

Ok so the general jacobian matrix of the system is given by

$$J = \begin{bmatrix} 1 + 10h + hy_n & -10h + hx_n \\ 30h + 2hx_n & 1 - 20h \end{bmatrix}$$

Now if we evaluate this matrix at the fixed point, then  $x_n = 0$  and  $y_n = 0$  giving us

$$J_{(0,0)} = \begin{bmatrix} 1 + 10h & -10h \\ 30h & 1 - 20h \end{bmatrix}$$

We need to find the eigenvalues of this matrix which are given by solving  $\text{Determinant}(J_{0,0} - \lambda I) = 0$

This gives us

$$(1 + 10h - \lambda)(1 - 20h - \lambda) + 300h^2 = 0$$

and expanding this out and solving for  $\lambda$  we get two solutions

1.  $\lambda_1 = -5i\sqrt{3}h - 5h + 1$
2.  $\lambda_2 = 5i\sqrt{3}h - 5h + 1$

There is a theorem that states a fixed point is stable if and only if the real parts of the eigenvalues are negative. The real parts of both of the eigenvalues is given by  $-5h + 1$

So solving  $-5h + 1 < 0$  and keeping in mind that  $h > 0$  by assumption we get that  $h > \frac{1}{5}$ . So the system is stable provided  $h > \frac{1}{5}$  and unstable if  $0 < h < \frac{1}{5}$