

Let $f = x + a_2x^2 + a_3x^3 + \dots$

$$\begin{aligned}
f^{[n]} &= x + na_2x^2 + ((n-1)na_2^2 + na_3)x^3 + \left(\frac{(n-1)n(2n-3)}{2}a_2^3 + 5\frac{(n-1)n}{2}a_2a_3 + na_4\right)x^4 + \\
&\left(\frac{n(3n^3-13n^2+18n-8)}{3}a_2^4 + \frac{n(26n^2-63n+37)}{6}a_2^2a_3 + 3(n-1)na_2a_4 + 3\frac{(n-1)n}{2}a_3^2 + na_5\right)x^5 + \dots \\
ilog(f) &= a_2x^2 + (a_3 - a_2^2)x^3 + \left(\frac{3}{2}a_2^3 - \frac{5}{2}a_2a_3 + a_4\right)x^4 + \left(-\frac{8}{3}a_2^4 + \frac{37}{6}a_2^2a_3 - 3a_2a_4 - \frac{3}{2}a_3^2 + a_5\right)x^5 + \\
&\left(\frac{31}{6}a_2^5 - \frac{46}{3}a_2^3a_3 + 8a_2^2a_4 + \frac{49}{6}a_2a_3^2 - \frac{7}{2}a_2a_5 - \frac{7}{2}a_3a_4 + a_6\right)x^6 + \\
&\left(-\frac{157}{15}a_2^6 + \frac{572}{15}a_2^4a_3 - \frac{62}{3}a_2^3a_4 - \frac{193}{6}a_2^2a_3^2 + 10a_2^2a_5 + \frac{62}{3}a_2a_3a_4 - 4a_2a_6 + \frac{7}{2}a_3^3 - 4a_3a_5 - 2a_4^2 + a_7\right)x^7 + \dots
\end{aligned}$$

Let $g = b_2x^2 + b_3x^3 + b_4x^4 + \dots$

$$\begin{aligned}
iexp(g) &= x + b_2x^2 + (b_2^2 + b_3)x^3 + \left(b_2^3 + \frac{5}{2}b_2b_3 + b_4\right)x^4 + \left(b_2^4 + \frac{13}{3}b_2^2b_3 + 3b_2b_4 + \frac{3}{2}b_3^2 + b_5\right)x^5 + \\
&\left(b_2^5 + \frac{77}{12}b_2^3b_3 + 6b_2^2b_4 + \frac{35}{6}b_2b_3^2 + \frac{7}{2}b_2b_5 + \frac{7}{2}b_3b_4 + b_6\right)x^6 + \\
&\left(b_2^6 + \frac{87}{10}b_2^4b_3 + 10b_2^3b_4 + \frac{85}{6}b_2^2b_3^2 + 8b_2^2b_5 + \frac{46}{3}b_2b_3b_4 + 4b_2b_6 + \frac{5}{2}b_3^3 + 4b_3b_5 + 2b_4^2 + b_7\right)x^7 + \dots
\end{aligned}$$

Forget about previous g . And let $g = x + b_2x^2 + b_3x^3 + \dots$

$$\begin{aligned}
f \circ g &= x + (a_2 + b_2)x^2 + (a_3 + b_3 + 2a_2b_2)x^3 + (a_4 + b_4 + a_2b_2^2 + 3a_3b_2 + 2a_2b_3)x^4 + \\
&(a_5 + b_5 + 2a_2b_4 + 3a_3b_3 + 4a_4b_2 + 3a_3b_2^2 + 2a_2b_2b_3)x^5 + \\
&(a_6 + b_6 + 2a_2b_5 + 3a_3b_4 + 4a_4b_3 + 5a_5b_2 + a_2b_3^2 + a_3b_2^3 + 6a_4b_2^2 + 2a_2b_2b_4 + 6a_3b_2b_3)x^6 + \dots \\
iexp(ilog(f) + ilog(g)) &= x + (a_2 + b_2)x^2 + (a_3 + b_3 + 2a_2b_2)x^3 + \left(a_4 + b_4 + \frac{1}{2}a_2^2b_2 + \frac{1}{2}a_2b_2^2 + \frac{5}{2}a_2b_3 + \frac{5}{2}a_3b_2\right)x^4 + \\
&\left(a_5 + b_5 + 3a_2b_4 + 3a_3b_3 + 3a_4b_2 - \frac{1}{6}a_2b_2^3 - \frac{1}{6}a_2^3b_2 + \frac{4}{3}a_3b_2^2 + \frac{4}{3}a_2^2b_3 + \frac{1}{3}a_2^2b_2^2 + \frac{7}{6}a_2a_3b_2 + \frac{7}{6}a_2b_2b_3\right)x^5 + \dots \\
\lim_{n \rightarrow \infty} (f^{\lfloor \frac{1}{n} \rfloor} \circ g^{\lfloor \frac{1}{n} \rfloor})^{[n]} &= x + (a_2 + b_2)x^2 + (a_3 + b_3 + 2a_2b_2)x^3 + \\
&\left(a_4 + b_4 + \frac{1}{2}a_2^2b_2 + \frac{1}{2}a_2b_2^2 + \frac{5}{2}a_2b_3 + \frac{5}{2}a_3b_2\right)x^4 + \dots
\end{aligned}$$

