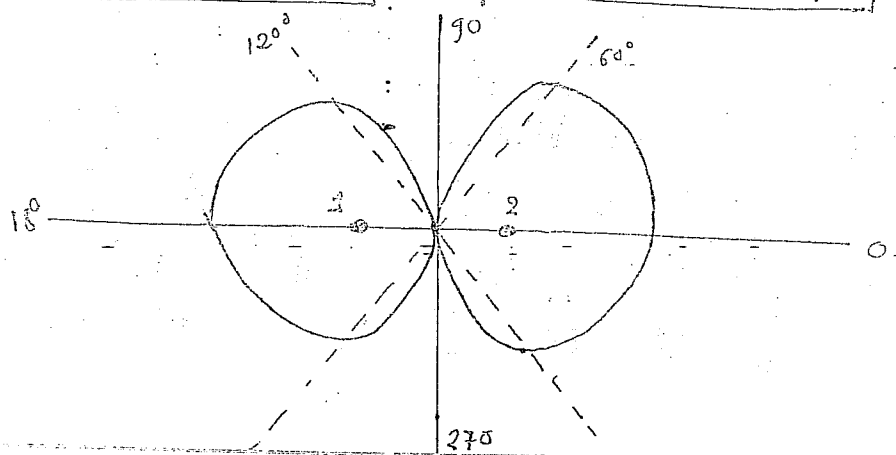
$$\sin (\pi \cos \phi) = \pm 1/\sqrt{2}$$

$$\pi \cos \phi = \pm (2n+1)\pi/4$$

$$\text{For } n=0, \pi \cos \phi = \pm \pi/4$$

$$\cos \phi = \pm 1/2$$

$$\therefore \phi = 60^\circ \text{ \& } 120^\circ$$



# The source are arranged as in case 1

$$\text{Total field } E = E_0 e^{j\psi/2} + E_0 e^{-j\psi/2}$$

$$E = 2E_0 \cos \psi/2$$

we have  $\psi = \beta d \cos \phi + \alpha$ ,  $\alpha = 90^\circ (\pi/2)$

$$\therefore E = 2E_0 \cos \left( \frac{\beta d \cos \phi}{2} + \frac{\pi}{4} \right)$$

For  $d = \lambda/2$  & normalizing

$$E = \cos \left[ \frac{\pi}{2} (\cos \phi + \frac{\pi}{4}) \right]$$

maxima

$$\cos \left( \frac{\pi}{2} \cos \phi + \frac{\pi}{4} \right) = \pm 1$$

$$\frac{\pi}{2} \cos \phi + \frac{\pi}{4} = \pm n\pi$$

For  $n=0$ ,  $\frac{\pi}{2} \cos \phi = -\frac{\pi}{4}$

$$\cos \phi = -\frac{1}{2}$$

$$\therefore \phi = 120^\circ, 240^\circ$$

minima

$$\cos \left( \frac{\pi}{2} \cos \phi + \frac{\pi}{4} \right) = 0$$

$$\frac{\pi}{2} \cos \phi + \frac{\pi}{4} = \pm (2n+1) \frac{\pi}{2}$$

For  $n=0$ ,

$$\frac{\pi}{2} \cos \phi + \frac{\pi}{4} = \pm \frac{\pi}{2}$$

$$\frac{\pi}{2} \cos \phi = \frac{\pi}{4} \text{ or } -\frac{3\pi}{4}$$

$$\cos \phi = \frac{1}{2} \text{ or } -\frac{3}{2}$$

$$\therefore \phi = 60^\circ \text{ \& } 300^\circ$$

(invalid)

Half power pt

$$\cos \left( \frac{\pi}{2} \cos \phi + \frac{\pi}{4} \right) = \pm \frac{1}{2}$$

$$\frac{\pi}{2} \cos \phi + \frac{\pi}{4} = \pm (2n+1) \frac{\pi}{2}$$

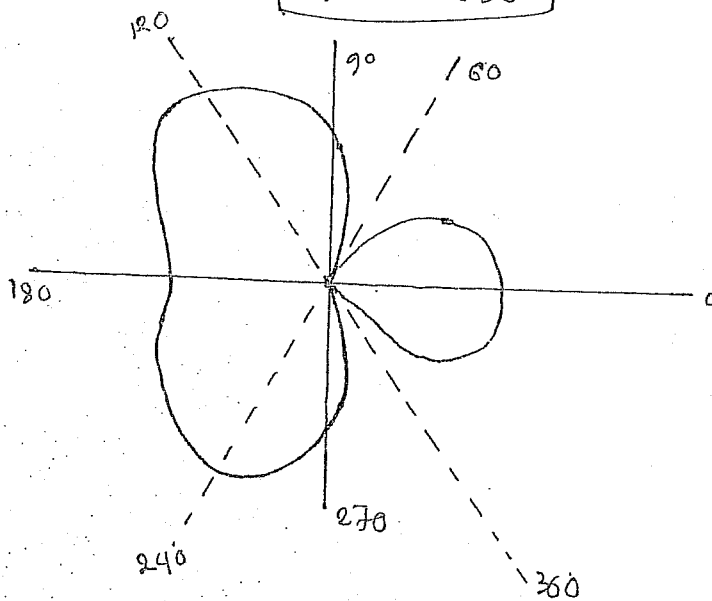
For  $n=0$

$$\frac{\pi}{2} \cos \phi + \frac{\pi}{4} = \pm \frac{\pi}{4}$$

$$\frac{\pi}{2} \cos \phi = 0 \text{ or } -\frac{\pi}{2}$$

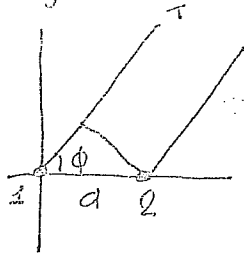
$$\therefore \cos \phi = 0 \text{ or } -1$$

$$\therefore \phi = 90^\circ \text{ \& } 270^\circ \text{ \& } 180^\circ$$



and any phase.

# This is the more general case involving two isotropic point sources with unequal amplitude and any phase difference  $\alpha$ .



# Let's take source 1 as reference for phase with amplitude of source 1 as  $E_1$  and source 2 as  $E_2$ . &  $E_1 > E_2$

# Then total phase difference is  

$$\psi = \beta d \cos \phi + \alpha$$

# Now the total field is given by

$$E = E_1 e^{j0} + E_2 e^{j\psi}$$

$$= E_1 \left[ 1 + \frac{E_2}{E_1} e^{j\psi} \right]$$

$$= E_1 [1 + k e^{j\psi}] \quad \text{where } k = \frac{E_2}{E_1}$$

$$= E_1 [1 + k (\cos \psi + j \sin \psi)] \quad \text{since } E_1 > E_2 \therefore 0 \leq k \leq 1$$

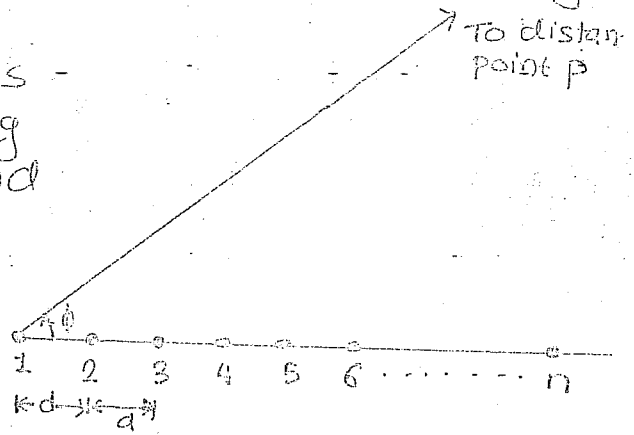
$$E = E_1 \sqrt{(1 + k \cos \psi)^2 + (k \sin \psi)^2} \angle \phi$$

where 
$$\phi = \tan^{-1} \frac{k \sin \psi}{1 + k \cos \psi}$$

Q.007 Derive the expr for total electric field radiated by an antenna array.

# Consider  $n$  isotropic point sources of equal amplitude and spacing arranged as a linear array and having an uniform progressive phase shift.

# Source 1 is taken as the reference for phase, thus at distant point  $P$  in  $\phi$  direction



the field from source 2 is advanced in phase wrt source 1 by  $\psi$ , field from source 3 is advanced in phase wrt source 1 by  $2\psi$  & so on.

# The total electric field is

$$E_T = E_0 + E_0 e^{j\psi} + E_0 e^{2j\psi} + E_0 e^{3j\psi} + \dots + E_0 e^{(n-1)j\psi}$$

$$E_T = E_0 [1 + e^{j\psi} + e^{2j\psi} + e^{3j\psi} + \dots + e^{(n-1)j\psi}] \quad \text{--- (1)}$$

multiplying by  $e^{j\psi}$

$$E_T e^{j\psi} = E_0 [e^{j\psi} + e^{2j\psi} + e^{3j\psi} + e^{4j\psi} + \dots + e^{nj\psi}] \quad \text{--- (2)}$$

Subtracting (2) from (1)

$$E_T (1 - e^{j\psi}) = E_0 [1 - e^{jn\psi}]$$

$$\therefore E_T = E_0 \frac{(1 - e^{jn\psi})}{(1 - e^{j\psi})}$$

$$= E_0 \left( \frac{-e^{jn\psi/2}}{-e^{j\psi/2}} \right) \frac{(-e^{-jn\psi/2} + e^{jn\psi/2}) \times 2j}{(-e^{-j\psi/2} + e^{j\psi/2}) \times 2j}$$

$$= E_0 \frac{\sin n\psi/2}{\sin \psi/2} \cdot e^{j \frac{(n-1)}{2} \psi}$$

If the reference pt is shifted to the centre of array the term  $e^{j \frac{(n-1)}{2} \psi}$  can be eliminated

$$\therefore E_T = E_0 \left( \frac{\sin(n\psi/2)}{\sin \psi/2} \right) \quad \text{--- (3)}$$

$E_0$  represents individual source pattern or primary pattern &  $\left( \frac{\sin n\psi/2}{\sin \psi/2} \right)$  represents array factor or secondary factor.



$$E_0 \frac{\sin \psi/2}{\sin \psi/2} \quad \text{--- (4)}$$

Applying L-Hopital rule for RHS of (4) with limit  $\psi \rightarrow 0$

$$\propto \lim_{\psi \rightarrow 0} \frac{d/d\psi \sin(n\psi/2)}{d/d\psi \sin(\psi/2)}$$

$$\propto \lim_{\psi \rightarrow 0} \frac{n/2 \cos(n\psi/2)}{1/2 \cos \psi/2}$$

$$= n$$

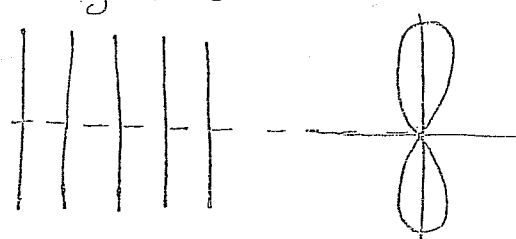
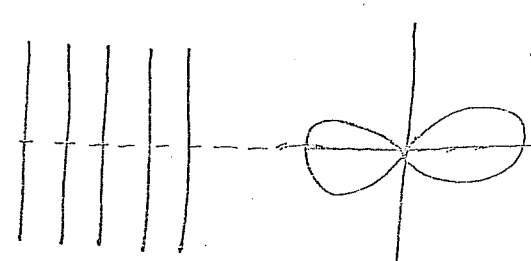
Thus  $\frac{\sin(n\psi/2)}{\sin(\psi/2)}$  has maximum value 'n' at  $\psi=0$

Thus normalizing (4) [dividing by max<sup>n</sup> value]  $\frac{E_t}{E_0}|_{\max} = 1$

$$E_n = \frac{E_t}{E_0} = \frac{1}{n} \frac{\sin(n\psi/2)}{\sin(\psi/2)}$$

## BROAD SIDE ARRAY & END FIRE ARRAY

2007 Distinguish Broad side and End Fire Array. Derive  
2009 Expression for Beam width in both types.

#	BROAD SIDE	END FIRE
①	The radiation pattern of broad side array is perpendicular to line of array axis.	① The radiation pattern of end fire is along the axis of antenna array
②		② 
③	Each individual antennas (or elements) are equally spaced and each element is fed with current of equal magnitude and all in same phase =	③ Each individual elements in end fire array are equally spaced and is fed with current of equal magnitude but their phase varies progressively along the line of antenna to generate the desired radiation pattern along axis of antenna array

# Consider a broadside array of 4 elements spaced  $\lambda/2$ . i.e.  $n=4$ ,  $d=\lambda/2$ ,  $\alpha=0$

we have,

$$E_n = \frac{1}{n} \frac{\sin(n\psi/2)}{\sin(\psi/2)}$$

Here,

$$\psi = \beta d \cos \phi + \alpha$$

$$\psi = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} \cos \phi + 0$$

$$\psi = \pi \cos \phi$$

$$\therefore E_n = \frac{1}{4} \frac{\sin\left(4 \cdot \frac{\pi \cos \phi}{2}\right)}{\sin\left(\frac{\pi \cos \phi}{2}\right)}$$

$$= \frac{1}{4} \frac{\sin(2\pi \cos \phi)}{\sin(\pi/2 \cos \phi)}$$

$$= \frac{2 \sin(\pi \cos \phi) \cdot \cos(\pi \cos \phi)}{4 \sin(\pi/2 \cos \phi)} \quad \left[ \because \sin 2\theta = 2 \sin \theta \cdot \cos \theta \right]$$

$$= \frac{2 \cdot 2 \sin(\pi/2 \cos \phi) \cdot \cos(\pi/2 \cos \phi) \cos(\pi \cos \phi)}{4 \sin(\pi/2 \cos \phi)}$$

$$E_n = \cos(\pi/2 \cos \phi) \cdot \cos(\pi \cos \phi)$$

maxima

$E_n$  will be max<sup>m</sup> when

$$\cos(\pi/2 \cos \phi) \cdot \cos(\pi \cos \phi) = 1$$

$$\therefore \cos(\pi/2 \cos \phi) = 1 \quad \& \quad \cos(\pi \cos \phi) = 1$$

This requirement is satisfied when

$$\phi = (2k+1)\pi/2 \quad k = 0, 1, \dots$$

$$\therefore \phi = 90^\circ, 270^\circ$$

MINIMUM

For minimum

$$\cos(\pi/2 \cos \phi) \cos(\pi \cos \phi) = 0$$

Either  $\cos(\pi/2 \cos \phi) = 0$

OR  $\cos(\pi \cos \phi) = 0$

$$\boxed{\phi = K\pi}$$

OR  $\boxed{\phi = 60^\circ, 120^\circ, 240^\circ}$

### Beamwidth

# Let  $\phi$  be the direction of null &  $\theta$  be the complementary angle of  $\phi$ .

Then  $2\theta$  is the beamwidth of primary lobe.

# For obtaining null in field strength,

$$E_n = 0$$

$$\text{or } \frac{1}{n} \frac{\sin(n\psi/2)}{\sin(\psi/2)} = 0$$

$$\text{or } \sin(n\psi/2) = 0 \quad \text{--- (1)}$$

we have,

$$\psi = \beta d \cos \phi = \frac{2\pi}{\lambda} \cdot d \cos(90^\circ - \theta) \quad [\because \phi = 90^\circ - \theta]$$

$$\therefore \psi = \frac{2\pi}{\lambda} d \sin \theta$$

putting value of  $\psi$  in eqn (1)

$$\sin\left(n \frac{2\pi d}{\lambda} \sin \theta\right) = 0$$

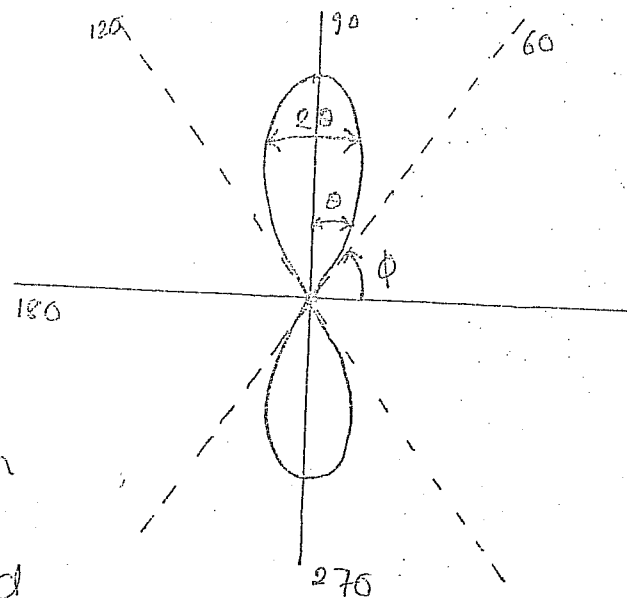
$$\sin\left(\frac{n\pi d \sin \theta}{\lambda}\right) = 0$$

$$\frac{n\pi d \sin \theta}{\lambda} = k\pi, \quad k = 0, 1, \dots$$

$$\sin \theta = \frac{k\lambda}{nd}$$

$$\therefore \theta = \sin^{-1}\left(\frac{k\lambda}{nd}\right)$$

$$\boxed{2\theta = 2 \sin^{-1}\left(\frac{k\lambda}{nd}\right)}$$



$$\therefore 2\theta = 2 \sin^{-1} \left( \frac{\lambda}{n} \right)$$

For specific case of  $d = \lambda/2$

$$\therefore 2\theta = 2 \sin^{-1} \left( \frac{\lambda}{n} \right)$$

For  $n = 2$ ,  $2\theta = 60^\circ$

$n = 10$ ,  $2\theta = 23^\circ$

$\therefore$  Beamwidth decreases with number of radiator.  
Hence directivity increases with increase in number of radiators.

### \* END FIRE ARRAY

# In End Fire array there exists certain phase difference bet<sup>n</sup> the adjacent element & the max<sup>m</sup> radiation pattern occurs in the direction of array i.e. at  $\phi = 0$ .

We know,  $\psi = \beta d \cos \phi + \alpha$

or,  $0 = \beta d \cos 0 + \alpha$  [ $\because$  For end fire array  $\phi = 0$  &  $\psi = 0$ ]

$$\boxed{\alpha = -\beta d}$$

$$\boxed{\alpha = -\frac{2\pi}{\lambda} d}$$
 for  $\theta = 0$  &  $\boxed{\alpha = \frac{2\pi}{\lambda} d}$  for  $\theta = 180^\circ$

# This means the phase bet<sup>n</sup> adjacent element is retarded progressively by the same amount as the spacing between sources in radians. Thus if spacing bet<sup>n</sup> is  $\lambda/4$ , source 2 should lag source 1 by  $90^\circ$ , source 3 should lag source 2 by  $90^\circ$  and so on.

# consider an end fire array of 4 elements a  $\lambda/2$  apart, i.e.  $n=4$  &  $d=\lambda/2$

$$\psi = \beta d \cos \phi + \alpha$$

$$= \frac{2\pi}{\lambda} \cdot \lambda/2 \cos \phi + \left(-\frac{2\pi}{\lambda} \cdot \lambda/2\right) \quad \left[\because \alpha = -\frac{2\pi d}{\lambda}\right]$$

$$\boxed{\psi = \pi (\cos \phi - 1)}$$

we have,

$$E_n = \frac{1}{n} \frac{\sin(n\psi/2)}{\sin(\psi/2)} = \frac{1}{n} \frac{\sin\left[\frac{n\pi}{2} (\cos \phi - 1)\right]}{\sin\left[\frac{\pi}{2} (\cos \phi - 1)\right]}$$

$$= \frac{1}{4} \frac{\sin[2\pi (\cos \phi - 1)]}{\sin\left[\frac{\pi}{2} (\cos \phi - 1)\right]} \quad [\because n=4]$$

$$= \frac{1}{4} \cdot 2 \cdot \frac{\sin[\pi (\cos \phi - 1)] \cdot \cos[\pi (\cos \phi - 1)]}{\sin\left[\frac{\pi}{2} (\cos \phi - 1)\right]}$$

$$= \frac{1}{2} \cdot 2 \frac{\sin\left[\frac{\pi}{2} (\cos \phi - 1)\right] \cdot \cos\left[\frac{\pi}{2} (\cos \phi - 1)\right] \cdot \cos[\pi (\cos \phi - 1)]}{\sin\left[\frac{\pi}{2} (\cos \phi - 1)\right]}$$

$$\boxed{E_n = \cos\left[\frac{\pi}{2} (\cos \phi - 1)\right] \cdot \cos[\pi (\cos \phi - 1)]}$$

\* maxima

#  $E_n$  will be max<sup>m</sup> when,

$$\cos\left[\frac{\pi}{2} (\cos \phi - 1)\right] \cdot \cos[\pi (\cos \phi - 1)] = 1$$

# For this cond<sup>n</sup> both  $\cos\left[\frac{\pi}{2} (\cos \phi - 1)\right]$  &  $\cos[\pi (\cos \phi - 1)]$  should be unity.

