Toegepaste Stochastiek
Homework assignment 4

Hand in before lecture of 12 June

Problem 4.1 (8 points). Prove that for every fixed $\varepsilon > 0$, there is a fixed $\lambda > 0$ such that the following holds a.a.s.: for any two disjoint subsets of $G_{n,\lambda/n}$ with at least εn vertices, there must be an edge between them.

Problem 4.2 (7 points). Use a Chernoff bound[1] to prove that the minimum and maximum degrees of dense Erdős-Rényi random graphs satisfy the following inequalities. Assuming $np = \omega(\log n)$, then for any fixed $\varepsilon > 0$ it holds a.a.s. that $(1-\varepsilon)np \leq \delta(G_{n,p})$ and $\Delta(G_{n,p}) \leq (1+\varepsilon)np$.

Problem 4.3 (15 points, a:8 b:7). Recall that at each step of the preferential attachment process, a new vertex of degree m is added, with each of its incident edges joined to the existing graph independently at random with probability proportional to the existing degree distribution.

(a) Show that the preferential attachment process with $m = x$ is equivalent to running the preferential attachment process with $m = 1$ and then merging successive blocks of exactly x vertices.

(b) Prove that the degree of the ith vertex added in the preferential attachment process tends to infinity a.a.s. as $t \to \infty$.

Problem 4.4 (5 points). Prove that the total number of configurations on $[m]$ (m even) is $(m-1) \cdot (m-3) \cdots 3 \cdot 1$.

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[1] Say, if X has binomial distribution with parameters n and p, then for any $0 < \varepsilon \leq 3/2$, $\Pr(|X - \mathbb{E}(X)| \geq \varepsilon\mathbb{E}(X)) \leq 2\exp(-\varepsilon^2\mathbb{E}(X)/3)$