$$X^A = \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\} \begin{matrix} \to c_0 \\ \to c_1 \end{matrix}$$

$$X^B = \left\{ \begin{bmatrix} -1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} \right\} \begin{matrix} \to c_0 \\ \to c_1 \end{matrix}$$

Które cechy są najlepsze: $c_0 c_1$, c_0 czy c_1?

$c_0 c_1$:

$$C = \frac{1}{n} (\underline{\underline{X}} - \underline{\underline{\mu}})(\underline{\underline{X}} - \mu)^T$$

$$\underline{\underline{X}}^A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \qquad \underline{\mu}^A = \begin{bmatrix} \frac{1}{3}(0+1+1) \\ 1/3(1+1+0) \end{bmatrix} = \begin{bmatrix} 2/3 \\ 2/3 \end{bmatrix}$$

$$C^A = 1/3 \left(\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} - \begin{bmatrix} 2/3 & 2/3 & 2/3 \\ 2/3 & 2/3 & 2/3 \end{bmatrix} \right) \cdot \left(\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} - \begin{bmatrix} 2/3 & 2/3 & 2/3 \\ 2/3 & 2/3 & 2/3 \end{bmatrix} \right)^T =$$

$$= \frac{1}{3} \begin{bmatrix} -2/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & -2/3 \end{bmatrix} \cdot \begin{bmatrix} -2/3 & 1/3 \\ 1/3 & 1/3 \\ 1/3 & -2/3 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 2/3 & -1/3 \\ -1/3 & 2/3 \end{bmatrix}$$

$$\underline{\underline{X}}^B = \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & -1 \end{bmatrix} \qquad \underline{\mu}^B = \begin{bmatrix} -2/3 \\ -2/3 \end{bmatrix}$$

$$C^B = 1/3 \left(\begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & -1 \end{bmatrix} - \begin{bmatrix} -2/3 & -2/3 & -2/3 \\ -2/3 & -2/3 & -2/3 \end{bmatrix} \right) \cdot \left(\begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & -1 \end{bmatrix} - \begin{bmatrix} -2/3 & -2/3 & -2/3 \\ -2/3 & -2/3 & -2/3 \end{bmatrix} \right)^T =$$

$$= \frac{1}{3} \begin{bmatrix} -1/3 & 2/3 & -1/3 \\ 2/3 & -1/3 & -1/3 \end{bmatrix} \cdot \begin{bmatrix} -1/3 & 2/3 \\ 2/3 & -1/3 \\ -1/3 & -1/3 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 2/3 & -1/3 \\ -1/3 & 2/3 \end{bmatrix}$$

$$F_{c_0 c_1} = \frac{\| \mu^A - \mu^B \|}{\det C^A + \det C^B} = \frac{\sqrt{(2/3 + 2/3)^2 + (2/3 + 2/3)^2}}{2 \cdot 1/9} =$$

$$\det C^A = 1/3 (2/3 \cdot 2/3 - 1/3 \cdot 1/3) = = 1/3 \cdot 1/3 = \det C^B$$

$$= \sqrt{\frac{2 \cdot 16}{9}} \cdot \frac{9}{2} = \frac{\cancel{2} \cdot 4\sqrt{2}}{\cancel{9}\,3} \cdot \frac{9}{2} = 6\sqrt{2} \approx 8{,}5$$

c_0:

$$F_{c_0} = \frac{|\mu^A - \mu^B|}{\sigma^A + \sigma^B} = \frac{2/3 + 2/3}{1/3 \left(\sqrt{(-2/3)^2 + (1/3)^2 + (1/3)^2} + \sqrt{(-1/3)^2 + (2/3)^2 + (-1/3)^2} \right)} = \frac{4/3}{1/3 \left(\sqrt{2/3} + \sqrt{2/3} \right)} =$$

$$= \frac{2}{\sqrt{\frac{2}{3}}} \approx 2{,}45$$

c_1:

$$F_{c_1} = \frac{|\mu^A - \mu^B|}{\sigma^A + \sigma^B} = \frac{2/3 + 2/3}{1/3 \left(\sqrt{2/3} + \sqrt{2/3} \right)} = 2 / \sqrt{2/3} \approx 2{,}45$$

Najlepszym wyborem jest $c_0 c_1$ bo $F_{c_0 c_1}$ jest największy.

Jaki jest kierunek potencjalnie najlepszej separacji próbek?

$$C = \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix}$$

Kierunki wektorów własnych macierzy C:

$$\underline{\underline{C}}\,\underline{x} = \lambda\,\underline{x} \longrightarrow \det(A - \lambda \cdot \underline{\underline{1}}) = 0$$

$$\begin{bmatrix} 4-\lambda & 1 \\ 1 & 1-\lambda \end{bmatrix}$$

$$(4-\lambda)(1-\lambda) - 1 = 0$$

$$4 - 4\lambda - \lambda + \lambda^2 - 1 = 0$$

$$3 - 5\lambda + \lambda^2 = 0$$

$$\lambda^2 - 5\lambda + 3 = 0$$

$$\Delta = 13 \qquad \sqrt{\Delta} = \sqrt{13} \approx 3{,}5$$

$$\lambda_1 = \frac{5-3{,}5}{2} = \frac{1{,}5}{2} = 0{,}75$$

$$\lambda_2 = \frac{5+3{,}5}{2} = 4{,}3$$

$$\begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix} \cdot \underline{V} = 4{,}3 \cdot \underline{V} \longrightarrow \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = 4{,}3 \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 4x + y \\ x + y \end{bmatrix} = \begin{bmatrix} 4{,}3x \\ 4{,}3y \end{bmatrix}$$

$$\boxed{V_2 = \begin{bmatrix} 1 \\ 0{,}3 \end{bmatrix}} \Longleftarrow \begin{cases} 4x + y = 4{,}3x \Rightarrow y = 0{,}3x \\ x + y = 4{,}3y \Rightarrow x = 3{,}3y \end{cases}$$

$$x^A = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad x^B = \begin{bmatrix} 0 \\ 0{,}5 \end{bmatrix}$$

rzutowanie punktów x^A i x^B na prostą V_2

$$x^{AT} \cdot \underline{V_2} = [2\ 1] \cdot \begin{bmatrix} 1 \\ 0{,}3 \end{bmatrix} = 2.3$$

$$x^{BT} \cdot \underline{V_2} = [0\ 0{,}5] \cdot \begin{bmatrix} 1 \\ 0{,}3 \end{bmatrix} = 0{,}3$$