# This condition is satisfied when  $\boxed{\phi = k\pi}$  where  $k = 0, 1, \dots$

#  $\therefore$  Direction of max<sup>m</sup> field is in direction of radiator/elements itself

# For min<sup>th</sup> value of  $E_n$ .

$$\cos\left[\frac{\pi}{2}(\cos\phi - 1)\right] \cdot \cos[\pi(\cos\phi - 1)] = 0$$

Either

$$\cos\left[\frac{\pi}{2}(\cos\phi - 1)\right] = 0$$

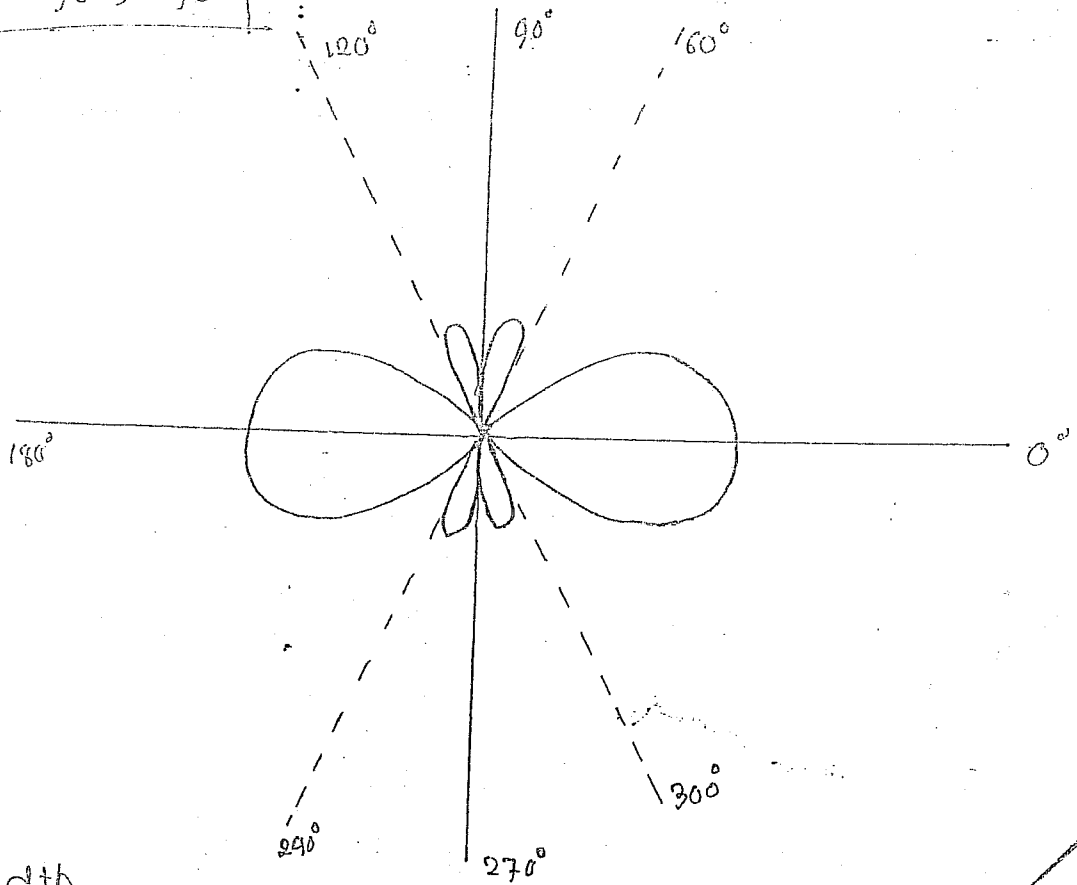
or

$$\cos[\pi(\cos\phi - 1)] = 0$$

$$\therefore \phi = (2k+1)\frac{\pi}{2}$$

$$\therefore \phi = 90^\circ, 270^\circ$$

$$\therefore \phi = 60^\circ, 120^\circ, 240^\circ, 300^\circ$$



### Beamwidth

# We have  $\phi$  is the direction of null and thus  $2\phi$  is the beamwidth of the primary lobe.

# For obtaining null field

$$E_n = 0$$

$$\text{or } \frac{1}{n} \frac{\sin\left(\frac{n\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)} = 0$$

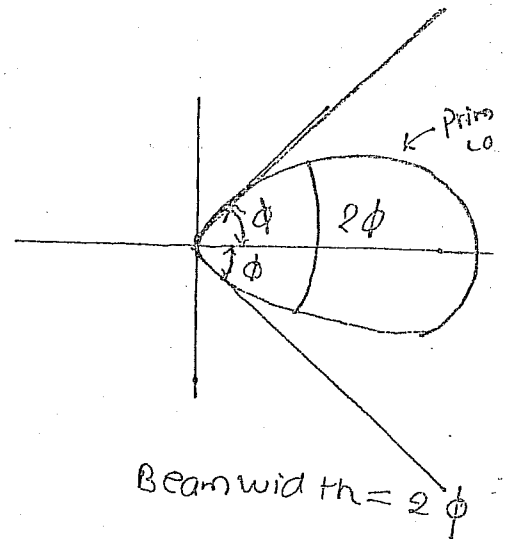
$$\text{or, } \sin\left(\frac{n\psi}{2}\right) = 0$$

$$\frac{n\psi}{2} = \pm k\pi$$

$$\psi = \frac{2k\pi}{n}$$

For end fire array  $\psi = \frac{2\pi}{\lambda} d (\cos\phi - 1)$

$$\therefore \frac{2k\pi}{n} = \frac{2\pi}{\lambda} d (\cos\phi - 1)$$



Beamwidth =  $2\phi$

$$2 \sin^2 \frac{\phi}{2} = \frac{\frac{n\lambda}{2}}{\frac{n\lambda}{2}}$$

$$\sin \frac{\phi}{2} = \sqrt{\frac{\frac{n\lambda}{2}}{\frac{n\lambda}{2}}}$$

$$\frac{\phi}{2} = \sin^{-1} \left( \sqrt{\frac{\frac{n\lambda}{2}}{\frac{n\lambda}{2}}} \right)$$

$$\therefore \phi = 2 \sin^{-1} \left( \sqrt{\frac{\frac{n\lambda}{2}}{\frac{n\lambda}{2}}} \right)$$

$$\therefore 2\phi = 4 \sin^{-1} \left( \sqrt{\frac{\frac{n\lambda}{2}}{\frac{n\lambda}{2}}} \right)$$

For first null  $k = 1$

$$2\phi = 4 \sin^{-1} \left( \sqrt{\frac{1}{2nd}} \right)$$

In End Fire array also When no of sources 'n' increases  $\rightarrow$  beamwidth decreases  $\rightarrow$  Thus directivity increases.

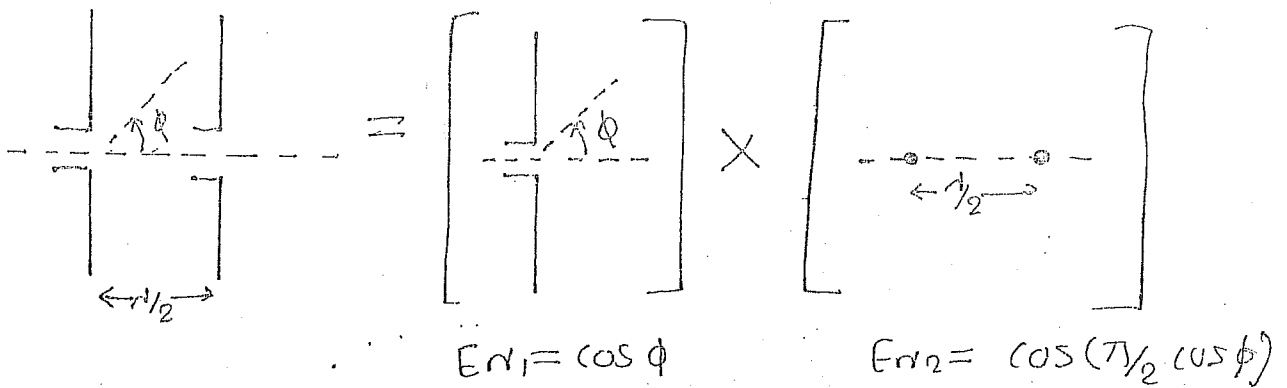
## 2.002. Process of multiplication of patterns

\* It is one of the methods of obtaining radiation pattern of an array.

\* It states that:

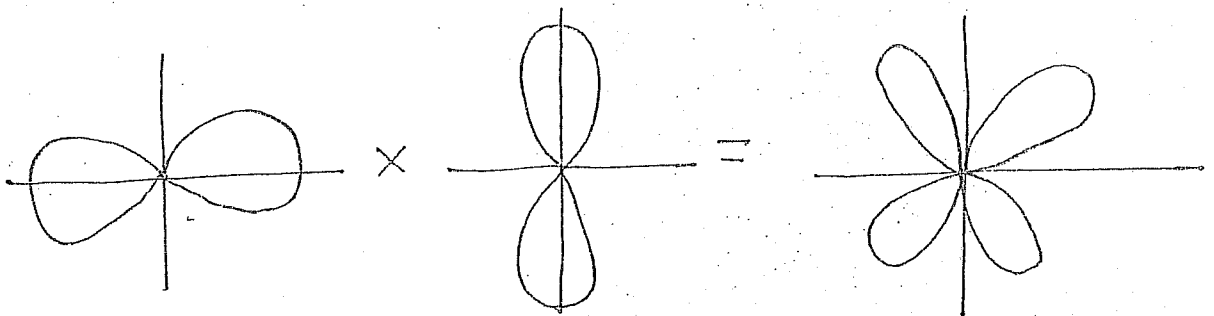
The total field of an array of non isotropic but similar sources is the product of individual source pattern and the pattern of an array of isotropic point sources each located at the phase centre of individual sources and having the same relative amplitude and phase.

\* Eg.


$$E_{N1} = \cos \phi$$
$$E_{N2} = \cos(\pi/2 \cos \phi)$$

The resultant field will be

$$E_N = E_{N1} \times E_{N2}$$
$$= \cos \phi \cdot \cos(\pi/2 \cos \phi)$$





2024

spaced  $\frac{r}{2}$  apart

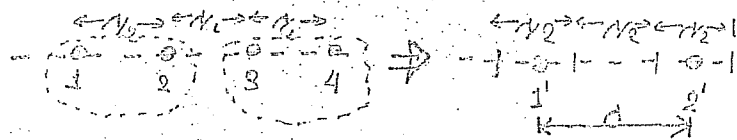


fig (a) 4 isotropic elements spaced  $\frac{r}{2}$  apart

fig (b)

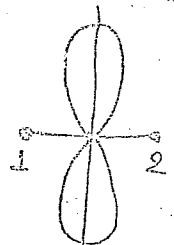


fig (c) pattern of two point sources spaced  $\frac{r}{2}$  apart

# Consider element 1 and 2 as one unit and is placed midway of the elements. And element 3 & 4 operate as another unit as shown in fig (b)

# Two point sources spaced  $\frac{r}{2}$  apart fed in phase has pattern as shown in fig (c)

# The radiation pattern for two point sources separated by a distance apart (of fig b) is as shown in fig (d)

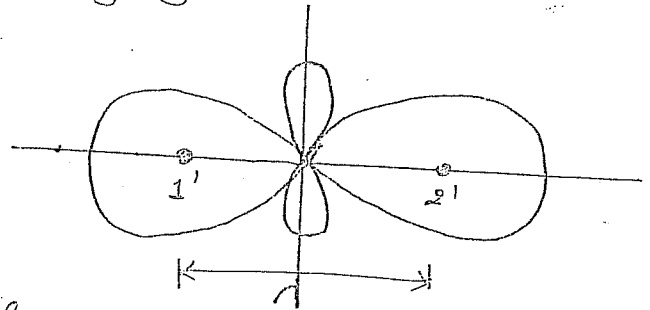
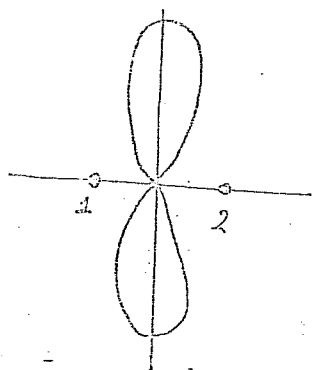


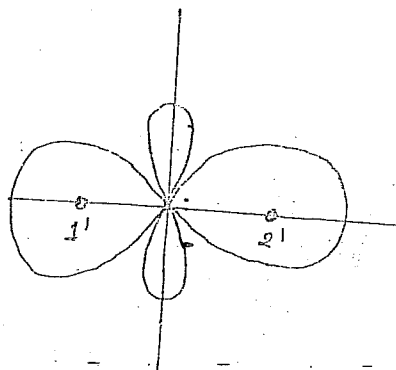
fig (d) Radiation pattern for two point sources a distance apart

# Now the radiation pattern of four isotropic elements can be obtained by multiplying radiation pattern of fig (c) & (d)



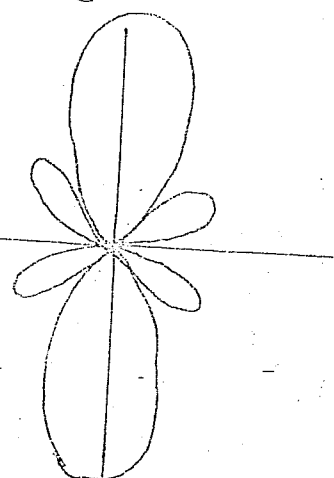
Individual / unit pattern (due to two individual elements)

X



Group Pattern (due to array of two isotropic point sources)

$\Rightarrow$



Resultant pattern (of 4 isotropic sources)



# ELECTROMAGNETIC PROPAGATION & ANTENNA

## CHAPTER-3

### ANTENNA PROPAGATION

By  
Rajan Sharma

\* Transmission Loss Between Antennas.  
(Fundamental eq<sup>n</sup> for Free space) (Friis Transmission Formula)

- Ques 2009 Derive free space Transmission formula.  
Ques 2006 Derive Friis Transmission Formula. Also derive eq<sup>n</sup> for basic Transmission Loss.  
Ques 2005 For Tx Rx system derive the expression for free space Loss (FSL) in dB.

# Friis Transmission formula gives the power received over a radio communication link.

# Let  $W_T$  be the power fed to transmitting Antenna of effective area  $A_{et}$ .

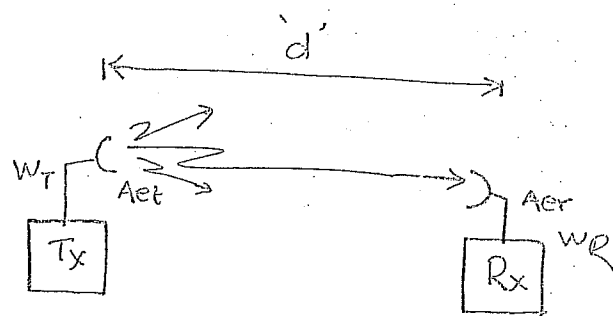


Fig: communication system from transmitting Antenna to receiving antenna separated by distance 'd'

# Assuming transmitting Antenna to be isotropic, power per unit area (power density) at distance  $d$  is at receiver antenna is

$$\frac{W_T}{4\pi d^2}$$

# If the Transmitting Antenna has gain  $G_T$ , the power density available at receiving antenna is

$$\text{Power density} = \frac{G_T W_T}{4\pi d^2}$$

# At a distance 'd' a receiving antenna of effective aperture ' $A_{er}$ ' some of the power radiated by transmitting Antenna, the power collected by receiving antenna is,

Power received ( $W_R$ ) = power density  $\times$  effective area

$$W_R = \frac{G_T W_T}{4\pi d^2} \cdot A_{er}$$

We have Effective Area  $A_{er} = \frac{d^2}{4\pi} G_R$

$$W_R = \frac{G_T W_T}{4\pi d^2} \times \frac{A_R}{4\pi} \cdot G_R$$

$$\therefore \frac{W_R}{W_T} = G_T G_R \left( \frac{d}{4\pi d} \right)^2$$

This is Friis Transmission Formula.

where  $d$  &  $d$  are in meters.

OR  
In terms of effective Aperture

$$\frac{W_R}{W_T} = \frac{A_{eT} A_{eR}}{d^2 \lambda^2}$$

# The basic transmission loss  $L_b$  is defined as the reciprocal of Friis eq<sup>n</sup> expressed in decibel

$$\therefore L_b = 10 \log \left( \frac{W_T}{W_R} \right)$$

For isotropic Antenna  $G_T = G_R = 1$

$$\therefore L_b = 10 \log \left( \frac{4\pi d}{\lambda} \right)^2$$

$$L_b = 20 \log \left( \frac{4\pi d}{\lambda} \right)$$

$$L_b = 20 \log \left( \frac{4\pi d f}{c} \right) \quad \because \lambda = c/f$$

expressing  $d$  in km &  $f$  in MHz we get

$$L_b = 32.45 + 20 \log F + 20 \log d$$

Always take care of units of  $F$  in MHz &  $d$  in km.

For directive antennas If gains are given in dB

$$L_{dB} = G_T + G_R - L_b$$

If gains are not given in dB

$$G_T = 10 \log g_1$$

$$G_R = 10 \log g_2$$

## NUMERICALS

2008

Q1. What is the max<sup>m</sup> power received at a distance of 10 km over a free space given a 1000 MHz circuit consist of a transmitting antenna with 25 dB gain and receiving antenna with 20 dB gain with respect to isotropic antenna.

The input power to transmitting antenna is 150 W

Soln

Given,  $d = 10 \text{ km}$

$f = 1000 \text{ MHz}$

$$G_T = 25 \text{ dB} = 316.23 \quad [\because 10 \log x = 25]$$

$$G_R = 20 \text{ dB} = 100$$

$$W_T = 150 \text{ dB} = 1 \times 10^{15} \text{ W}$$

$$\lambda = c/f = \frac{3 \times 10^8}{1000 \times 10^6} = 0.3 \text{ m}$$

Then power received will be

$$\begin{aligned} W_R &= W_T \cdot G_T \cdot G_R \left( \frac{\lambda}{4\pi d} \right)^2 \\ &= 1 \times 10^{15} \times 316.23 \times 100 \left( \frac{0.3}{4 \times 3.14 \times 10 \times 10^3} \right)^2 \\ &= 1.8 \times 10^8 \text{ W} \\ &= 10 \log (1.8 \times 10^8) \end{aligned}$$

$$\boxed{W_R = 82.56 \text{ dB}}$$

OR

$$W_R(\text{dB}) = W_T(\text{dB}) + G_T(\text{dB}) + G_R(\text{dB}) - L_s(\text{dB})$$

$$\begin{aligned} L_s(\text{dB}) &= 32.45 + 20 \log F + 20 \log d \\ &= 32.45 + 20 \log 1000 + 20 \log 10 \end{aligned}$$

[F in MHz  
d in km]

$$\therefore W_R(\text{dB}) = 150 + 25 + 20 - 112.45$$

$$\boxed{W_R = 82.56 \text{ dB}}$$

In a microwave communication link two identical antennas operating at 100 GHz is used with power gain of 40 dB. If Tx power is 1 W, find received power if range of link is 30 km.

Soln

$$f = 100 \text{ GHz} = 100000 \text{ MHz}$$

$$G_T = 40 \text{ dB}$$

$$G_R = 40 \text{ dB}$$

$$d = 30 \text{ km}$$

$$W_T = 1 \text{ W} = 10 \log 1 = 0 \text{ dB}$$

$$W_R = ?$$

$$\text{we have, } W_R(\text{dB}) = W_T(\text{dB}) + G_T(\text{dB}) + G_R(\text{dB}) - L_s$$

$$\begin{aligned} L_s &= 32.45 + 20 \log f + 20 \log d \\ &= 32.45 + 20 \log 100000 + 20 \log 30 \\ &= 161.99 \end{aligned}$$

$$\therefore W_R(\text{dB}) = 0 + 40 + 40 - 161.99$$

$$W_R(\text{dB}) = -81.99 \text{ dB}$$

$$10 \log(W_R) = -81.99$$

$$W_R = 6.3 \times 10^{-9} \text{ W}$$

Q3. Two planes 15 km apart are in radio comm. The transmitting plane delivers 500 W its antenna gain being 10. The power absorbed is 2  $\mu$ W by receiving antenna. Find the effective area of receiving antenna.

Soln

$$d = 15 \text{ km}$$

$$W_T = 500 \text{ W}$$

$$G_T = 10$$

$$W_R = 2 \mu\text{W}$$

we have

Received power = power density  $\times$  Effective area of Rx antenna

$$W_R = \frac{G_T W_T}{4\pi d^2} \times A_{\text{eff}}$$

$$\therefore A_{\text{eff}} = \frac{W_R \times 4\pi d^2}{G_T W_T} = \frac{2 \times 10^{-6} \times 4 \times 3.14 \times (15 \times 10^3)^2}{10 \times 500}$$

$$\therefore A_{\text{eff}} = 1.13 \text{ m}^2$$

Q4 2005

Find the basic path loss for a communication from earth to the moon. The earth operating at 4000 MHz. Assume distance betn moon & earth is 384000 km.

Soln

$$\begin{aligned} \text{path loss} &= 32.45 + 20 \log F(\text{MHz}) + 20 \log d(\text{km}) \\ &= 32.45 + 20 \log 4000 + 20 \log 384000 \\ &= 216.17 \text{ dB.} \end{aligned}$$

Ans

Q5 2005 [10 marks] [LINK BUDGET DESIGN]

consider the case of synchronous satellite relay where 6 GHz is used for Ground to satellite link (uplink) and 4 GHz is used for downlink. Consider 30m diameter ground Antenna & 0.3 m diameter satellite antenna. Assuming 7% of effective area and distance of satellite 36000 km from earth station. Find the following

1. Basic Transmission loss
2. maxm directive gain of antenna
3. with ground transmitted power of 12 kW, find power received at the satellite
4. with satellite Transmitted power of 1 W. find the power received at ground station

Soln

$$\text{uplink} = 6 \text{ GHz} = 6000 \text{ MHz}$$

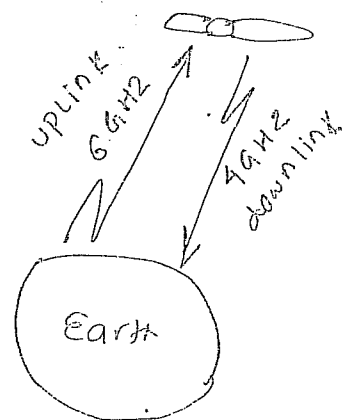
$$\text{downlink} = 4 \text{ GHz} = 4000 \text{ MHz}$$

$$d = 36000 \text{ km.}$$

$$A_e = 7\%$$

$$\text{Ground antenna diameter} = 30 \text{ m}$$

$$\text{Satellite " " " } = 0.3 \text{ m.}$$



(1) Basic TX loss

$$\begin{aligned} \text{a) For uplink } L_b(\text{uplink}) &= 32.45 + 20 \log 6000 + 20 \log 36000 \\ &= 199.13 \text{ dB} \end{aligned}$$

$$\begin{aligned} \text{b) For downlink } L_b(\text{downlink}) &= 32.45 + 20 \log 4000 + 20 \log 36000 \\ &= 195.61 \text{ dB} \end{aligned}$$

(i) Earth Station

(a) uplink  $\Rightarrow G_{ES}(\text{uplink}) = 10 \log(G_1)$

$$= 10 \log \left( \frac{G_{ES} \times 4\pi}{r^2} \right)$$

$$= 10 \log \left[ \frac{0.67 \times \frac{17d^2}{4} \times 4\pi}{\left( \frac{c}{f_{\text{uplink}}} \right)} \right]$$

$$= 10 \log \left[ \frac{0.67 \times 3.14 \times \frac{30^2}{4} \times 4 \times 3.14}{\left( \frac{3 \times 10^8}{6 \times 10^9} \right)} \right]$$

$$= 63.76 \text{ dB.}$$

(b) downlink  $\Rightarrow G_{ES}(\text{downlink}) = 10 \log \left[ \frac{0.67 \times 3.14 \times \frac{30^2}{4} \times 4 \times 3.14}{\left( \frac{3 \times 10^8}{4 \times 10^9} \right)} \right]$

$$= 60.25 \text{ dB.}$$

(ii) Satellite

(a) uplink  $\Rightarrow G_{\text{Sat}}(\text{uplink}) = 10 \log \left[ \frac{0.67 \times 3.14 \times \frac{0.3^2}{4} \times 4 \times 3.14}{\left( \frac{3 \times 10^8}{6 \times 10^9} \right)} \right]$

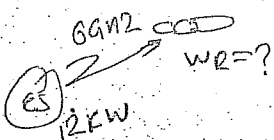
$$= 23.76 \text{ dB}$$

(b) Downlink  $\Rightarrow G_{\text{Sat}}(\text{downlink}) = 10 \log \left[ \frac{0.67 \times 3.14 \times \frac{0.3^2}{4} \times 4 \times 3.14}{\left( \frac{3 \times 10^8}{4 \times 10^9} \right)} \right]$

$$= 20.24 \text{ dB}$$

(8) If  $W_T(\text{gnd}) = 12 \text{ kW} = 10 \log(12000) = 40.79 \text{ dB}$   
 $W_R(\text{sat}) = ?$

$$W_R(\text{sat}) = W_T(\text{dB}) + G_{ES}(\text{uplink}) + G_{\text{Sat}}(\text{uplink}) - L_{\text{b uplink}}$$
$$= 40.79 + 63.76 + 23.76 - 199.13 \text{ dB}$$
$$= -70.8 \text{ dB}$$



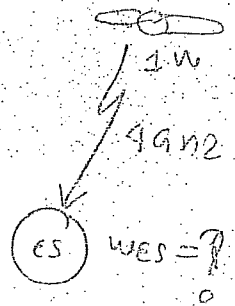


$$W_R(ES) = ?$$

$$W_{R_{ES}}(dB) = W_{Tsat}(dB) + G_{ES \text{ downlink}} + G_{sat \text{ downlink}} - L_{b \text{ downlink}}$$

$$= 0 + 60.25 + 20.24 - 195.61$$

$$= -115.12 \text{ dB.}$$



Ans

Q6 2007 2005

A Geostationary TV satellite & RX antenna have freq link 13.78 GHz at distance 36000 km. The Transmit power  $P_T$  is 110 W & Transmit antenna gain  $G_T = 30 \text{ dB}$ . calculate

- i) power density (watt/m<sup>2</sup>) at Receive antenna
- ii) what should be antenna Area  $S_A$  & antenna gain  $G_A$  in dB if Receiver antenna receives a power  $P_R$  which exceeds a threshold  $P_0 = 2 \times 10^{-11} \text{ W}$ .

