

# Can mathematics be dangerous?

Mathematics deals with finite and infinite collections of objects. Its strength and beauty is based on exact reasoning. Thus from a mere logical point of view, mathematics is utmost fragile: A tiny little mistake ends up with a collapse of the whole building.

Recall that mathematical induction reaches infinity within two steps:

1. the inductive anchorage,
2. the inductive move.

The following example shows how induction, if misapplied, is able to remove any finite number of objects from the scene.

**Theorem.** *For a given property  $P$ , and a finite set  $M$ , every element of  $M$  has property  $P$ .*

*Proof.* We proceed by induction on the number  $n$  of elements of  $M$ . For  $n = 0$ , there are no elements of  $M$ , so there is nothing to prove. Assume that the theorem holds for  $n$ -element sets, and consider a set  $M = \{x_1, \dots, x_{n+1}\}$  with  $n + 1$  elements. Thus by our inductive hypothesis, the  $n$ -element subsets  $M_1 := \{x_1, \dots, x_n\}$  and  $M_2 := \{x_2, \dots, x_{n+1}\}$  meet the claim, i. e. their elements have property  $P$ . Consequently, the elements of the union  $M = M_1 \cup M_2$  also have property  $P$ , which completes the inductive step from  $n$  to  $n + 1$ . Whence the theorem is proved.  $\square$

**Remark.** The theorem becomes dangerous if property  $P$  is chosen to be “non-existent”. With this property  $P$ , the theorem yields:

**Corollary.** *Every finite set  $M$  is empty.*

What has been proved? “Parvus error in principio magnus est in fine” - Logically, there is no difference between small and big mistakes! Yes - logically. However, the sense to recognize mathematical beauty is the same that prevents mathematicians from ending up in a chaos of mistakes - it’s intuition! Just like in music: Rules are the indispensable framework to cultivate intuition. Without intuition, mathematical rules appear to be frightening. - In the light of intuition, they reveal mathematical beauty.