Again, forget about previous f, g .

And let $f = a_2x^2 + a_3x^3 + \dots$, $g = b_2x^2 + b_3x^3 + b_4x^4 + \dots$

$$\begin{aligned}
ilog(iexp(f) \circ iexp(g)) &= (a_2 + b_2)x^2 + (a_3 + b_3)x^3 + \left(a_4 + b_4 + \frac{1}{2}(a_3b_2 - a_2b_3)\right)x^4 + \\
&\left(a_5 + b_5 - a_2b_4 + a_4b_2 + \frac{1}{6}a_3b_2^2 + \frac{1}{6}a_2^2b_3 - \frac{1}{6}a_2a_3b_2 - \frac{1}{6}a_2b_2b_3\right)x^5 +
\end{aligned}$$

$$(a_6 + b_6 - \frac{3}{2}a_2b_5 - \frac{1}{2}a_3b_4 + \frac{1}{2}a_4b_3 + \frac{3}{2}a_5b_2 - \frac{1}{12}a_2b_3^2 - \frac{1}{12}a_3^2b_2 + \frac{1}{2}a_4b_2^2 + \frac{1}{2}a_2^2b_4 \\ + \frac{1}{12}a_2a_3b_3 - \frac{1}{2}a_2a_4b_2 - \frac{1}{2}a_2b_2b_4 + \frac{1}{12}a_3b_2b_3 - \frac{1}{4}a_2a_3b_2^2 + \frac{1}{4}a_2^2b_2b_3)x^6 + \dots$$

$$Let \ A(X) = \begin{pmatrix} a_{11}x + a_{12}y \\ a_{21}x + a_{22}y \end{pmatrix}, \ B(X) = \begin{pmatrix} b_{11}x + b_{12}y \\ b_{21}x + b_{22}y \end{pmatrix}.$$

Let $M(\bullet)$ is corresponding matrix of linear vector field $\bullet$.

$$M(A) = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \ M(B) = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}, \quad \nabla_A B = \begin{pmatrix} (a_{11}b_{11} + a_{21}b_{12})x + (a_{12}b_{11} + a_{22}b_{12})y \\ (a_{11}b_{21} + a_{21}b_{22})x + (a_{12}b_{21} + a_{22}b_{22})y \end{pmatrix},$$

$$\nabla_B A = \begin{pmatrix} (a_{11}b_{11} + a_{12}b_{21})x + (a_{11}b_{12} + a_{12}b_{22})y \\ (a_{21}b_{11} + a_{22}b_{21})x + (a_{21}b_{12} + a_{22}b_{22})y \end{pmatrix}$$

$$M(\nabla_A B) = \begin{pmatrix} a_{11}b_{11} + a_{21}b_{12} & a_{12}b_{11} + a_{22}b_{12} \\ a_{11}b_{21} + a_{21}b_{22} & a_{12}b_{21} + a_{22}b_{22} \end{pmatrix} = M(B)M(A)$$

$$M(\triangle_A B) = M(\nabla_B A) = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{pmatrix} = M(A)M(B)$$

(where $\triangle_A B := \nabla_B A$)