Soln Try yourself based on the derivation of Friis eq<sup>n</sup>

hint: Power density =  $\frac{W_T G_T}{4\pi d^2}$

Receive power =  $\frac{W_T G_T}{4\pi d^2} \times A_e$  (or  $S_A$ ) find  $A_e$  from this

$$\frac{W_R}{W_T} = \frac{G_T G_R d^2}{(4\pi d)^2}$$

## \* Transmission Loss as a function of frequency

# The variation of transmission loss with frequency depends on the circumstance of the problem.

### ① Vehicular Communication: (Antenna with fixed directional gain)

For air to ground links and navigation systems, it is normally required that both antennas have omnidirectional coverage (i.e. they have fixed directional gain).

$$\therefore \frac{W_R}{W_T} = \frac{d^2 G_T G_R}{(4\pi d)^2} = \frac{G_T G_R}{(4\pi d)^2} \times \frac{c^2}{f^2} \quad [\because d = c/f]$$

In this case Received power is inversely proportional to square of frequency & Transmission loss ( $W_T/W_R$ ) is directly proportional to square of frequency.

### ② Earth-satellite communication:

In this case, antenna at earth would be directive [i.e. not fixed gain] and antenna at satellite is isotropic. Assuming Earth as Rx & satellite as Tx

$$\begin{aligned} \frac{W_R}{W_T} &= \frac{d^2 G_T G_R}{(4\pi d)^2} = \frac{d^2 G_T}{(4\pi d)^2} \cdot \frac{4\pi \cdot A_{er}}{d^2} \quad [\because G_R = \frac{4\pi \cdot A_{er}}{d^2}] \\ &= \frac{G_T A_{er}}{4\pi d^2} \end{aligned}$$

$A_{er}$  is effective area of Earth station antenna.

If Earth as Tx & satellite - Rx then,

$$\frac{W_R}{W_T} = \frac{G_R A_{er}}{4\pi d^2}$$

In both case Transmission loss is independent of frequency.

### ③ microwave link:

In this case both antennas are made directional

$$\begin{aligned} \frac{W_R}{W_T} &= \frac{d^2 G_T G_R}{(4\pi d)^2} = \frac{d^2}{(4\pi d)^2} \cdot \frac{4\pi \cdot A_{er}}{d^2} \cdot \frac{4\pi \cdot A_{et}}{d^2} \quad \left[ \because \begin{aligned} G_T &= \frac{4\pi \cdot A_{er}}{d^2} \\ G_R &= \frac{4\pi \cdot A_{et}}{d^2} \end{aligned} \right] \\ &= \frac{A_{er} A_{et}}{d^2 \cdot c^2} \cdot f^2 \end{aligned}$$

$\therefore$  Received Power is directly proportional to square of frequency and Transmission loss is inversely proportional to square of frequency.

# \* Antenna Temperature and signal to noise Ratio

2004 Find the expression for antenna temperature & SNR  
 2006 For receive transmit system, derive the expression for  
 2009 SNS for receiving system.

# Every object with physical temperature above absolute zero radiates energy. The amount of energy radiated is usually represented by an equivalent temperature  $T_b$  known as brightness temperature.

# The brightness temperature emitted by different sources is intercepted by antennas and it appears at their terminals as antenna temperature.

# The noise power per unit B.W available at terminals of resistor of resistance  $R$  at a temperature  $T_r$  is given by relation

$$P = k T_r$$

where

$P$  = Power per unit bandwidth

$k$  = Boltzmann's constant  $= 1.38 \times 10^{-23} \text{ J/K}$

$T_r$  = absolute temp, K

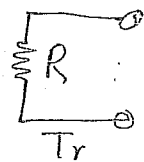


fig (A) Resistor at temp  $T_r$

# If the resistor  $R$  is replaced by a lossless antenna of radiation resistance  $R$  in an anechoic chamber at temp  $T_c$ , the noise power per unit B.W available at antenna terminal is same [provided  $T_c = T_r$ ]

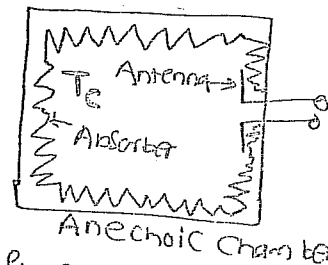


fig (B) Antenna in an anechoic chamber

# Now if antenna is removed from anechoic chamber and pointed at sky of temperature  $T_s$ , the noise power per unit bandwidth is still same [provided  $T_s = T_r$ ]. And we can say that antenna has a noise temp  $T_A$  equal to sky temperature  $T_s$ .

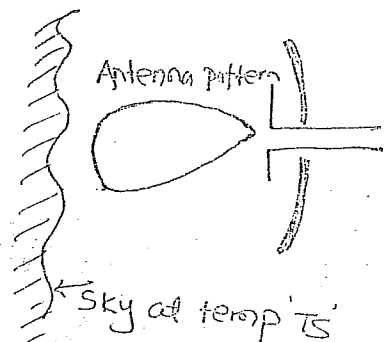


fig (C) Antenna observing sky at temp  $T_s$

# Thus antenna noise-temperature may be used to measure the distant or sky temperature  $T_s$

# Thus for antenna, the noise power per unit bandwidth is given by

$$P = K T_A$$

$T_A$  = Antenna temperature.

# Total power is thus

$$\text{Total Power } P = K T_A B$$

where  $B$  is Bandwidth (Hz)

# Let the source flux density (power density per unit BW) be  $S$  and  $A_e$  be the effective area of antenna.

$$\therefore S = \frac{P}{A_e}$$

$$S = \frac{K T_A}{A_e}$$

$$\therefore T_A = \frac{S A_e}{K} \text{ } ^\circ K$$

which is the antenna temperature.

### Signal To Noise Ratio for receiving system of communication link

# If a transmitter radiates a power  $P_t$  isotropically and uniformly over a bandwidth  $\Delta f_t$ . It produces a flux density at distance  $r$  of  $\frac{P_t}{4\pi r^2 \Delta f_t}$ .

# A receiving antenna of effective aperture ' $A_{er}$ ' at distance  $r$  can collect power.

$$P_r = \frac{P_t A_{er} \Delta f_r}{4\pi r^2 \Delta f_t} \quad \Delta f_r = \text{receiver bandwidth}$$

# If transmitting antenna has directivity  $D = \frac{4\pi}{A_e} A_{et}$   
 $A_{et}$  = effective aperture of Tx antenna

$$\text{Then, } P_r = \frac{P_t A_{er} A_{et}}{r^2 \pi^2} \frac{\Delta f_r}{\Delta f_t} \quad \text{--- (1)}$$

# For  $\Delta f_r = \Delta f_t$  (Bandwidth matched) eq? (1) is reduced to

# The noise power is the sum of antenna noise and receiver noise.

$$P_n = K T_A B + K T_e B$$

$$P_n = K T_{sys} B$$

$T_e$  = effective noise temp of Rx

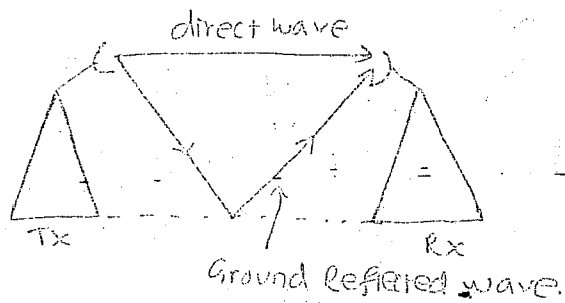
where  $T_{sys}$  is the system temperature.

Thus the signal to noise ratio for matched B.W is

$$\frac{S}{N} = \frac{P_r}{P_n}$$

$$\boxed{\frac{S}{N} = \frac{P_t A_e r A_e t}{r^2 n^2 K T_{sys} B}}$$

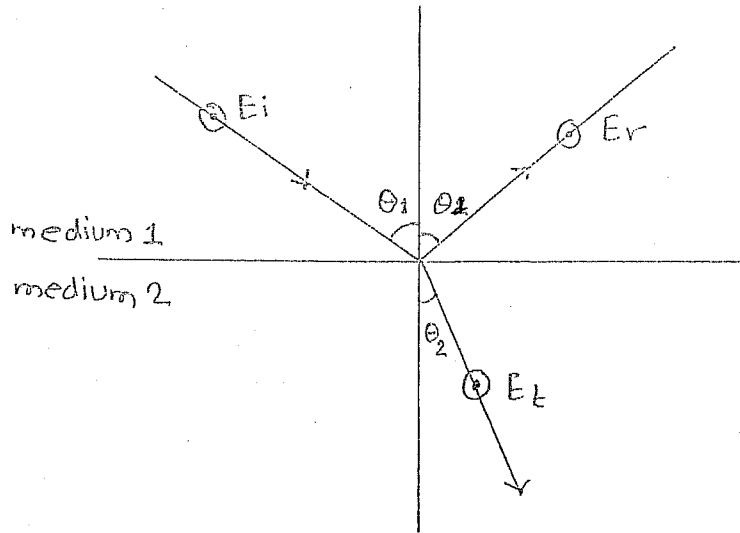
This is the signal to noise ratio for receiving system.



2007 2008 2009

### Case 1: (Horizontal Polarization)

when Electric vector is parallel to boundary surface and magnetic vector is perpendicular to boundary surface



# Applying boundary condition, the tangential component across the boundary is.

$$E_i + E_r = E_t$$

$$\therefore \frac{E_t}{E_i} = 1 + \frac{E_r}{E_i} \quad \text{--- (1)}$$

# Assuming permeability of medium to be same

$$\frac{\sin \theta_1}{\sin \theta_2} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

# From definition of Poynting Vector, Power transmitted per square meter is given by  $E^2/2$

# Power of incident wave is  $\frac{E_i^2 \cos \theta_1}{21}$

" " reflected wave is  $\frac{E_r^2 \cos \theta_1}{21}$

" " transmitted wave is  $\frac{E_t^2 \cos \theta_2}{22}$

conservation of energy

$$\frac{E_i^2 \cos \theta_1}{n_1} = \frac{E_r^2 \cos \theta_1}{n_1} + \frac{E_t^2 \cos \theta_2}{n_2}$$

Dividing by  $\frac{E_i^2 \cos \theta_1}{n_1}$

$$1 = \frac{E_r^2}{E_i^2} + \frac{n_1}{n_2} \cdot \frac{E_t^2}{E_i^2} \cdot \frac{\cos \theta_2}{\cos \theta_1}$$

$$1 - \frac{E_r^2}{E_i^2} = \frac{\sqrt{\mu/\epsilon_1}}{\sqrt{\mu/\epsilon_2}} \cdot \frac{E_t^2}{E_i^2} \cdot \frac{\cos \theta_2}{\cos \theta_1}$$

$$1 - \left(\frac{E_r}{E_i}\right)^2 = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \left(1 + \frac{E_r}{E_i}\right)^2 \frac{\cos \theta_2}{\cos \theta_1} \quad \left[ \because \frac{E_t}{E_i} = 1 + \frac{E_r}{E_i} \right]$$

$$\left(1 - \frac{E_r}{E_i}\right) \left(1 + \frac{E_r}{E_i}\right) = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \left(1 + \frac{E_r}{E_i}\right)^2 \frac{\cos \theta_2}{\cos \theta_1}$$

$$1 - \frac{E_r}{E_i} = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \left(1 + \frac{E_r}{E_i}\right) \frac{\cos \theta_2}{\cos \theta_1}$$

$$1 = \frac{E_r}{E_i} + \sqrt{\frac{\epsilon_2}{\epsilon_1}} \frac{\cos \theta_2}{\cos \theta_1} + \sqrt{\frac{\epsilon_2}{\epsilon_1}} \frac{\cos \theta_2}{\cos \theta_1} \cdot \frac{E_r}{E_i}$$

$$1 - \sqrt{\frac{\epsilon_2}{\epsilon_1}} \frac{\cos \theta_2}{\cos \theta_1} = \frac{E_r}{E_i} \left[ 1 + \sqrt{\frac{\epsilon_2}{\epsilon_1}} \frac{\cos \theta_2}{\cos \theta_1} \right]$$

$$\therefore R_h = \frac{E_r}{E_i} = \frac{1 - \sqrt{\epsilon_2/\epsilon_1} \frac{\cos \theta_2}{\cos \theta_1}}{1 + \sqrt{\epsilon_2/\epsilon_1} \frac{\cos \theta_2}{\cos \theta_1}}$$

when  $R_h$  is horizontal  
reflection coefficient

$$R_h = \frac{\cos \theta_1 \sqrt{\epsilon_1} - \sqrt{\epsilon_2} \cos \theta_2}{\cos \theta_1 \sqrt{\epsilon_1} + \sqrt{\epsilon_2} \cos \theta_2}$$

$$= \frac{\sqrt{\epsilon_1} \cos \theta_1 - \sqrt{\epsilon_2 - \epsilon_2 \sin^2 \theta_2}}{\sqrt{\epsilon_1} \cos \theta_1 + \sqrt{\epsilon_2 - \epsilon_2 \sin^2 \theta_2}} \quad \left[ \because \cos \theta_2 = \sqrt{1 - \sin^2 \theta_2} \right]$$

$$\therefore R_h = \frac{\sqrt{\epsilon_1} \cos \theta_1 - \sqrt{\epsilon_2 - \epsilon_1 \sin^2 \theta_1}}{\sqrt{\epsilon_1} \cos \theta_1 + \sqrt{\epsilon_2 - \epsilon_1 \sin^2 \theta_1}} \quad \left[ \because \frac{\sin \theta_1}{\sin \theta_2} = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \right]$$

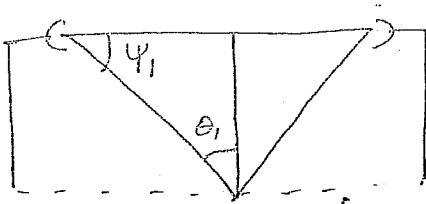
For earth surface where  $\sigma \rightarrow \infty$  for conductor we find  $\epsilon$  as

$$\begin{aligned}\nabla \times \vec{H} &= \epsilon j\omega_0 \vec{E} + \sigma \vec{E} \\ &= j\omega_0 \vec{E} \left( \epsilon + \frac{\sigma}{j\omega_0} \right) \\ &= \vec{E} \epsilon'\end{aligned}$$

$$\therefore \epsilon' = \epsilon + \frac{\sigma}{j\omega_0} \text{ is permittivity of earth.}$$

# In our case medium 1 is air,  $\therefore \epsilon_1 = \epsilon_r$  & medium 2 is earth,  $\therefore \epsilon_2 = \epsilon' = \left( \epsilon + \frac{\sigma}{j\omega_0} \right)$

$$\therefore R_n = \frac{E_r}{E_i} = \frac{\sqrt{\epsilon_r} \cos \theta_1 - \sqrt{\epsilon' - \epsilon_r \sin^2 \theta_1}}{\sqrt{\epsilon_r} \cos \theta_1 + \sqrt{\epsilon' - \epsilon_r \sin^2 \theta_1}} \quad \text{--- (2)}$$



$$\begin{aligned}\theta_1 &= 90^\circ - \psi_1 \\ \cos \theta_1 &= \cos(90^\circ - \psi_1) = \sin \psi_1 \text{ \& } \\ \sin \theta_1 &= \sin(90^\circ - \psi_1) = \cos \psi_1\end{aligned}$$

And

$$\epsilon' = \epsilon + \frac{\sigma}{j\omega_0}$$

$$\frac{\epsilon'}{\epsilon_r} = \frac{\epsilon}{\epsilon_r} + \frac{\sigma}{j\omega_0 \epsilon_r}$$

$$\therefore \frac{\epsilon'}{\epsilon_r} = \epsilon_{rel} - \frac{j\sigma}{\omega_0 \epsilon_r}$$

$\epsilon_{rel}$  = relative permittivity

$$\boxed{\frac{\epsilon'}{\epsilon_r} = \epsilon_{rel} - jX} \quad , X = \frac{\sigma}{\omega_0 \epsilon_r}$$

Then, dividing eqn (2) by  $\sqrt{\epsilon_r}$  on numerator & denominator.

$$R_n = \frac{E_r}{E_i} = \frac{\cos \theta_1 - \sqrt{\frac{\epsilon'}{\epsilon_r} - \sin^2 \theta_1}}{\cos \theta_1 + \sqrt{\frac{\epsilon'}{\epsilon_r} - \sin^2 \theta_1}}$$

$$\therefore R_n = \frac{\sin \psi_1 - \sqrt{(\epsilon_{rel} - jX) - \cos^2 \psi_1}}{\sin \psi_1 + \sqrt{(\epsilon_{rel} - jX) - \cos^2 \psi_1}}$$

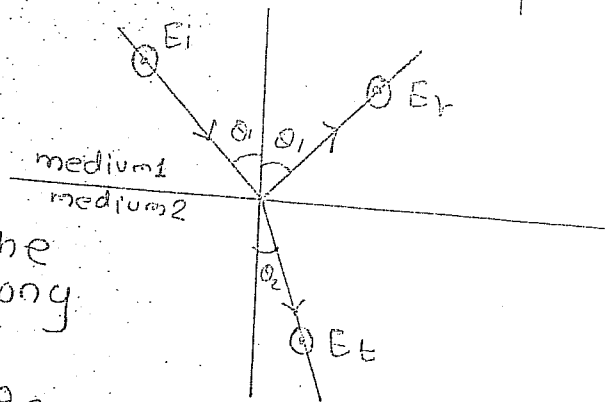


## CASE 2: Vertical Polarization

When electric field vector is perpendicular to boundary surface and magnetic field vector parallel to boundary surface.

# we have,

$$\frac{\sin \theta_1}{\sin \theta_2} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$



# Applying boundary condition on the tangential component of  $E$  along the boundary

$$(E_i - E_r) \cos \theta_1 = E_t \cos \theta_2$$

$$\therefore \frac{E_t}{E_i} = \left(1 - \frac{E_r}{E_i}\right) \frac{\cos \theta_1}{\cos \theta_2}$$

# From the conservation of energy (As in case 1)

$$1 - \frac{E_r^2}{E_i^2} = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \frac{E_t^2}{E_i^2} \frac{\cos \theta_2}{\cos \theta_1}$$

$$1 - \left(\frac{E_r}{E_i}\right)^2 = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \left(1 - \frac{E_r}{E_i}\right)^2 \frac{\cos \theta_1}{\cos \theta_2}$$

$$\left(1 + \frac{E_r}{E_i}\right) = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \left(1 - \frac{E_r}{E_i}\right) \frac{\cos \theta_1}{\cos \theta_2}$$

$$\frac{E_r}{E_i} \left(1 + \sqrt{\frac{\epsilon_2}{\epsilon_1}} \frac{\cos \theta_1}{\cos \theta_2}\right) = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \frac{\cos \theta_1}{\cos \theta_2} - 1$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\frac{\epsilon_2}{\epsilon_1}} \frac{\cos \theta_1}{\cos \theta_2} - 1}{\sqrt{\frac{\epsilon_2}{\epsilon_1}} \frac{\cos \theta_1}{\cos \theta_2} + 1}$$

we have,

$$\frac{E_r}{E_i} = R_V, \text{ where } R_V \text{ is vertical reflection coeff}$$

$$R_V = \frac{\sqrt{\epsilon_2} \cos \theta_1 - \sqrt{\epsilon_1} \cos \theta_2}{\sqrt{\epsilon_2} \cos \theta_1 + \sqrt{\epsilon_1} \cos \theta_2}$$

$$R_V = \frac{\sqrt{\epsilon_2} \cos \theta_1 - \sqrt{\epsilon_1} (1 - \sin^2 \theta_2)}{\sqrt{\epsilon_2} \cos \theta_1 + \sqrt{\epsilon_1} (1 - \sin^2 \theta_2)}$$

$$\therefore \cos \theta_2 = \sqrt{1 - \sin^2 \theta_2}$$

$$R_v = \frac{\sqrt{\epsilon_2} \cos \theta_1 - \sqrt{\epsilon_1 - \frac{\epsilon_1^2}{\epsilon_2} \sin^2 \theta_1}}{\sqrt{\epsilon_2} \cos \theta_1 + \sqrt{\epsilon_1 - \frac{\epsilon_1^2}{\epsilon_2} \sin^2 \theta_1}}$$

$$\left[ \because \frac{\sin \theta_1}{\sin \theta_2} = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \right]$$

dividing by  $\epsilon_1$  and multiplying by  $\sqrt{\epsilon_2}$

$$R_v = \frac{\frac{\epsilon_2}{\epsilon_1} \cos \theta_1 - \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2 \theta_1}}{\frac{\epsilon_2}{\epsilon_1} \cos \theta_1 + \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2 \theta_1}}$$

We have medium 1 is air and medium 2 is earth

$$\therefore \epsilon_1 = \epsilon_r$$

$$\epsilon_2 = \epsilon' = \epsilon + \frac{\sigma}{j\omega_0}$$

$$\therefore \frac{\epsilon_2}{\epsilon_1} = \frac{\epsilon'}{\epsilon_r} = \epsilon_{rel} - jx \quad [\text{as in first case}]$$

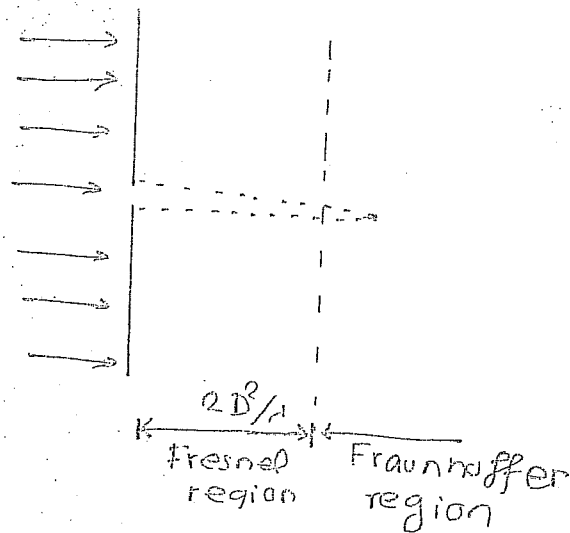
$$\text{and } \theta_1 = \theta_0 - \psi_1$$

$$\therefore R_v = \frac{(\epsilon_{rel} - jx) \sin \psi_1 - \sqrt{(\epsilon_{rel} - jx) - \cos^2 \psi_1}}{(\epsilon_{rel} - jx) \sin \psi_1 + \sqrt{(\epsilon_{rel} - jx) - \cos^2 \psi_1}}$$

➤ more the  $R_h$  or  $R_v$ , better is the radiation

## \* DIFFRACTION

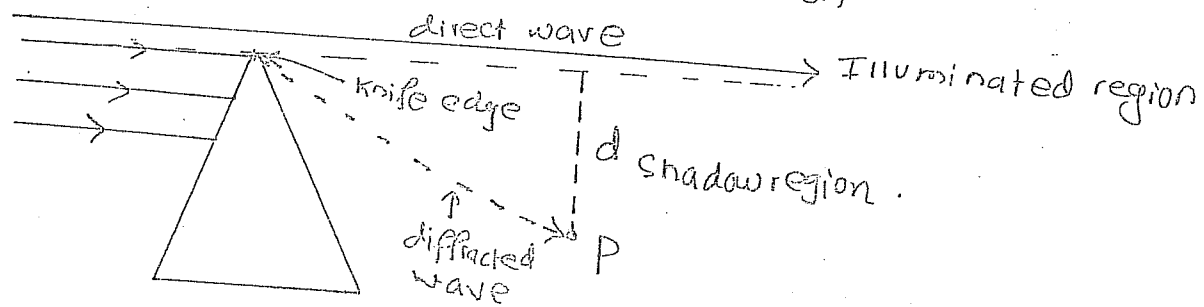
- # When waves pass through small opening or obstacles the wave deviate from straight line path and enters a region that would otherwise be shadowed.



- # Fresnel diffraction or near field diffraction is the diffraction pattern obtained close to diffracting edge.
- # Fraunhofer region is the diffraction pattern obtained far from the source of diffraction.

### Knife edge diffraction

The bending of electromagnetic waves around the obstacles when obstacle acts as sharp edge is known as knife edge diffraction.



$$P_{avg} = \frac{r_1}{4\pi^2 d^2}$$

## \*Ground Wave attenuation factor.

† The waves which are guided by the conducting surface of the earth along which they propagate are known as ground waves.

# As the waves travel along the ground, they get attenuated. The attenuation of ground waves as they travel along the surface of the earth is proportional to the frequency.

† The field strength for ground waves for flat earth is

$$E_g = \frac{E_0 A}{d}$$

where,  $E_0$  = Ground wave field strength at surface at unit distance

$A$  = Attenuation factor which accounts for losses in earth surface

$d$  = distance between Tx & Rx

# The factor  $A$  includes losses in the ground and is the function of frequency dielectric constant.

# ELECTROMAGNETIC PROPAGATION AND ANTENNA

## CHAPTER-4

### PROPAGATION IN RADIO FREQUENCY

By :Rajan Sharma

#### THE EARTH'S ATMOSPHERE

The earth's atmosphere is divided into three separate regions, or layers. They are the troposphere, the stratosphere, and the ionosphere.

##### 1. Troposphere

Almost all weather phenomena take place in the troposphere. The temperature in this region decreases rapidly with altitude. Clouds form, and there may be a lot of turbulence because of variations in the temperature, pressure, and density. These conditions have a profound effect on the propagation of radio waves.

##### 2. Stratosphere

The stratosphere is located between the troposphere and the ionosphere. The temperature throughout this region is almost constant and there is little water vapor present. Because it is a relatively calm region with little or no temperature change, the stratosphere has almost no effect on radio waves.

##### 3. Ionosphere

This is the most important region of the earth's atmosphere for long distance, point-to-point communications. Because the existence of the ionosphere is directly related to radiation emitted from the sun, the movement of the earth about the sun or changes in the sun's activity will result in variations in the ionosphere. These variations are of two general types: (1) those that more or less occur in cycles and, therefore, can be predicted with reasonable accuracy; and (2) those that are irregular as a result of abnormal behavior of the sun and, therefore, cannot be predicted. Both regular and irregular variations have important effects on radio-wave propagation.

#### Ionization

In ionization, high-energy ultraviolet light waves from the sun periodically enter the ionosphere, strike neutral gas atoms, and knock one or more electrons free from each atom. When the electrons are knocked free, the atoms become positively charged (positive ions) and remain in space, along with the negatively charged free electrons. The free electrons absorb some of the ultraviolet energy that initially set them free and form an ionized layer.

## Recombination

Recombination is the reverse process of ionization. It occurs when free electrons and positive ions collide, combine, and return the positive ions to their original neutral state. Like ionization, the recombination process depends on the time of day. Between early morning and late afternoon, the rate of ionization exceeds the rate of recombination. During this period the ionized layers reach their greatest density and exert maximum influence on radio waves. However, during the late afternoon and early evening, the rate of recombination exceeds the rate of ionization, causing the densities of the ionized layers to decrease. Throughout the night, density continues to decrease, reaching its lowest point just before sunrise.

## Ionospheric Layers

The ionosphere is composed of three distinct layers, designated from lowest level to highest level (D, E, and F). In addition, the F layer is divided into two layers, designated F1 (the lower level) and F2 (the higher level).

The presence or absence of these layers in the ionosphere and their height above the earth vary with the position of the sun. At high noon, radiation in the ionosphere above a given point is greatest, while at night it is minimum.

### D LAYER.

- Lowest region of ionosphere.
- Ionization in the D layer is low because less ultraviolet light penetrates to this level.
- Disappears at night
- At very low frequencies, the D layer and the ground act as a huge waveguide, making communication possible only with large antennas and high power transmitters.
- At low and medium frequencies, the D layer becomes highly absorptive, which limits the effective daytime communication range to about 200 miles.
- Signals passing through the D layer normally are not absorbed but are propagated by the E and F layers.

### E LAYER.

- Layer next to D layer
- The rate of ionospheric recombination in this layer is rather rapid after sunset, causing it to nearly disappear by midnight.
- The E layer permits medium-range communications on the low-frequency.
- The range of communication in sporadic-E often exceeds 1000 miles, but the range is not as great as with F layer propagation.

## F LAYER.

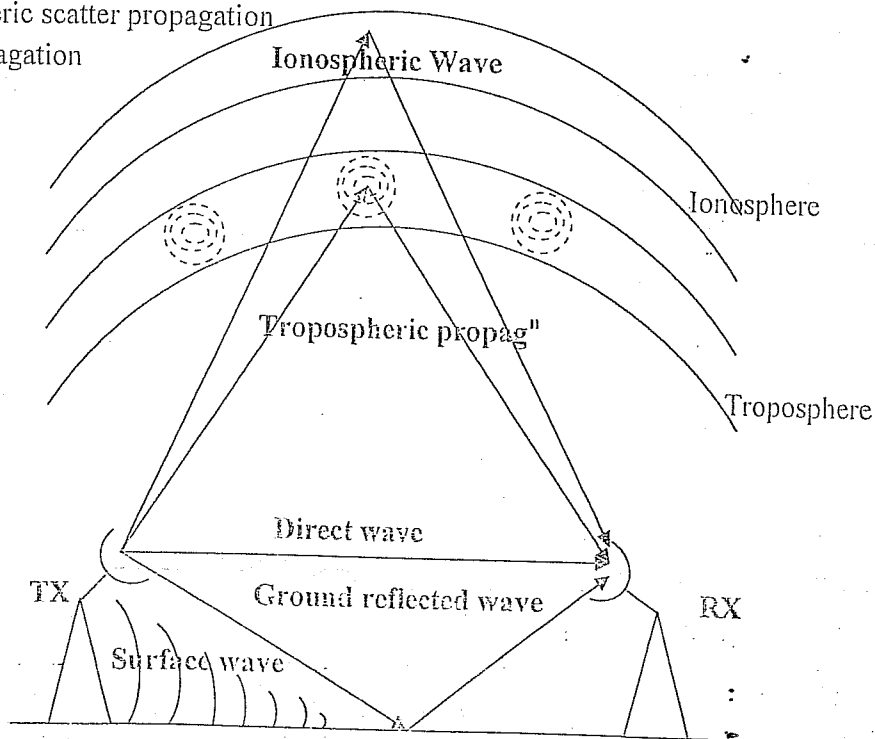
- This layer remains all the time irrespective of time.
- During daylight hours, the F layer separates into two layers, F1 and F2.
- During the night, the F1 layer usually disappears.
- The F layer produces maximum ionization during the afternoon hours, but the effects of the daily cycle are not as pronounced as in the D and E layers.
- Atoms in the F layer stay ionized for a longer time after sunset.
- Since the F layer is the highest of the ionospheric layers, it also has the longest propagation capability.
- For horizontal waves, the single-hop F2 distance can reach 3000 miles.
- The F layer is responsible for most high frequency, long-distance communications.

## MODES OF WAVE PROPAGATION

[2004,2009 PU: Explain different radio wave propagation methods]

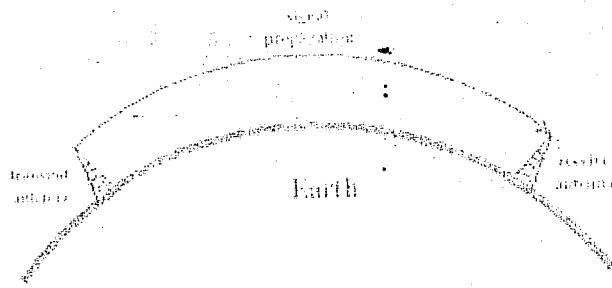
The methods by which radio waves propagate from transmitter to receiver can be of following types:

1. Ground wave or surface wave propagation
2. Sky wave propagation or ionospheric propagation
3. Space wave propagation
4. Tropospheric scatter propagation
5. Duct propagation



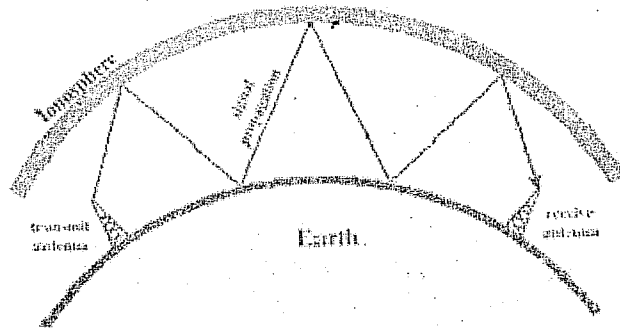
## 1. Ground wave or Surface wave propagation

- In ground wave propagation a vertically polarized EM wave is radiated at zero or small angle with earth surface. These waves are guided by the conducting surface of the earth along which they are propagated. Such waves are called Ground wave or surface wave.
- The ground wave is guided along the surface of the earth just as an electromagnetic wave is guided by a waveguide or transmission line.
- Surface wave permits the propagation around the curvature of the earth.
- The attenuation of ground wave is directly proportional to the frequency of waves. Thus ground wave is applicable in low frequency communication
- Frequency **up to 2 MHz**
- Example – AM radio



## 2. Sky wave propagation or ionospheric propagation

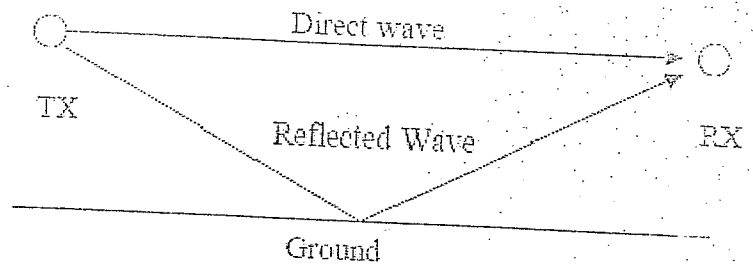
- The sky waves are of practical importance at medium and high frequencies for very long distance radio communications.
- Applicable to frequency range of **2MHz to 30 MHz**
- In this mode of propagation electromagnetic waves reach the receiving point after reflection from the ionized region in the upper atmosphere called ionosphere
- Signal can travel a number of hops, back and forth between ionosphere and earth's surface.
- Ionosphere contains large concentration of charge gaseous ions, free electrons, neutral molecules etc. These large concentrations tend to bend the passing EM wave through process of refraction.
- The deviation of EM wave depends on frequency, angle of incidence, density of charged particles, thickness of ionosphere etc
- Examples – Military Comm., Amateur radio.



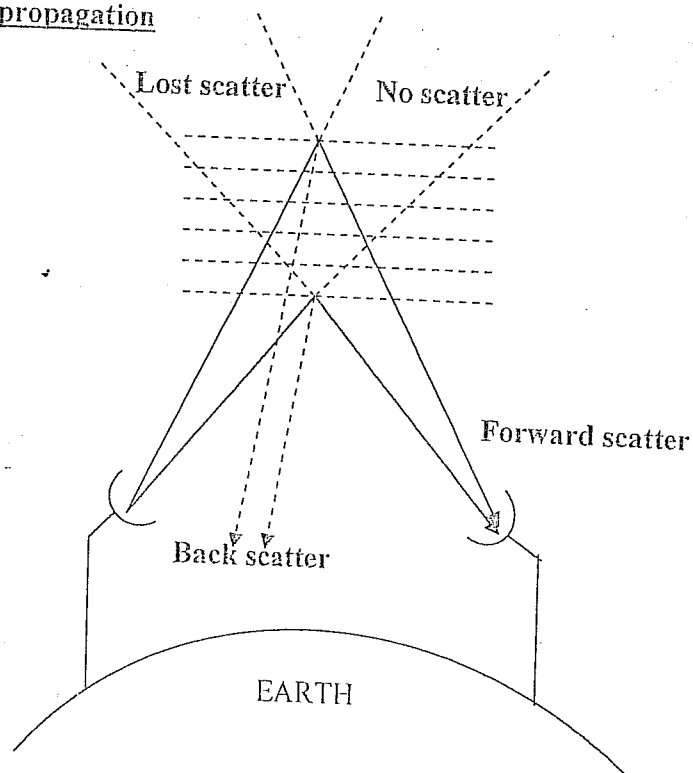


### 3. Space wave propagation

- In this mode of propagation, electromagnetic waves from the transmitting antenna reach the receiving antenna either directly or after reflections from the ground.
- Transmitting and receiving antennas must be within line of sight.
- EM waves above 30 MHz are not reflected by the ionosphere. Thus VHF and UHF communication is not possible through ionospheric propagation. So for this type of communication we use Space wave propagation.
- Frequency **above 30 MHz**
- The height of transmitter and receiver can improve the communication.
- Examples: TV, satellite, FM radio.

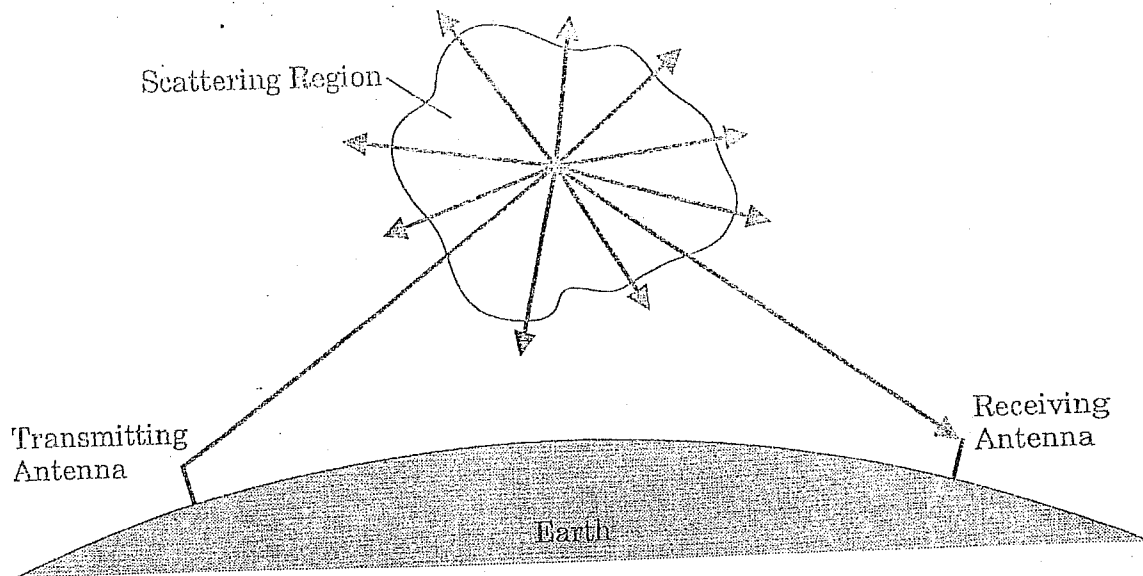


### 4. Tropospheric scatter propagation



- This mode uses certain properties of troposphere.
- Troposphere contains certain blocks of high density particles and when EM waves falls on these blocks it gets scattered and reflected to the receiver
- This mode can propagate much beyond than LOS propagation.

- Forward scatter propagation or simply <sup>Scatter</sup> propagation is of practical importance at VHF, UHF and microwaves
- It provides reliable communication across large stretches of water e.g inland lakes, islands and offshore islands.
- It also reduces the number of stations required to cover a given large distance as compared to radio links.



#### 5. Duct propagation [2008 PU short notes]

- It is also known as **super refraction**
- A duct is something that will confine whatever is traveling along it into a narrow 'pipe'.
- The atmosphere can assume a structure that will produce a similar effect on radio waves. When a radio wave enters a duct it can travel with low loss over great distances. The atmosphere will then act in the manner of a giant optical fiber, trapping the radio wave within the layer of high refractive index.
- A wave trapped in a duct can travel beyond the radio horizon with very little loss.
- In atmosphere the air is frequently turbulent and there are layers of air one above another having different temperature and water vapor contents.
- When layers of warm air form above layers of cold air, the condition known as temperature inversion develops. This phenomenon causes ducts or channels to be formed, by sandwiching cool air either between the surface of the earth and a layer of warm air, or between two layers of warm air. If the radio wave enters the duct at a very low angle of incidence, VHF and UHF transmissions may be propagated far beyond normal line-of-sight distances (thousand of KMs).

- These long distances are possible because of the different densities and refractive qualities of warm and cool air. The sudden change in densities when a radio wave enters the warm air above the duct causes the wave to be refracted back toward earth. When the wave strikes the earth or a warm layer below the duct, it is again reflected or refracted upward and proceeds on through the duct with a multiple-hop type of action.

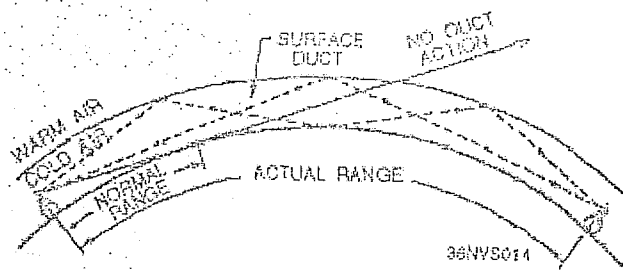
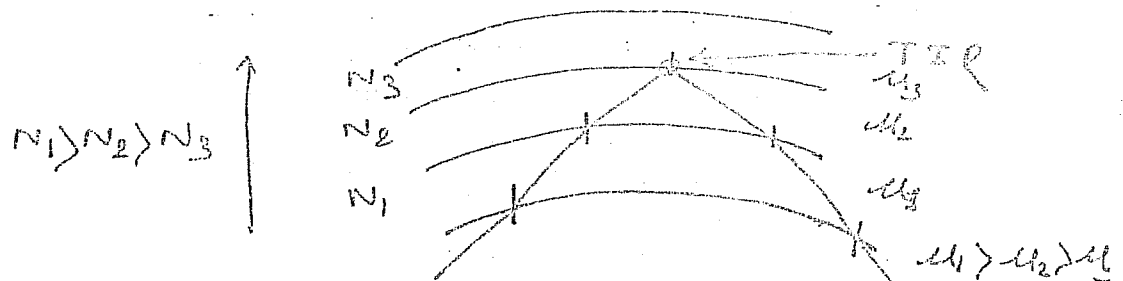


Figure 1-14.—Duct effect caused by temperature inversion.

## PROPAGATION OF RADIO WAVES THROUGH IONOSPHERE

### REFLECTION BY IONOSPHERE [VVVIMPI]

[2005, 2006, 2007, 2009, 2010 PU, Derive Refractive index of Ionosphere and MUF]



# In Ionosphere the angle by which the waves deviates depends upon the following

1. Frequency of Radio wave
2. Angle of incidence at which wave enters the ionosphere
3. Density of charged particles in the ionosphere

# Let Electric field of value

$E = E_m \sin \omega t$  (V/m) is acting on a cubic meter of space of ionosphere

Force exerted =  $-eE$  Newton

Again  $F = ma$

$$-eE = m \frac{dv}{dt}$$

where  $m = \text{mass of } e^-$   
 $e = \text{charge of } e^-$

we know,

$$(\text{velocity}) \quad v = - \int \frac{eE}{m} dt$$

$$= - \frac{e}{m} \int E_m \sin \omega t dt$$

$$= - \frac{e E_m}{m} \frac{-\cos \omega t}{\omega}$$

$$v = \frac{e E_m \cos \omega t}{m \omega} \quad \text{--- (1)}$$

current distribution by  $N$  electron moving with instantaneous velocity  $v$  is.

$$i_e = -Ne v_i$$

$$= -Ne \frac{e E_m \cos \omega t}{m \omega}$$

$$\therefore i_e = -\frac{Ne^2}{m\omega} E_m \cos \omega t \quad \text{--- (1)}$$

which shows  $i_e$  lags behind by  $90^\circ$ . Thus this current is inductive.

# Beside this, usual capacitive current  $i_c$  exists

$$i_c = \frac{dD}{dt} = \frac{d(\epsilon_0 E)}{dt} = \epsilon_0 \frac{d}{dt} (E_m \sin \omega t)$$

$$\therefore i_c = \epsilon_0 \omega E_m \cos \omega t$$

# Now, Total current  $i = i_c + i_e$

$$i = \omega E_m \cos \omega t \left[ \epsilon_0 - \frac{Ne^2}{m\omega^2} \right]$$

# The term  $\left[ \epsilon_0 - \frac{Ne^2}{m\omega^2} \right]$  is effective dielectric constant of ionosphere  $\therefore \epsilon = \epsilon_0 - \frac{Ne^2}{m\omega^2}$

# Relative dielectric constant,

$$\epsilon_r = \frac{\epsilon}{\epsilon_0} = 1 - \frac{Ne^2}{m\omega^2 \epsilon_0}$$

Now,

$$\mu = \sqrt{\epsilon_r} = \sqrt{1 - \frac{Ne^2}{m\omega^2 \epsilon_0}}$$

Thus we get,

$$\mu = \sqrt{1 - \frac{81N}{f^2}} < 1.$$

$$\begin{aligned} m &= \text{mass of } e^- \\ &= 9.107 \times 10^{-31} \text{ kg} \\ e &= 1.67 \times 10^{-19} \\ \epsilon_0 &= 8.854 \times 10^{-12} \\ N &= \text{electron density} \\ &\quad (\text{per } m^3) \end{aligned}$$

This is the required expression for refractive Index of ionosphere.

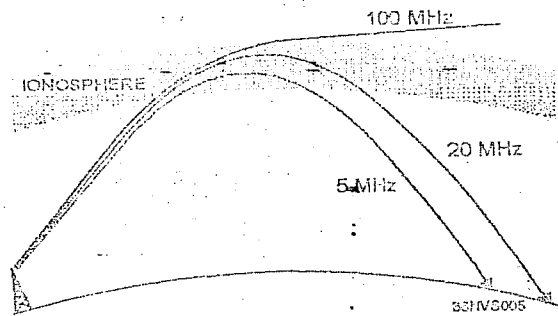
3 If  $(1 - \frac{81N}{f^2})$  is +ve i.e.  $> 0$  there will be refraction of EM wave

3 If  $(1 - \frac{81N}{f^2})$  is -ve i.e.  $< 0$ , EM wave will be reflected back to earth

3 If  $(1 - \frac{81N}{f^2}) = 0$ , neither reflection nor refraction, it will dissipate in ionised layer

## CRITICAL FREQUENCY [2007 PU]

- The lower the frequency of a radio wave, the more rapidly the wave is refracted by a given degree of ionization.
- Figure shows three separate waves of differing frequencies entering the ionosphere at the same angle. The 5-MHz wave is refracted quite sharply, while the 20-MHz wave is refracted less sharply and returns to earth at a greater distance than the 5-MHz wave. Notice that the 100-MHz wave is lost into space.
- For any given ionized layer, there is a maximum frequency at which the wave can be reflected back to earth at vertical incidence. This frequency is called the critical frequency.
- Critical frequency  $f_c$  corresponds to the maximum electron density  $N_{max}$
- We have,



$$\mu = \frac{\sin i}{\sin r} = \sqrt{1 - \frac{81N}{f^2}}$$

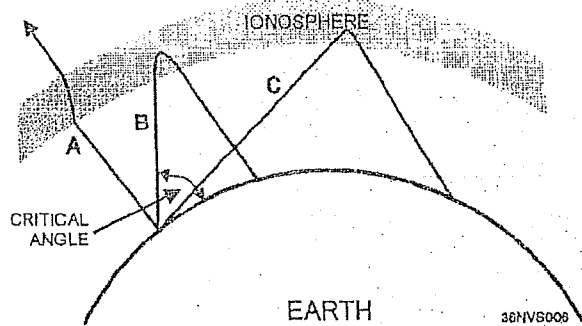
By definition,  $i=0$ ,  $N=N_{max}$  and  $f=f_c$

$$\mu \frac{\sin 0}{\sin r} = \sqrt{1 - \frac{81N_{max}}{f_c^2}} = 0$$

$$f_c = \sqrt{81N_{max}}$$

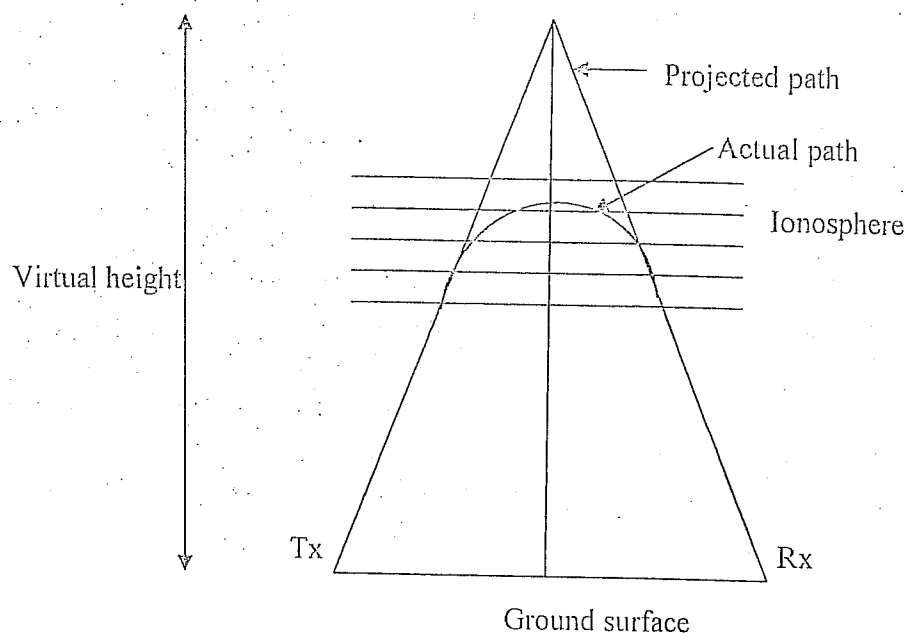
## CRITICAL ANGLE

- When a radio wave encounters a layer of the ionosphere, that wave is returned to earth at the same angle (roughly) as its angle of incidence.
- Figure shows three radio waves of the same frequency entering a layer at different incidence angles. The angle at which wave A strikes the layer is too nearly vertical for the wave to be refracted to earth. However, wave B is refracted back to earth.
- The angle between wave B and the earth is called the critical angle. Any wave, at a given frequency, that leaves the antenna at an incidence angle greater than the critical angle will be lost into space. This is why wave A was not refracted. Wave C leaves the antenna at the smallest angle that will allow it to be refracted and still return to earth.
- The critical angle for radio waves depends on the layer density and the wavelength of the signal.



### VERTICAL HEIGHT

- Virtual height is the height which the wave would reach if it were to propagate in a straight line in the ionosphere at the speed of light and then be refracted by the plane mirror like surface.
- Virtual height is always greater than actual height



### MAXIMUM USABLE FREQUENCY [2005,2008,2009 PU]

[2009 PU, Derive expression relating critical frequency and MUF]

- The higher the frequency of a radio wave, the lower the rate of refraction by the ionosphere.
- Therefore, for a given angle of incidence and time of day, there is a maximum frequency that can be used for communications between two given locations. This frequency is known as the **MAXIMUM USABLE FREQUENCY (MUF)**.
- Waves at frequencies above the MUF are normally refracted so slowly that they return to earth beyond the desired location or pass on through the ionosphere and are lost.
- Critical frequency is for vertical incidence whereas MUF is for specific angle of incidence.
- For the sky wave to return to earth, angle of reflection =  $90^\circ$ .

$$\mu = \frac{\sin i}{\sin r} = \sqrt{1 - \frac{81N}{f^2}}$$

$$\text{or, } \mu = \frac{\sin i}{\sin 90} = \sqrt{1 - \frac{81N_{\max}}{f_{\text{muf}}^2}}$$

$$\text{or, } \sin^2 i = \sqrt{1 - \frac{81 N_{max}}{f_{muf}^2}}$$

$$\text{or, } \sin^2 i = 1 - \frac{81 N_{max}}{f_{muf}^2}$$

$$\text{or, } \sin^2 i = 1 - \frac{f_c^2}{f_{muf}^2} \quad \therefore f_c = \text{critical frequency} = \sqrt{81 N_{max}}$$

$$\text{or, } \frac{f_c^2}{f_{muf}^2} = 1 - \sin^2 i$$

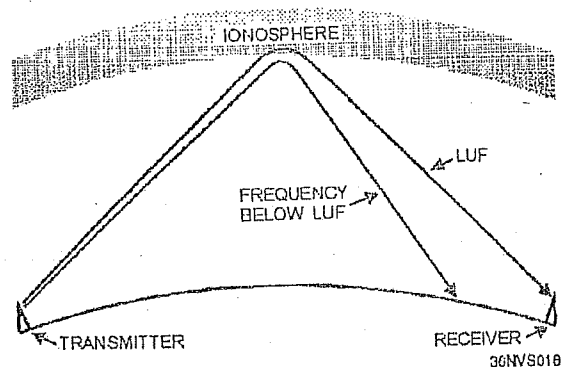
$$\text{or, } \frac{f_c^2}{f_{muf}^2} = \cos^2 i$$

$$\therefore f_{muf} = f_c \sec i$$

This is known as Secant law.

### LOWEST USABLE FREQUENCY

- Just as there is a MUF that can be used for communications between two points, there is also a minimum operating frequency that can be used known as the LOWEST USABLE FREQUENCY (LUF).
- As the frequency of a radio wave is lowered, the rate of refraction increases. So a wave whose frequency is below the established LUF is refracted back to earth at a shorter distance than desired, as shown in figure.
- As a frequency is lowered, absorption of the radio wave increases. A wave whose frequency is too low is absorbed to such an extent that it is too weak for reception.
- Atmospheric noise is also greater at lower frequencies. A combination of higher absorption and atmospheric noise could result in an unacceptable signal-to-noise ratio.
- For a given angle, ionospheric conditions, of incidence and set of the LUF depends on the refraction properties of the ionosphere, absorption considerations, and the amount of noise present.



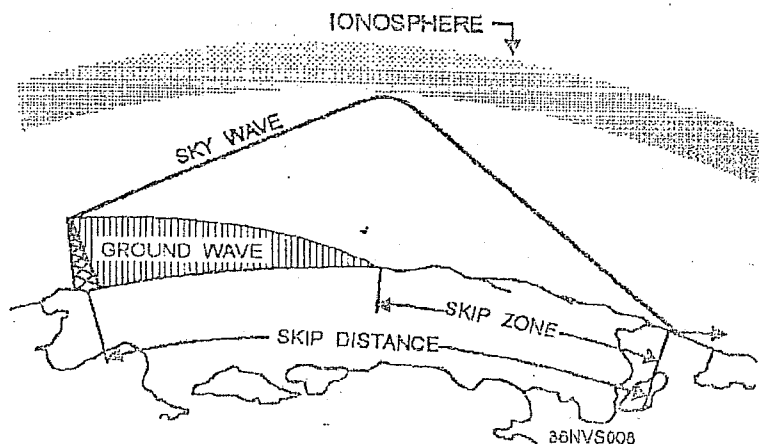


### OPTIMUM USABLE FREQUENCY [2007 PU]

- In practical radio communication for satisfactory reception of signal at receiving point it is essential that the frequency should be less than MUF and more than LUF such that absorption of waves by ionosphere be small.
- It should be high enough to avoid the problems of multipath fading, absorption, and noise encountered at the lower frequencies; but not so high as to be affected by the adverse effects of rapid changes in the ionosphere.
- A frequency that meets the above criteria is known as the OPTIMUM WORKING FREQUENCY
- The Optimum Working Frequency is roughly about 85% of the MUF, but the actual percentage varies and may be considerably more or less than 85 percent.

### SKIP DISTANCE [2006,2008 PU]

- The **skip distance** is the distance from the transmitter to the point where the sky wave first returns to the earth. i.e it is the nearest distance from the transmitter where the receiver can be placed.
- The skip distance depends on the wave's frequency and angle of incidence, and the degree of ionization.
- The **skip zone** is a zone of silence between the point where the ground wave is too weak for reception and the point where the sky wave is first returned to earth.
- The outer limit of the skip zone varies considerably, depending on the operating frequency, the time of day, the season of the year, sunspot activity, and the direction of transmission.



## Calculation of MUF and Skip Distance

CASE 1: Assuming Earth to be Flat

We have formula,

$$f_{muf} = f_{cr} \sec i$$

Here,

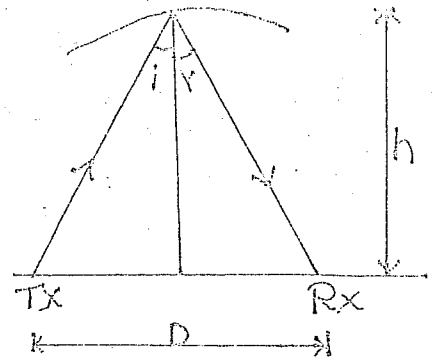
$$\tan i = \frac{D/2}{h} = \frac{D}{2h}$$

$$\therefore f_{muf} = f_{cr} \sqrt{1 + \tan^2 i}$$

$$f_{muf} = f_{cr} \sqrt{1 + \left(\frac{D}{2h}\right)^2}$$

Add Skip distance,

$$D = 2h \sqrt{\frac{f_{muf}^2}{f_{cr}^2} - 1}$$



CASE 2: When Earth is considered to be curve

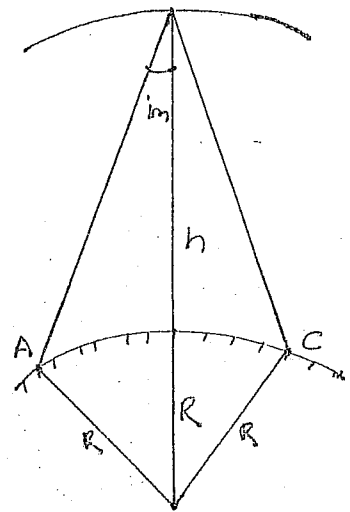
Then

$$f_{muf} = f_{cr} \sqrt{\frac{D^2/4 + \left[h + \frac{D^2}{8R}\right]^2}{\left(h + \frac{D^2}{8R}\right)^2}}$$

And

skip distance is

$$D = 2 \left[ \left( h + \frac{D^2}{8R} \right) \sqrt{\frac{f_{muf}^2}{f_{cr}^2} - 1} \right]$$



Q The reflection takes place at a height of 350 km and maximum density in the ionosphere corresponds to a 0.75 Refractive index at 10 MHz. What is the range for which MUF is 12 MHz. (Assume the earth to be flat)

Soln

$$\mu = \sqrt{1 - \frac{81 N_{max}}{f^2}}$$

$$0.75 = \sqrt{1 - \frac{81 N_{max}}{(10 \times 10^6)^2}}$$

$$\therefore N_{max} = 0.54 \times 10^{12}$$

$$f_{cr} = \sqrt{81 N_{max}}$$

$$= \sqrt{81 \times 0.54 \times 10^{12}}$$

$$= 6.61 \times 10^6$$

$$= 6.61 \text{ MHz}$$

Thus,

$$\text{Range, } D = 2h \sqrt{\frac{f_{muf}^2}{f_{cr}^2} - 1}$$

$$= 2 \times 350000 \sqrt{\frac{12^2}{6.61^2} - 1}$$

$$= 1.06 \times 10^6 \text{ m}$$

$$= 1060 \text{ km.}$$

2007. An Ionosphere has max<sup>m</sup>  $e^-$  density  $9 \times 10^{12} \text{ e/m}^3$ .  
Virtual height of layer is 125 km. For Flat  
earth Find MUF for Rx situated at 100 km distance.

Soln

Given,  $N_{max} = 9 \times 10^{12}$

$$h = 125000 \text{ m}$$

$$D = 100,000 \text{ m.}$$

we have,  $f_{cr} = \sqrt{81 N_{max}} = 2.7 \times 10^7 \text{ Hz} = 27 \text{ MHz}$

$$D = 2h \sqrt{\frac{f_{muf}^2}{f_{cr}^2} - 1}$$

$$100000 = 2 \times 125000 \sqrt{\frac{f_{muf}^2}{27^2} - 1}$$

$$0.4 = \sqrt{\frac{f_{muf}^2}{27^2} - 1}$$

$$\boxed{\therefore f_{muf} = 29.0798 \text{ MHz}}$$

## IRREGULAR VARIATIONS IN IONOSPHERE

- The ionosphere is highly dependent on the sun and hence its conditions vary continuously.
- The variations may be of regular and irregular type.
- Ionospheric predictions are therefore needed in planning of communication system.
- The irregular variations in the ionosphere are caused by following :

### 1. Gaseous movements

- The ionospheric layers are by no means stable.
- Strong horizontal and vertical movements of the gaseous masses causes the fluctuations in all kind of observations.

### 2. Sudden Ionospheric Disturbances (SID) [vvvimp, almost every year asked in PU]

- The occurrence of SID is caused by a bright solar eruption producing an unusually intense burst of ultraviolet light that is not absorbed by the F1, F2, or E layers. Instead, it causes the D-layer ionization density to greatly increase.
- As a result, frequencies above 1 or 2 megahertz are unable to penetrate the D layer and are completely absorbed.
- Commonly known as SID, these disturbances may occur without warning and may last for a few minutes to several hours.
- When SID occurs, long-range HF communications are almost totally blanked out. The radio operator listening during this time will believe his or her receiver has gone dead.

### 3. Ionospheric Storms

- Ionosphere storm is concerned with many other solar and terrestrial phenomena like magnetic storm.
- Cause of these storms is thought to be the emission of bursts of charged particle from the sun.
- The storms affect mostly the F2 layer, reducing its ion density and causing the critical frequencies to be lower than normal.
- What this means for communication purposes is that the range of frequencies on a given circuit is smaller than normal and that communications are possible only at lower working frequencies.
- This phenomenon lasts for several days at a time.

### 4. Polar cap absorption

- Occurring only in polar regions during a period of sun spot .

### 5. Sporadic E

- Irregular cloud-like patches of unusually high ionization, called the sporadic E, often forms near the normal E layer.
- Their exact cause is not known and their occurrence cannot be predicted.
- Sporadic E can appear and disappear in a short time during the day or night and usually does not occur at same time for all transmitting or receiving stations.

- The sporadic E-layer can be so thin that radio waves penetrate it easily and are returned to earth by the upper layers, or it can be heavily ionized and extend up to several hundred miles into the ionosphere.
- This condition may be either harmful or helpful to radio-wave propagation.
- On the harmful side, sporadic E may blank out the use of higher more favorable layers or cause additional absorption of radio waves at some frequencies. It can also cause additional multipath problems and delay the arrival times of the rays of RF energy.
- On the helpful side, the critical frequency of the sporadic E can be greater than double the critical frequency of the normal ionospheric layers. This may permit long-distance communications with unusually high frequencies. It may also permit short-distance communications to locations that would normally be in the skip zone.

## Formula for VHF Propagation (LOS)

### (Range of space wave Propagation)

# Space wave communication takes place upto the LOS distance. This distance depends on the height of Transmitting and receiving antennas.

# Let  $d$  be distance bet<sup>n</sup> Tx & Rx. The height of receiving and transmitting antenna be  $h_r$  &  $h_t$ .

# From the figure the LOS distance

$$d = d_1 + d_2$$

$$\begin{aligned} d_1 &= \sqrt{(h_t + r)^2 - r^2} \\ &= \sqrt{h_t^2 + r^2 + 2h_tr - r^2} \\ &= \sqrt{h_t^2 + 2h_tr} \end{aligned}$$

Similarly

$$d_2 = \sqrt{h_r^2 + 2h_rr}$$

where  $r$  = radius of earth  
 $= 6370 \text{ km}$

$$\therefore d = d_1 + d_2$$

$$= \sqrt{h_t^2 + 2h_tr} + \sqrt{h_r^2 + 2h_rr} \text{ m}$$

Since  $r \gg h_t, h_r$

$$\therefore d = \sqrt{2h_tr} + \sqrt{2h_rr} \text{ m}$$

$$d = \sqrt{2r} (\sqrt{h_t} + \sqrt{h_r}) \text{ m}$$

### # Using the concept of effective Earth Radius.

→ Since the RF waves are refracted in atmosphere, the radio wave travelling horizontally in earth's atmosphere follows a slightly downward curvature path.

→ It permits the direct rays to reach point slightly beyond the horizon as found by straight line or LOS path.

→ This effect is obtained by considering an effective radius of earth which is bit greater than actual radius

where  $k$  is found to be

$$k = \frac{1}{1 - r \frac{du}{dh}}$$

The value of  $\frac{du}{dh}$  corresponds to  $0.04 \times 10^{-6}/m$

&  $r = 6370$ . putting these values we get,

$$k = \frac{4}{3}$$

Thus we found  $d$  as

$$\begin{aligned} d &= \sqrt{2r} (\sqrt{h_t} + \sqrt{h_r})^m \\ &= \sqrt{2 \times \frac{4}{3} \times 637000} (\sqrt{h_t} + \sqrt{h_r})^m \\ &= 4121.48 (\sqrt{h_t} + \sqrt{h_r})^m \end{aligned}$$

$$d = 4.12 (\sqrt{h_t} + \sqrt{h_r}) \text{ km}$$

$\Rightarrow h_t$  &  $h_r$  in m.

### NUMERICAL

Q. A. T.V antenna has a height of 256 m & the receiving antenna has a height of 25 m. what is the max distance through which the TV signal could be received by space wave propagation. what is radio horizon in this case.

Soln.

$$\begin{aligned} d &= 4.12 (\sqrt{h_t} + \sqrt{h_r}) \text{ km} \\ &= 4.12 (\sqrt{256} + \sqrt{25}) \text{ km} \\ d &= 86.52 \text{ km.} \end{aligned}$$

$$\begin{aligned} \text{Radio Horizon} &= \sqrt{2r} h_t = (4.12 \cdot \sqrt{h_t}) \text{ km} \\ &= 4.12 \cdot \sqrt{256} \\ &= 65.92 \text{ km.} \end{aligned}$$



## Field Strength of space wave

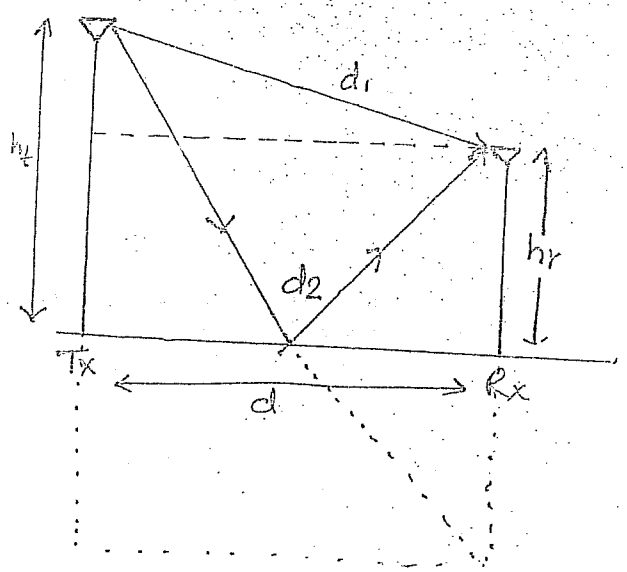
$h_t$  = height of Tx antenna

$h_r$  = " " Rx "

$d$  = distance bet<sup>n</sup> Tx & Rx ( $h_t - h_r$ )

$d_1$  = direct ray path

$d_2$  = Indirect " "



### APPROXIMATION

- (1) The Reflection is according to rule of geometrical optics
- (2) No Earth loss on reflection
- (3) Earth is flat for order of distance considered.

# The field strength received at the receiving point is the vector sum of the fields of the two rays.

# From the fig,

$$(h_t - h_r)^2 + d^2 = d_1^2 \quad \&$$

$$(h_t + h_r)^2 + d^2 = d_2^2$$

$$\therefore d_1 = [d^2 + (h_t - h_r)^2]^{1/2}$$

$$= d \left[ 1 + \left( \frac{h_t - h_r}{d} \right)^2 \right]^{1/2}$$

$$= d \left[ 1 + \frac{1}{2} \left( \frac{h_t - h_r}{d} \right)^2 + \dots \right]$$

$$= d \left[ 1 + \frac{1}{2} \left( \frac{h_t - h_r}{d} \right)^2 \right] \quad [\because \text{ignoring higher powers}]$$

$$\therefore d_1 = d + \frac{(h_t - h_r)^2}{2d}$$

Similarly,  $d_2 = d + \frac{(h_t + h_r)^2}{2d}$

# The path difference bet<sup>n</sup> direct & indirect ray is

$$p.d = d_2 - d_1$$

$$p.d = \frac{2h_t h_r}{d}$$

# Thus phase difference =  $\frac{2\pi}{\lambda} \times p.d$

$$\Delta = \frac{4\pi h_t h_r}{d\lambda}$$

# Beside the phase difference there is  $180^\circ$  phase difference due to reflection from ground.

∴ Total phase difference,  $\boxed{\Psi = 180 + \alpha}$

# If  $E_d$  &  $E_r$  are field strength due to direct and reflect ray at Rx antenna, the resultant field strength will be

∴  $E_T = E_D + E_R e^{-j\Psi}$

# we have assumed that amplitude doesn't decrease on reflection,

∴  $E_D = E_R = E_s$

∴  $E_T = E_s (1 + e^{-j\Psi})$

$E_T = E_s (1 + \cos\Psi - j\sin\Psi)$

$|E_T| = E_s \sqrt{(1 + \cos\Psi)^2 - (j\sin\Psi)^2}$

$= E_s \sqrt{1 + \cos^2\Psi + 2\cos\Psi + \sin^2\Psi}$

$= E_s \sqrt{2 + 2\cos\Psi}$

$= E_s \sqrt{2(1 + \cos\Psi)}$

$= E_s \sqrt{2 \cdot 2 \cos^2\Psi/2}$

$\boxed{|E_T| = 2 E_s \cos\Psi/2}$

$|E_T| = 2 E_s \cos\left(\frac{\alpha + \pi}{2}\right)$   $[\because \Psi = \alpha + \pi]$

$= 2 E_s \cos(\pi/2 + \alpha/2)$

$= 2 E_s \sin\alpha/2$

$= 2 E_s \sin\left(\frac{4\pi h_t h_r}{2d\lambda}\right)$   $[\because \alpha = \frac{4\pi h_t h_r}{d\lambda}]$

$= 2 E_s \sin\left(\frac{2\pi h_t h_r}{d\lambda}\right)$

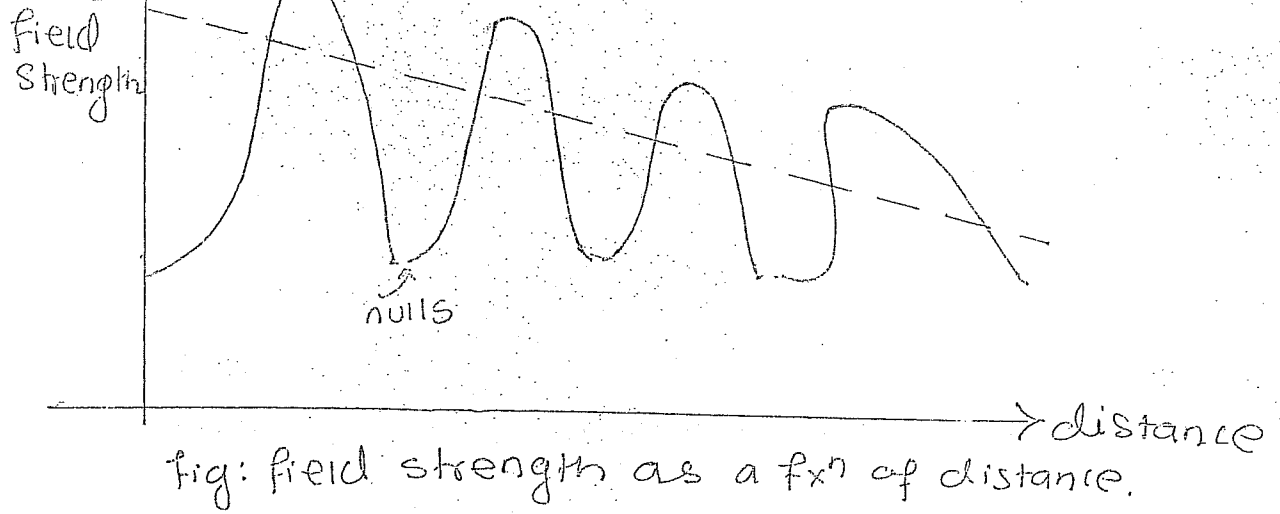
If  $E_0$  be field strength of ray at unit distance

$|E_T| = \frac{2 E_0}{d} \sin\left(\frac{2\pi h_t h_r}{d\lambda}\right)$

Since,  $d \gg h_t h_r$

$\boxed{|E_T| = \frac{4\pi h_t h_r}{\lambda d^2}}$

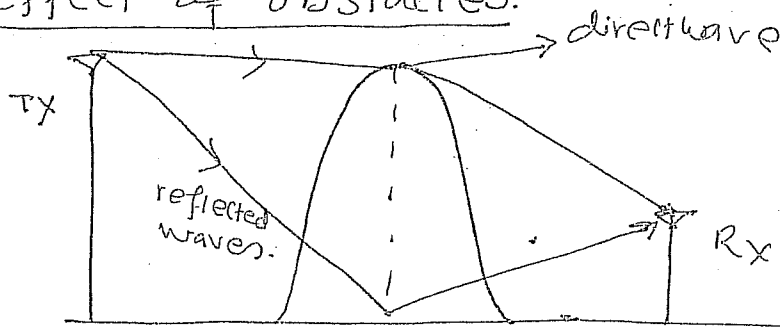
$[\because \sin\theta \approx \theta, \text{ when } \theta \text{ is small}]$



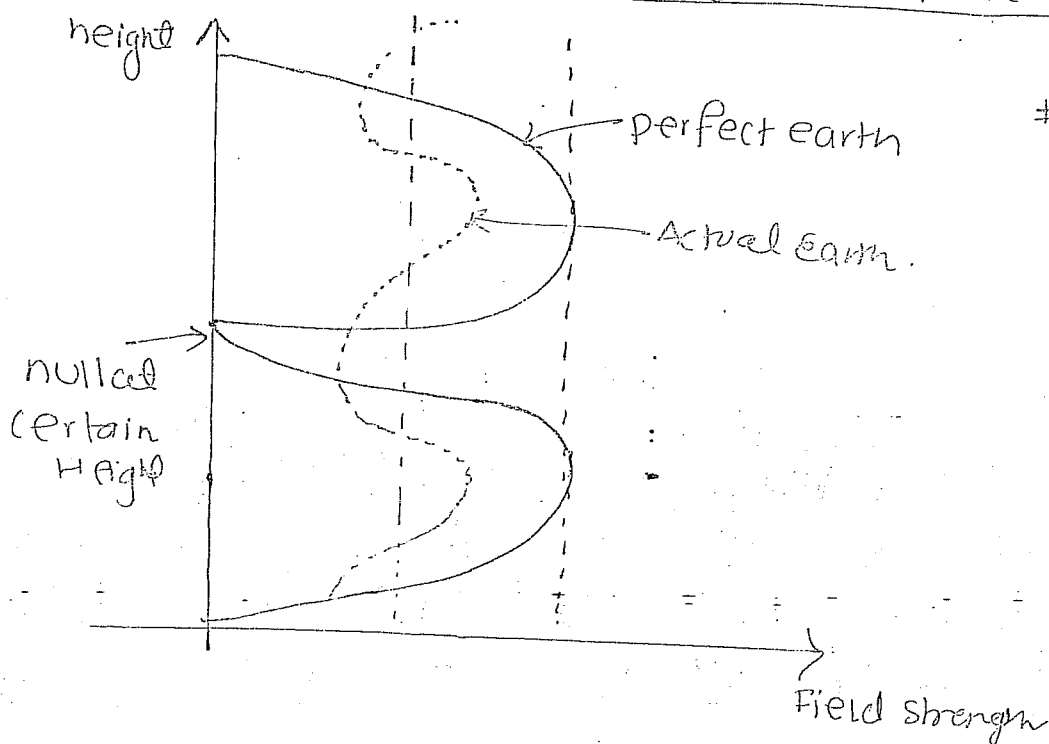
### \* various considerations in space wave propagation

- ① Effect of Earth roughness & imperfections on field strength in interface zone.
  - due to finite conductivity of earth
  - Attenuation more in horizontal polarization

### ② Effect of obstacles.



### ③ variation of field strength with height



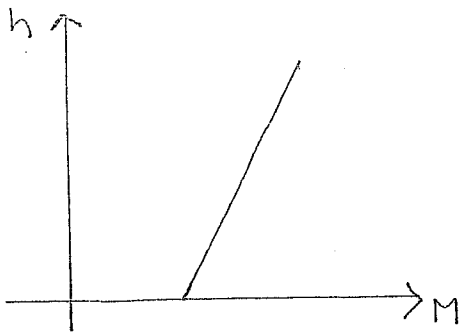
- # Location of Nulls & maxima depends upon
- (a) Height of Tx antenna
  - (b) Frequency
  - (c) distance.

# Atmospheric conditions

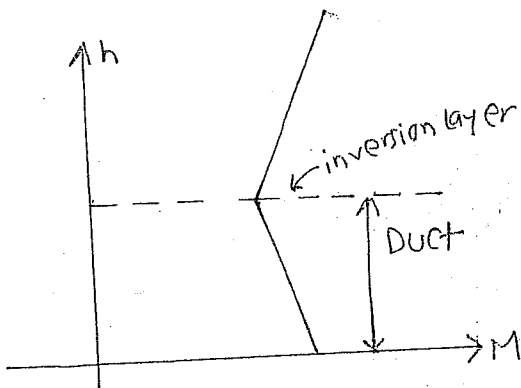
- # wave propagation depends upon the change of season, atmospheric condition, raining & various other phenomenon
- # The path of ray travelling in the atmosphere depends on the refractive index ( $n$ ) of the air:
- # In wave propagation the actual refractive index is modified into a new value  $M$  defined as

$$M = (n - 1 + h/a) \times 10^6$$

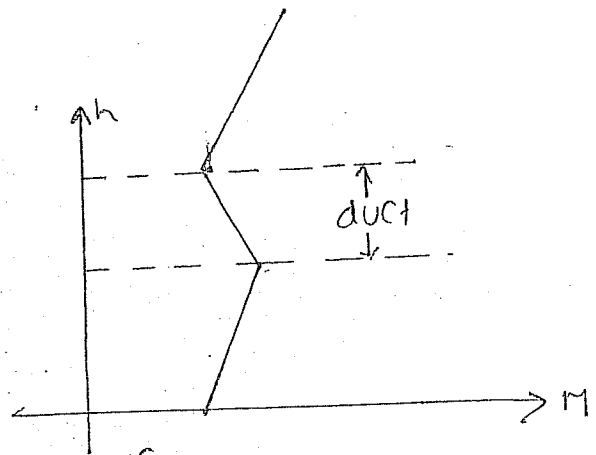
where  $n$  = refractive index  
 $h$  = height above ground  
 $a$  = radius of earth.



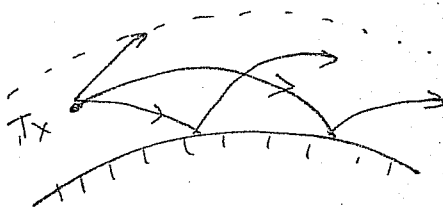
① Standard atm



② Ground based duct



③ elevated duct



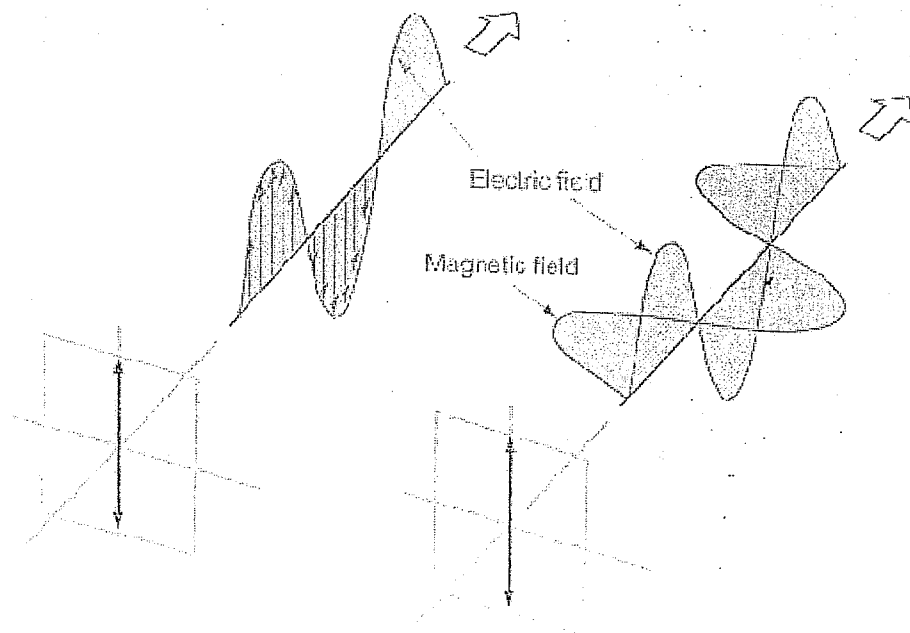
Propagation inside duct

## OTHER TOPICS THAT ARE ASKED IN PU EXAM

### WAVE POLARIZATION

- Polarization of wave is the orientation of electric field in the certain direction being radiated by the transmitting system..
- The plane of polarization of a radio wave is the plane in which the E-field propagates with respect to the Earth.
- The polarization of transmitting antenna and receiving antenna must be the same for maximum signal energy to be induced in receiving system.
- Antenna polarization is an important consideration when selecting and installing antennas. Most wireless communication systems use either linear (vertical, horizontal) or circular polarization.

#### 1. Linear polarization

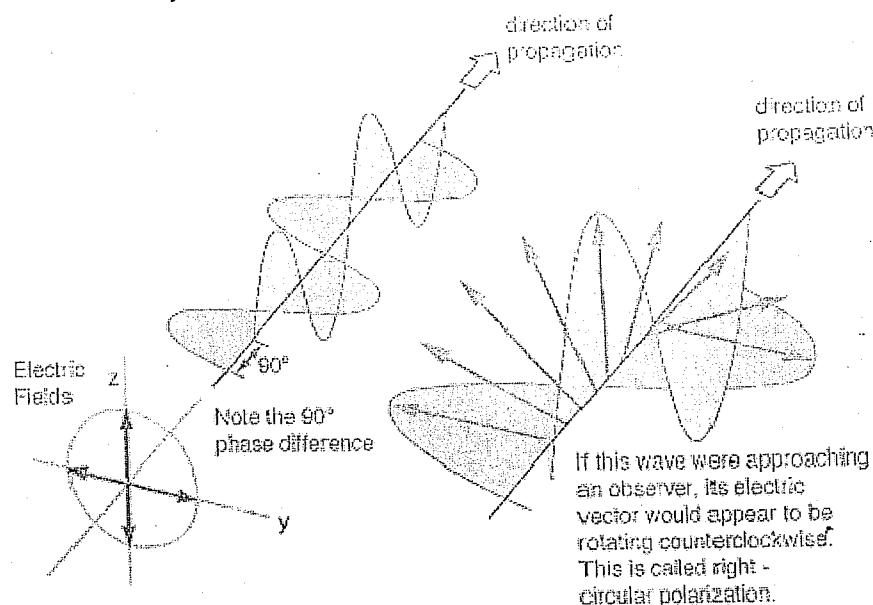


- If the electric field vector at that point is always oriented along the same straight line then it is called the linear polarization.
- Linear polarization may be horizontally polarized or vertically polarized.
- If the E-field component of the radiated wave travels in a plane perpendicular to the Earth's surface (vertical), the radiation is said to be VERTICALLY POLARIZED.

- If the E-field propagates in a plane parallel to the Earth's surface (horizontal), the radiation is said to be HORIZONTALLY POLARIZED.

## 2. CIRCULAR POLARIZATION

- If the electric field vector at that point traces a circle as a function of time then it is called circular polarization.
- In circular polarization the electric field orientation is not fixed horizontally or vertically but is constantly rotating.



- Circular polarization is one of the cases of elliptical polarization.

## Advantages of circular polarization over linear polarization.

### 1. Reflectivity:

Radio signals are reflected or absorbed depending on the material they come in contact with. Because linear polarized antennas are able to "attack" the problem in only one plane, if the reflecting surface does not reflect the signal precisely in the same plane, that signal strength will be lost. Since circular polarized antennas send and receive in all planes, the signal strength is not lost, but is transferred to a different plane and are still utilized.

2. **Absorption:** As stated above, radio signal can be absorbed depending on the material they come in contact with. Different materials absorb the signal from different planes. As a result, circular polarized antennas give you a higher probability of a successful link because it is transmitting on all planes.

**3. Phasing Issues:**

High-frequency systems (i.e. 2.4 GHz and higher) that use linear polarization typically require a clear line-of sight path between the two points in order to operate effectively. Such systems have difficulty penetrating obstructions due to reflected signals, which weaken the propagating signal. Reflected linear signals return to the propagating antenna in the opposite phase, thereby weakening the propagating signal. Conversely, circularly-polarized systems also incur reflected signals, but the reflected signal is returned in the opposite orientation, largely avoiding conflict with the propagating signal. The result is that circularly-polarized signals are much better at penetrating and bending around obstructions.

**4. Multi-path:**

Multi-path is caused when the primary signal and the reflected signal reach a receiver at nearly the same time. This creates an "out of phase" problem. The receiving radio must spend its resources to distinguish, sort out, and process the proper signal, thus degrading performance and speed. Linear Polarized antennas are more susceptible to multi-path due to increased possibility of reflection. Out of phase radios can cause dead-spots, decreased throughput, distance issues and reduce overall performance

**5. Inclement Weather:**

Rain and snow cause a microcosm of conditions explained above (i.e. reflectivity, absorption, phasing, multi-path and line of sight) Circular polarization is more resistant to signal degradation due to inclement weather conditions for all the reason stated above.

**6. Line-of-Sight:**

When a line-of-sight path is impaired by light obstructions (i.e. foliage or small buildings), circular polarization is much more effective than linear polarization for establishing and maintaining communication links.

## FADING [2006, 2008, 2009]

- When the radio frequency waves travel from transmitter to receiver there will be change in the signal intensity due to different factors and signal gets attenuated. This condition is called fading.
- Fading is the most troublesome and frustrating problem in receiving radio signals.
- There are basically 4 types of fading.

### 1. Interference fading

- Interference fading occurs due to phase interference of two or more waves from same source coming over different paths, producing path difference.
- Ionosphere disturbances can also cause interference fading.

### 2. Polarization fading

- The difference in polarization in transmitter and receiver antenna system cause polarization fading
- Polarization fading is rapid at high frequencies

### 3. Absorption fading

- Absorption fading is the result of absorption of EM waves in the ionosphere.
- Sudden ionospheric disturbances (SID) also results in heavy absorption and extreme fading.

### 4. Skip fading

- When the EM waves skip from the ionosphere instead of returning back to the earth it results in skip fading.
- Skipping and receiving of signal can take place due to MUF oscillating about actual MUF.
- Skipping is more prominent near sunset or sunrise when ionic density of layers change rapidly.



## ASSIGNMENT (Important Questions)

Last Date of submission: 26 march, 2012, (Monday) 9 AM Sharp.

Assignment copied from friends, incomplete assignment and late submission will not be considered for evaluation.

1. Differentiate between Broadside and End fire array.
2. Plot the radiation pattern for two element array having  $d=\lambda$  and  $\alpha=0^\circ$
3. Find the expression for effective area of monopole quarter wave antenna.
4. Show that the unattenuated radiation field at the surface of earth of quarter wave monopole is given by  $E=(6.14/r)\sqrt{W}$  mv/m where  $r$  is in miles and  $W$  in watt.
5. What are Group, Unit and resultant pattern ? Explain with suitable example.
6. Obtain the radiation pattern of 8 element uniform array using multiplication of pattern.
7. Derive Friis equation and explain its significance in communication system?
8. Calculate the power received by an receiving antenna of gain 60 dB at a distance of 100 km from the transmitting antenna whose gain is 50 dB. The transmitter radiates 100 watt of power at frequency of 1000 MHz.
9. How do transmission loss vary with frequency ? state different cases.
10. Discuss briefly the various modes of wave propagation.
11. Explain in detail about tropospheric scattering.
12. Derive Attenuation factor for ionospheric propagation.
13. If the reflection takes place at a height of 200 km and maximum density in the ionosphere corresponds to the refractive index of 0.85 at 10 MHz. Assuming the earth to be flat what will be the range for which maximum usable frequency is 13 MHz.
14. Discuss the wave bending phenomenon through ionosphere.
15. How signal is transmitted through optical fibre? Why is it preferred over other cable for signal transmission?



# ELECTROMAGNETIC PROPAGATION AND ANTENNA

## CHAPTER-5

### INTRODUCTION TO OPTICAL FIBRES

By :Rajan Sharma

- ✓ Optic Fiber is the transparent material, along which we can transmit light.
- ✓ Fiber optics is the system, or branch of engineering concerned with using the optic fibers. Optic fiber is therefore used in a fiber optic system.

### ✓ ADVANTAGES OF OPTICAL FIBERS (PU EXAM)

#### 1. Immunity from electrical interference

Optic fibers can run comfortably through areas of high level electrical noise such as near machinery and discharge lighting.

#### 2. No crosstalk

When copper cables are placed side by side for a long distance, electromagnetic radiation from each cable can be picked up by the others and so the signals can be detected on surrounding conductors. This effect is called crosstalk. In a telephone circuit it results in being able to hear another conversation in the background. Crosstalk can easily be avoided in optic fibers even if they are closely packed.

#### 3. Glass fibers are insulators

Being an insulator, optic fibers are safe for use in high voltage areas. They will not cause any arcing and can be connected between devices which are at different electrical potentials.

Where:  $n_1$  and  $n_2$  are the refractive indices of the two materials, and  $\theta_1$  and  $\theta_2$  are the angles of incidence and refraction respectively.

### CRITICAL ANGLE

When light travels from more dense medium to less dense medium ( $n_1 > n_2$ ) as shown in fig, As the angle of incidence in the first material is increased, there will come a time when, eventually, the angle of refraction reaches  $90^\circ$  and the light is refracted along the boundary between the two materials. The angle of incidence which results in this effect is called the **critical angle**.

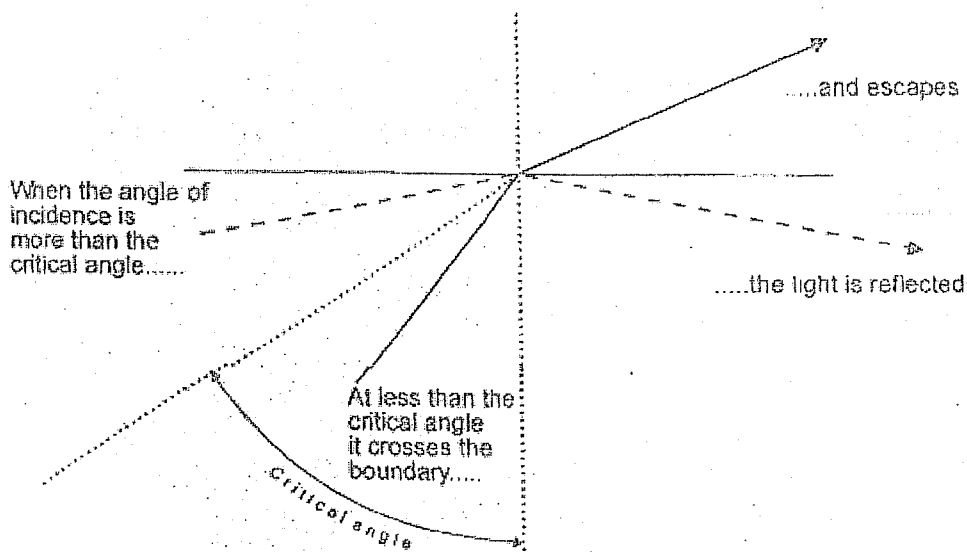
We can calculate the value of the critical angle by assuming the angle of refraction to be  $90^\circ$

From Snell's law:

$$n_1 \sin \theta_c = n_2 \sin 90^\circ$$

$$\theta_c = \sin^{-1}(n_2/n_1)$$

$\theta_c$  is the critical angle.



## TOTAL INTERNAL REFLECTION

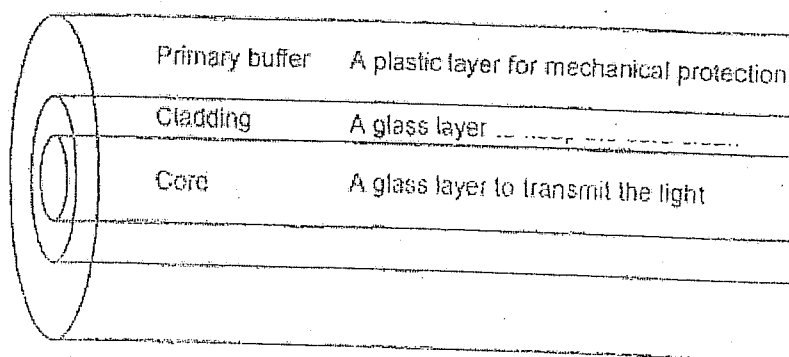
When light travel from denser to rarer medium, At angles of incidence less than the critical angle, the ray is refracted normally. However, if the light approaches the boundary at an angle greater than the critical angle, the light is actually reflected from the boundary region back into the first material. The boundary region simply acts as a mirror. This effect is called total internal reflection (TIR).

## STRUCTURE OF OPTICAL FIBER (PU EXAM)

An optical fiber is a thin, flexible, transparent fiber that acts as a "light pipe", to transmit light between the two ends of the fiber. Optical fiber typically consists of a transparent core surrounded by a transparent cladding material with a lower index of refraction. Light is kept in the core by total internal reflection. This causes the fiber to act as a waveguide.

The basic structure of an optical fiber consists of three parts;

1. the core,
2. the cladding, and
3. the coating or buffer.



- The basic structure of an optical fiber is shown in figure.

- The core is a cylindrical rod of dielectric material. Light propagates mainly along the core of the fiber. The core is generally made of glass with refractive index  $n_1$ .
- The core is surrounded by a layer of material called the cladding. The cladding layer is made of a dielectric material with an index of refraction  $n_2$ . The index of refraction of the cladding material is less than that of the core material. The cladding is generally made of glass or plastic.
- For extra protection, the cladding is enclosed in an additional layer called the coating or buffer. The coating or buffer is a layer of material used to protect an optical fiber from physical damage. The material used for a buffer is a type of plastic.

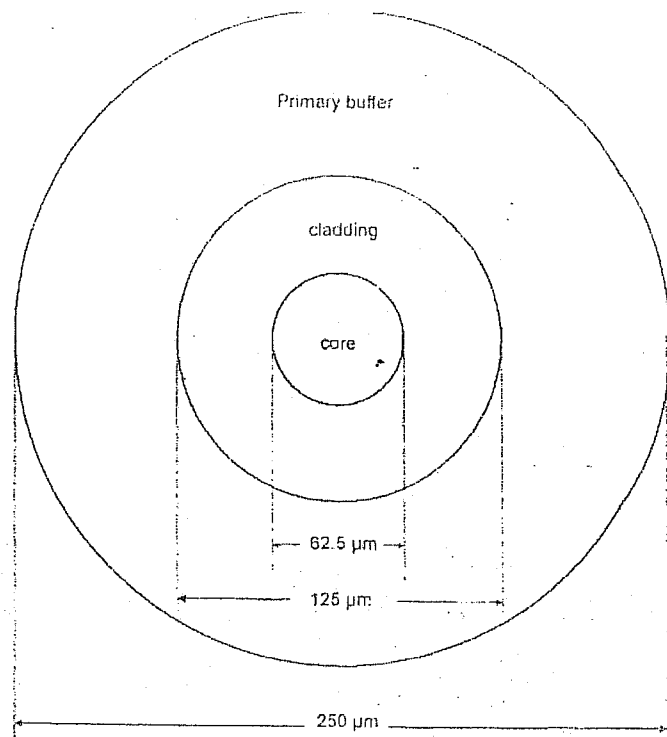
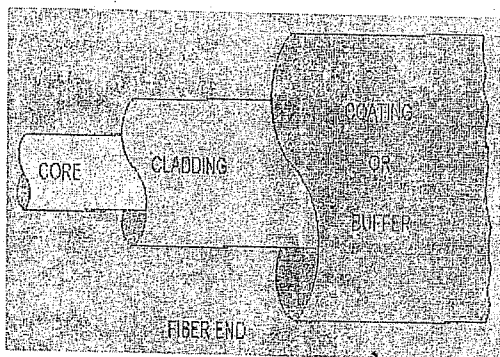
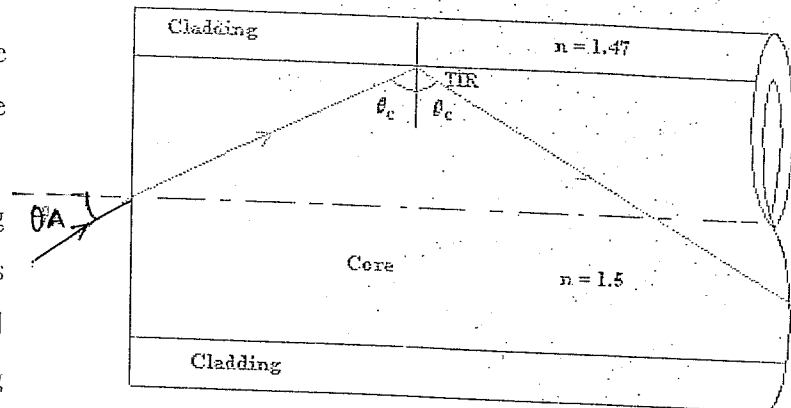


Fig : typical size of optical fiber

(PU EXAM)

## PRINCIPLE OF OPERATION/PROPAGATION OF LIGHT IN THE FIBRE:

- The angle  $\theta_A$  in the Figure is called the Acceptance Angle.
- Any light entering the fibre at less than this angle will meet the cladding at an angle greater than critical angle.



- If light meets the inner surface of the cladding (the core - cladding interface) at greater than critical angle then TIR occurs. So all the energy in the ray of light is reflected back into the core and none escapes into the cladding. The ray then crosses to the other side of the core and, because the fiber is more or less straight, the ray will meet the cladding on the other side at an angle which again causes TIR. The ray is then reflected back across the core again and the same thing happens.
- In this way the light travels its way along the fiber. This means that the light will be transmitted to the end of the fiber.

### ACCEPTANCE ANGLE (PU EXAM)

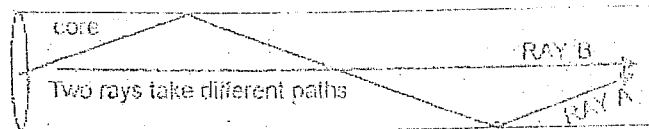
- It is the angle of incidence that causes TIR inside the fiber. It is the conical half angle as shown in fig. and the acceptance angle in the figure is  $14.18^\circ$
- It is denoted by  $\theta_A$

### Cone of acceptance

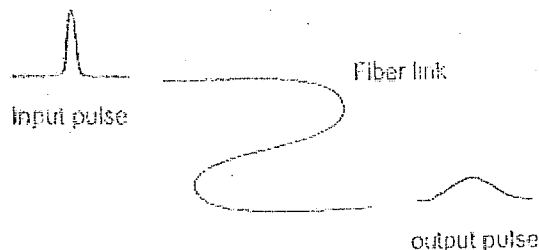
The cone of acceptance is the angle within which the light is accepted into the core and is able to travel along the fiber with TIR.

This spreading effect is called dispersion

Ray B will arrive first

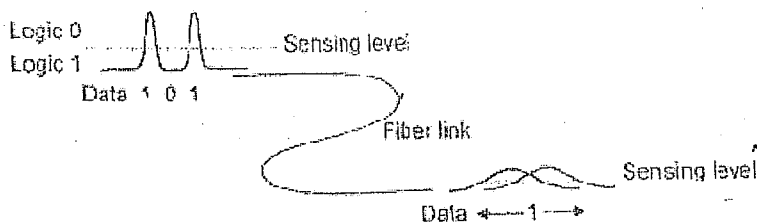


The pulse spreads out



The effect of dispersion is as shown in fig:

Dispersion has caused the pulses to merge



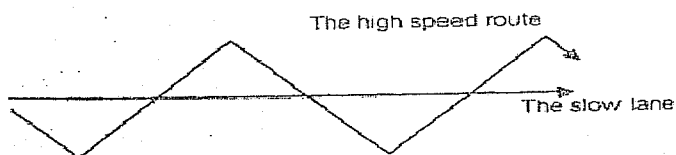
## How to overcome intermodal dispersion?

### 1 Using Graded index fiber

This design of fiber eliminates about 99% of intermodal dispersion.

The solution to our problem is to change the refractive index progressively from the center of the core to the outside. If the core center has the highest refractive index and the outer edge has the least, the ray will increase in speed as it moves away from the center.

They arrive at (almost) the same time



### 2 Single mode (SM) fiber



## INTRAMODAL (OR CHROMATIC) DISPERSION

A light source produces light of a many wavelength. In fact it produces a range of wavelengths. Even though it is far fewer for LASER than is produced by the LED

This is unfortunate as each component wavelength travels at a slightly different speed in the fiber. This causes the light pulse to spread out as it travels along the fiber — and hence causes dispersion. The effect is called chromatic dispersion.

## LOSSES IN OPTICAL FIBER (PU EXAM)

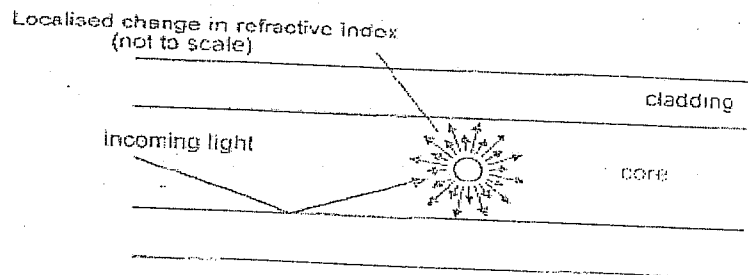
### 1. Absorption losses

Any impurities that remain in the fiber after manufacture will block some of the light energy. The worst culprits are hydroxyl ions and traces of metals.

### 2. Rayleigh scatter

This is the scattering of light due to small localized changes in the refractive index of the core and the cladding material.

The light is scattered in all directions

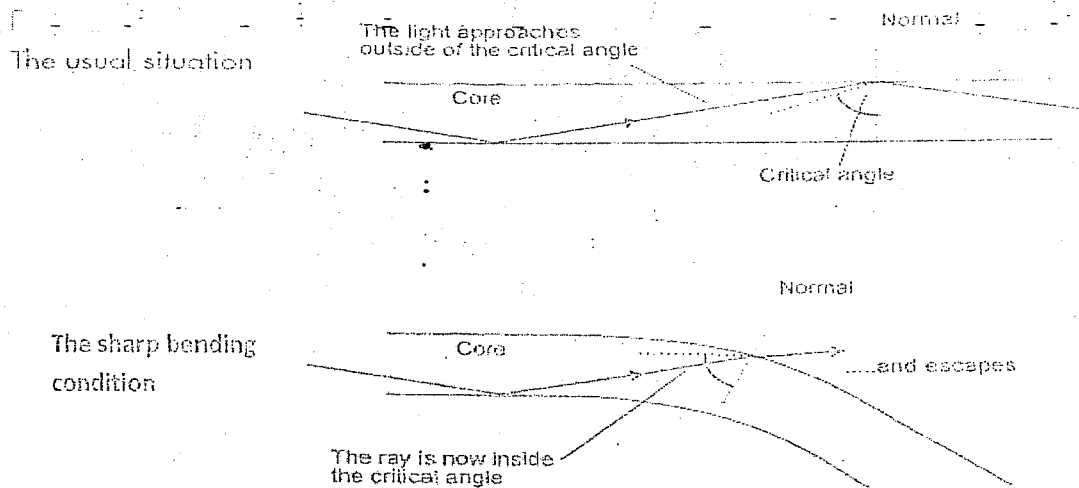


### 3. Fresnel reflection

This loss is due to the reflection from the entrance and exit surface of the fiber. Special coupling can be applied to remove this loss.

### 4. Bending losses

A sharp bend in a fiber can cause significant losses as well as the possibility of mechanical failure.

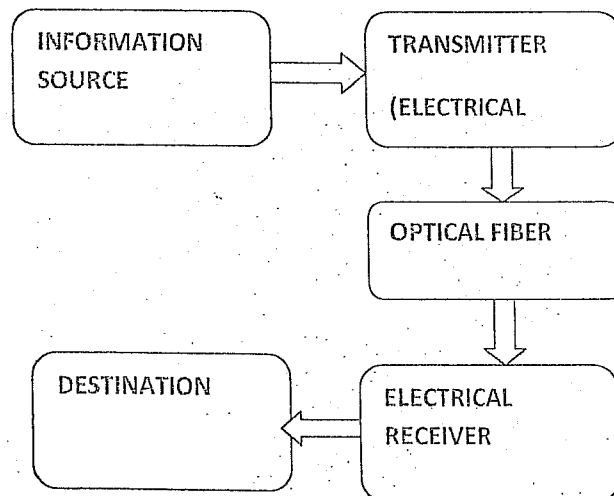


5. Connector loss

6. Splice loss

### BLOCK DIAGRAM OF OPTICAL FIBER COMMUNICATION SYSTEM (VVV IMP)

Draw the block diagram of optical fiber communication system and explain each block in brief [2005,2006,2006,2007 PU] 2012



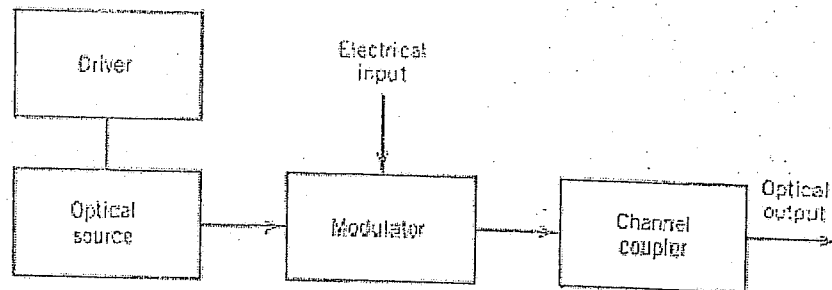
**Fig: Basic Block diagram of optical communication system**

### Information source

It is the source of information to be transmitted over the channel.

### Optical Transmitter

- The role of an optical transmitter is to convert the electrical signal into optical form and to launch the resulting optical signal into the optical fiber.
- Figure shows the block diagram of an optical transmitter. It consists of an optical source, a modulator, and a channel coupler.
- Semiconductor lasers or light-emitting diodes are used as optical sources because of their compatibility with the optical-fiber communication channel.



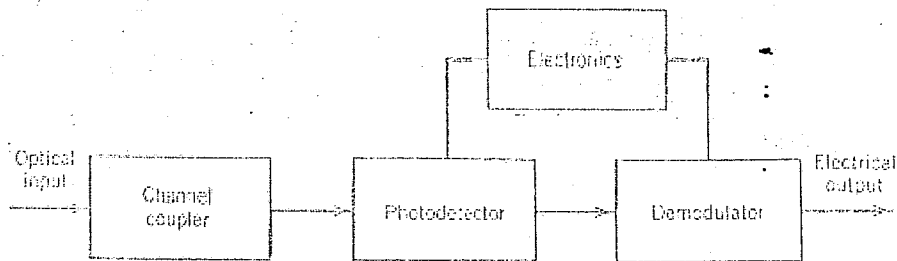
Components of an optical transmitter.

- The optical signal is generated by modulating the optical carrier wave.
- The output of a semiconductor optical source can be modulated directly by varying the injection current. Such a scheme simplifies the transmitter design and is generally cost-effective.
- The coupler is typically a microlens that focuses the optical signal onto the entrance plane of an optical fiber with the maximum possible efficiency.

### Optical channel/ Communication channel

The role of a communication channel is to transport the optical signal from transmitter to receiver without distorting it. Most light wave systems use optical fibers as the communication channel because silica fibers can transmit light with losses as small as 0.2 dB/km.

## Optical Receiver



Components of an optical receiver.

- An optical receiver converts the optical signal received at the output end of the optical fiber back into the original electrical signal.
- Fig shows the block diagram of an optical receiver. It consists of a coupler, a photodetector, and a demodulator.
- The coupler focuses the received optical signal onto the photodetector.
- Semiconductor photodiodes are used as photodetectors because of their compatibility with the whole system.
- The design of the demodulator depends on the modulation format used by the lightwave system.
- Most lightwave systems employ a scheme referred to as **“intensity modulation with direct detection” (IM/DD)**. Demodulation in this case is done by a decision circuit that identifies bits as 1 or 0, depending on the amplitude of the electric signal.

## OPTICAL LIGHT SOURCES (PU Exam)

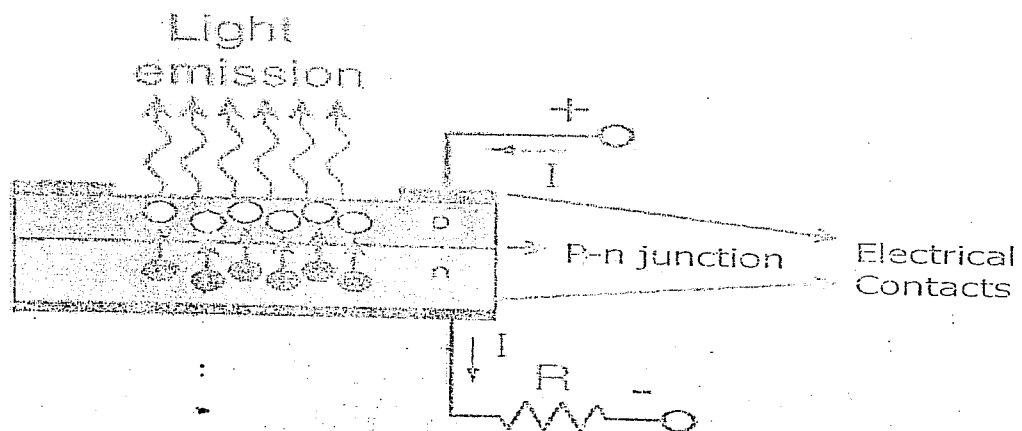
LED (Light Emitting Diode) AND LASER (Light Amplification by Stimulated Emission of Radiation) are the devices that are used widely as optical sources.

Light sources must have following properties to be used in optical communication system.

- Must have compatible size and configuration to effectively launch light into an optical fiber.
- Emit light at wavelength where fiber has low losses and low dispersion.
- Must have high intensity light output.
- Their light must be nearly monochromatic as much as possible.
- Allow direct modulation over wide bandwidth.

### LED

- A light-emitting diode (LED) is a semiconductor light source.
- Modern versions of LEDs are available across the visible, ultraviolet, and infrared wavelengths, with very high brightness.
- When a light-emitting diode is forward-biased (switched on), electrons are able to recombine with electron holes within the device, releasing energy in the form of photons. This effect is called electroluminescence and the color of the light (corresponding to the energy of the photon) is determined by the energy gap of the semiconductor.
- The LED consists of a chip of semiconducting material doped with impurities to create a p-n junction. As in other diodes, current flows easily from the p-side, or anode, to the n-side, or cathode, but not in the reverse direction. Charge-carriers—electrons and holes—flow into the junction from electrodes with different voltages. When an electron meets a hole, it falls into a lower energy level, and releases energy in the form of a photon.

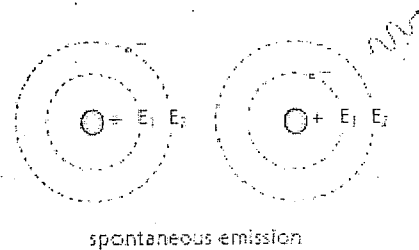


## LASER

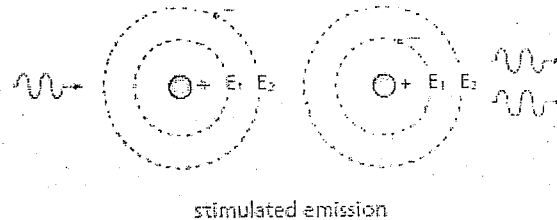
- A laser is a device that emits light (electromagnetic radiation) through a process of optical amplification based on the stimulated emission of photons.
- The term "laser" originated as an acronym for Light Amplification by Stimulated Emission of Radiation.
- Lasers are devices that produce intense beams of light which are monochromatic, and coherent. The wavelength (color) of laser light is extremely pure (monochromatic) when compared to other sources of light.
- Laser beams can be focused to very tiny spots, achieving a very high irradiance. Or they can be launched into a beam of very low divergence in order to concentrate their power at a large distance.
- Works on the principle of stimulated emission.

### SPONTANEOUS AND STIMULATED EMISSION

- In general, when an electron is in an excited energy state, it must eventually decay to a lower level, giving off a photon of radiation. This event is called "**spontaneous emission**," and the photon is emitted in a random direction and a random phase.



- On the other hand, if an electron is in energy state  $E_2$ , and its decay path is to  $E_1$ , but, before it has a chance to



spontaneously decay, a photon happens to pass by whose energy is approximately  $E_2 - E_1$ , there is a probability that the passing photon will cause the electron to decay in such a manner that a photon is emitted at exactly the same wavelength, in exactly the same direction, and with exactly the same phase as the passing photon. This process is called "**stimulated emission**."

## OPTICAL DETECTORS (PU EXAM)

- Optical detectors convert optical signal into an electrical signal.
- Optical detectors are the components that convert the light wave energy of fiber optic communications into electrical signals for recovery of data.
- When light strikes special types of materials, a voltage may be generated, a change in electrical resistance may occur, or electrons may be ejected from the material surface. As long as the light is present, the condition continues. It ceases when the light is turned off. Any of the above conditions may be used to change the flow of current or the voltage in an external circuit, and hence may be used to monitor the presence of the light and to measure its intensity.
- There are two broad classes of optical detectors: photon detectors and thermal detectors.
- Photon detectors rely on the action of quanta of light energy to interact with electrons in the detector material and to generate free electrons. To produce such effects, the quantum of light must have sufficient energy to free an electron.
- Thermal detectors respond to the heat energy delivered by the light. The response of these detectors involves some temperature-dependent effect, like a change of electrical resistance. Because thermal detectors rely on only the amount of heat energy delivered.
- The important characteristics of a photo detectors are:
  - Be compatible in size to low-loss optical fibers to allow for efficient coupling and easy packaging.
  - Have a high sensitivity at the operating wavelength of the optical source.
  - Have a sufficiently short response time (sufficiently wide bandwidth) to handle the system's data rate.
  - Contribute low amounts of noise to the system.
  - Maintain stable operation in changing environmental conditions, such as temperature.

## LOSS PROFILE AND OPTICAL WINDOWS

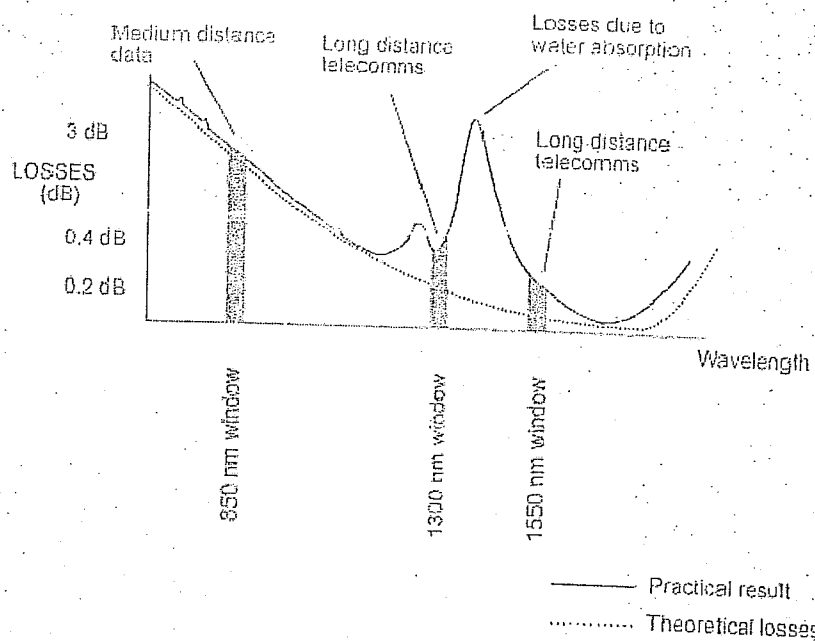
- ✓ Usable optical wavelength is 700nm-1600nm (IR)
- ✓ Optical fiber use IR band
  - Below 700 (visible range)→ Excessive loss
  - Above 1600(Invisible range)→ light degenerate into EM wave and loose photonic property and doesn't follows laws of reflection

High freq(low wavelength)→high BW→ high loss→ short distance

### Windows

- Having decided to use infrared light for (nearly) all communications, we are still not left with an entirely free hand.
- Some wavelengths are not desirable: 1380 nm for example. The losses at this wavelength are very high due to water within the glass. It is a real surprise to find that glass is not totally waterproof. Water in the form of hydroxyl ions is absorbed within the molecular structure and absorbs energy with a wavelength of 1380 nm. During manufacture it is therefore of great importance to keep the glass as dry as possible with water content as low as 1 part in 10<sup>9</sup>.
- It makes commercial sense to agree on standard wavelengths to ensure that equipment from different manufacturers is compatible. These standard wavelengths are called windows and we optimize the performance of fibers and light sources so that they perform at their best within one of these windows.
- The 1300 nm and 1550 nm windows have much lower losses and are used for long distance communications. The shorter wavelength window centered around 850 nm has higher losses and is used for shorter range data transmissions and local area networks (LANs), perhaps up to 10 km or so. The 850 nm window remains in use because the system is less expensive and easier to install.







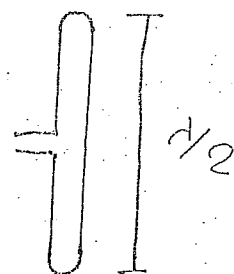
\* DIFFERENT TYPES OF ANTENNAS

By Rajan Sharma

\* FOLDED DIPOLE

# It is a dipole in which two half dipoles, one continuous and other split at the centre have been folded and joint together in parallel. The split dipole is fed at the centre by Transmission line

# For same power fed to both single and folded dipole, signal obtained is found to be 4 times more in folded than in single.



# power radiated by single dipole,

$$[Prad]_{dipole} = I_{rms}^2 \cdot [Rrad]_{dipole}$$

Power radiated by folded dipole is,

$$I_{rms}^2 \cdot [Rrad]_{folded} = 4 \times I_{rms}^2 \cdot [Rrad]_{dipole}$$

$$\therefore [Rrad]_{folded} = 4 \cdot [Rrad]_{dipole}$$

$$= 4 \times 73 \Omega$$

$$= 292 \Omega$$

\* YAGI-UDA ANTENNA

2005, 2006, 2009 : What is Active & parasitic elements  
discuss in detail about Yagi-uda antenna

Discuss the working principle of Yagi-uda Antenna.

# It is an array of active elements and one or more passive or parasitic elements.

# Driven element / Active Element

→ Direct current is fed to this element.

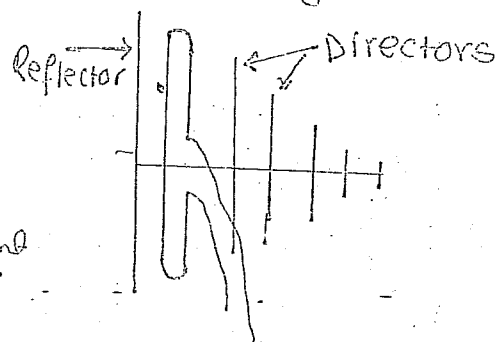


Fig: 7 element Yagi-uda

### # Parasitic element

- Current are not fed directly in parasitic elements but current flows due to mutual induction.
- Parasitic elements acquire their currents from mutual induction.
- Parasitic elements in the direction of beam are called directors and those in the backward directions are called reflectors.

# Yagi-Uda antenna works as an end fire array. i.e. there exists progressive phase difference between array elements.

### \* DESIGN

- # Length of dipole is set at slightly less than  $\lambda/2$  i.e.  $0.45\lambda$  to  $0.49\lambda$  to make it resonant so that its impedance be purely resistive.
- # Length of directors are made 5% smaller than dipole i.e.  $0.4$  to  $0.44\lambda$
- # Only one reflector is used and its length is 5% longer than dipole i.e.  $0.5 - 0.54\lambda$
- # Separation between directors is typically  $0.3 - 0.4\lambda$  and reflector is kept at a distance of  $0.25\lambda$ .

### \* Working Principle

- # Since length of each director is smaller than its corresponding resonating length, impedance of each will be capacitive resulting current to lead.
- # The impedance of reflector will be inductive since it is longer in length. So phase of current lags.
- # The total phase of the currents in directors & reflectors is not solely by their lengths but also by their separation to the adjacent elements.

# Thus properly spaced elements with their length slightly less than their corresponding resonant length will act as director since they form an array with currents approximately equal in magnitude & with equal progressive phase shift req<sup>d</sup> for formation of end fire array.

# Similarly a properly spaced element with a length of slightly greater will act as reflector.

## \* LOG PERIOD ARRAY / FREQ INDEPENDENT ANTENNA

(2007 2008) What is Freq Independent Antenna?  
Describe Log period antenna.

# Log periodic array provides good gain and wide Bandwidth.

# It is Based on Rumsey's Principle

≡ Rumsey's Principle states that the impedance and pattern properties of antenna will be frequency independent if antenna shape is specified in terms of angle.

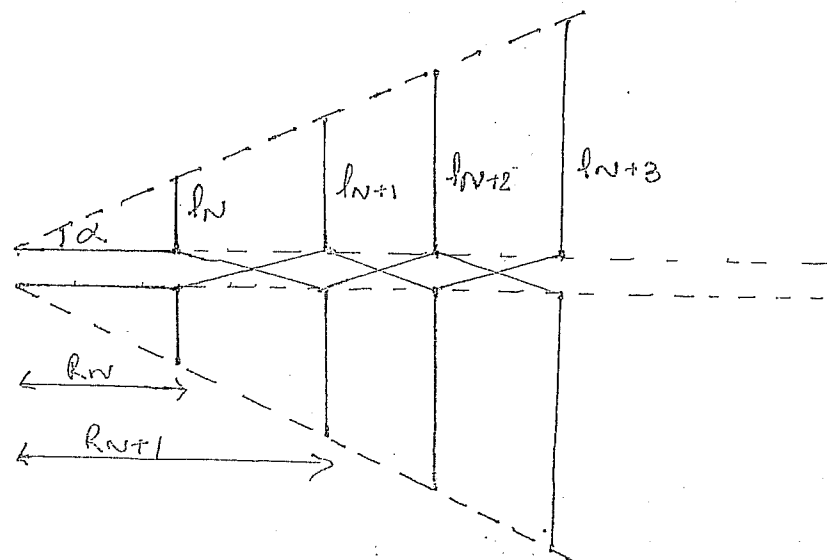


Fig Log Periodic antenna of criss cross connection.

# Log periodic antenna consists of an array of dipoles extended along a horizontal axis.

# The length of each dipole is shorter than preceding one

is connected to shortest dipole.

# Length and spacing of log periodic array are arranged in such a way that adjacent elements bear a constant ratio to each other.

$$\gamma = \frac{L_N}{L_{N+1}} = \frac{R_N}{R_{N+1}} \quad \gamma \text{ is design ratio factor.}$$

# It provides very wide Bandwidth than Yagi.

[2009] WORKING

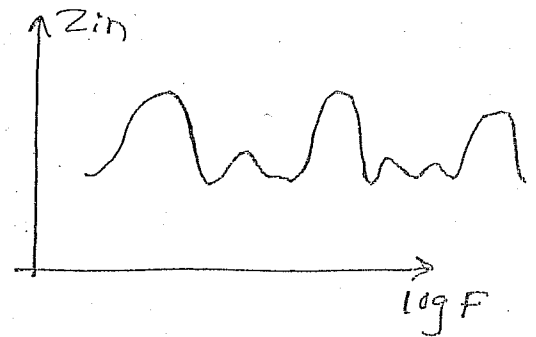
→ Log periodic antenna is the sum of many separate dipoles each tuned to slightly different frequencies.

→ Dipole adjacent to one that is resonant at a given frequency acts as a reflectors & directors

→ As frequency of interest shifts, the element acting as driven element changes & directors and reflectors also changes

→ The log periodic antenna can operate over 4 to 1 or greater freq ratio. i.e. highest usable freq is 4 times the lowest.

# The characteristics of the array fairly remains constant over wide frequency range. So it is called frequency independent antenna.



## \* APERATURE ANTENNA

- # An antenna having aperture with a certain geometrical shape is called aperture antenna.
- # The aperture may take a form of a waveguide or a horn.
- # They operate at microwave frequency so referred as microwave antenna.
- # They are mostly used in real life because:
  1. have Large gain
  2. Easy to flush mount to the surface of spacecraft or aircraft without disturbing aerodynamics.
  3. They are convenient to be covered with dielectrics to protect from unfavourable environmental conditions.
  4. They acts as a suitable feeding element for other antennas like reflector antennas

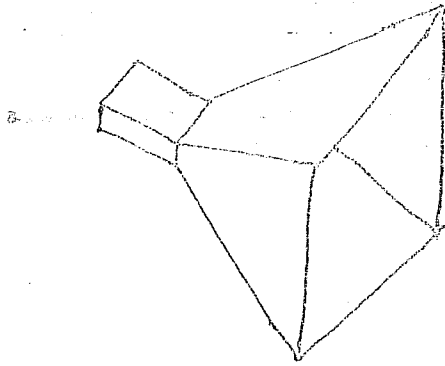
## \* HORN ANTENNA (2005, 2009)

- # A horn is a hollow pipe of different cross section which has been tapered to a large opening.
- # The opening may be a square, rectangular, circular etc.
- # The f<sup>n</sup> of horn is to produce uniform phase front with a larger aperture.
- # The open circuit is a discontinuity which matches to space poorly and beside diffraction around edges it will provide poor radiation and non directive pattern.  
In order to overcome it, the mouth of horn antenna is opened out for gradual transformation.

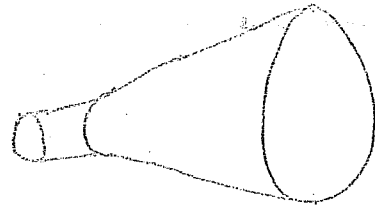
# Horn Antennas are widely used as antenna at UHF & microwave freq.

They are often used as feeder for larger antennas such as parabolic antennas

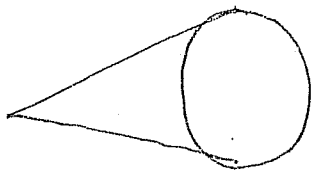
# They can operate over wide range of frequency



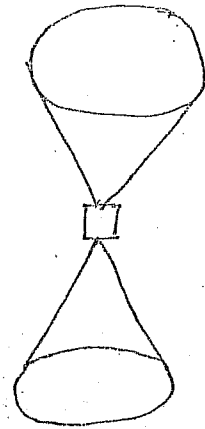
(a) Pyramidal horn.



(b) Conical horn.



(c) Circular horn



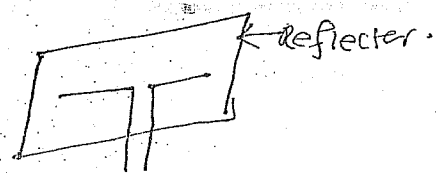
(d) Biconical horn.

### \* REFLECTOR ANTENNA

# Reflectors are widely used to modify radiation pattern of radiating elements.

For eg the backward radiation pattern from an antenna may be eliminated with a plane sheet reflector of large dimension.

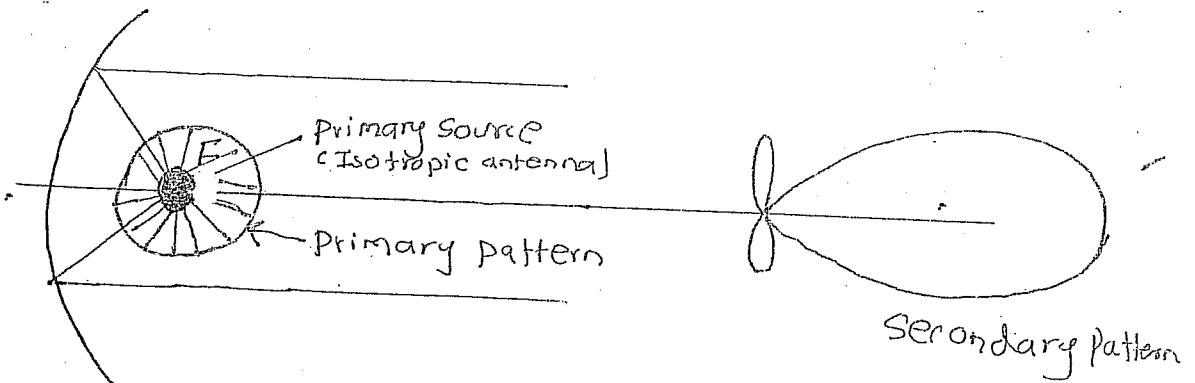
# Parabolic Reflector is most widely used Reflector in communication system.





## \* PARABOLIC ANTENNA TYPE AND CHARACTERISTICS

- # A parabolic antenna is an antenna that uses a parabolic reflector to direct radio waves.
- # It is often called dish antenna.
- # It consists of parabolic reflector which collects and concentrates an incoming parallel beam of radio waves and focuses them onto primary antenna placed at its focus.
- # The antenna at focus is also k/a feed antenna.
- # Generally Horn antenna can be used as a feed antenna for parabolic antenna.



- # The main advantage of parabolic antenna is its high directivity, large gain, & narrow beamwidth.
- # Parabolic antennas are used as high gain antennas for point to point communication eg microwave relay link, satellite communication, etc.

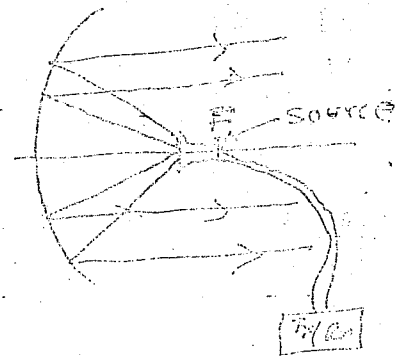
### WORKING

- # The operating principle of parabolic antenna is that a point source of radio waves at focal point of paraboloid reflector will be reflected into a plane wave beam along the axis of reflector.
- Conversely an incoming plane wave parallel to the axis will be focused to a point at the focal point.

# \* Feeding mechanism of Parabolic antenna

## ① Front Feed Mechanism / Axial Feed

# In front feed system, the primary source or active element is placed on the focus of parabolic reflector.



# In figure, horn antenna is used as primary antenna.

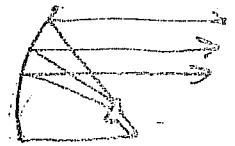
# This mechanism have two disadvantages.

1. The reflected wave by paraboloid is blocked by source itself
2. The reflected wave from parabola to source produce interaction & mismatching

# The modification of Front feed is the offaxis or offset feed.

↳ Here the reflector is assymetrical segment of paraboloid so the focus & feed antenna are located at one side of the dish.

↳ The purpose is to move feed structure out of beam path.



## ② Cassegrain Feed Mechanism

# In cassegrain system active element is placed on the surface of paraboloid rather than focal point

# The source directs the signal to another reflector centred on focal point, which in turn reflects the signal to entire dish surface

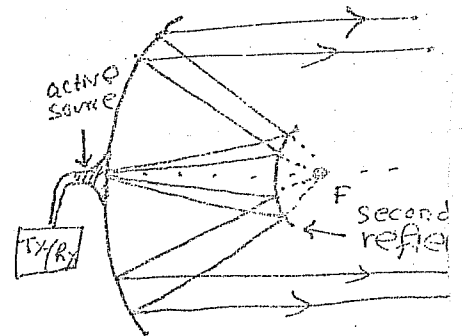


fig: cassegrain fee

Adv: ① Overcomes problem of Front Feed

② Since source is located outside, it is easier to install & adjust mechanically

Disadv ① use two reflective surface.

# If concave reflector is used as secondary reflector it is called Gregorian feed mechanism.



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