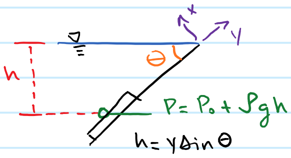
Midterm 2 Fluids Notes Back

Sunday, October 25, 2015 9:52 PM

Error Estimate $z = g(x, y, \ldots)$

$$\delta z^2 = \left(\frac{\partial g}{\partial x}\right)^2 \cdot dx^2 + \left(\frac{\partial g}{\partial y}\right)^2 dy^2 + \ldots$$



Shear Force: τ
Viscosity: μ
Pressure: P
Density: ρ
Resultant Force: F_R (Acts at y_R)
Centroid: y_c
Resultant distance: y_R
Bouyancy Force: F_B
Knudsen #: K_n
Nu ν**:** $\nu = \frac{\mu}{\rho}$
$\bar{V}$: velocity of fluid W.R.T. stationary obs
$\bar{W}_{c.v.}$: Velocity of C.V. W.R.T. stationary Frame
$\bar{V}_{R/c.v.}$: Velocity of fluid W.R.T. C.V.

Specific Weight

$\gamma = \rho g$

$SG = \frac{\rho}{\rho_{H_2O}}$

$\tau = \mu \cdot \frac{du}{dy}$

$-\nabla P + \rho \bar{g} = \rho \bar{a}$

$P_G = P - P_{atmo}$

$P_A = P$

$F_R = \int_A P\,dA = y_c \cdot w \cdot P_c$

$y_R = \frac{M_R}{F_R} = \frac{\int_A y P\,dA}{\int_A P\,dA}$

$y_c = \frac{1}{A}\int_A y\,dA$

$F_B = \rho_l \cdot g \cdot \forall$ Acts through Center of displaced liquid

$K_n = \frac{\lambda}{L} = \frac{M.F.P.}{length}$

$\bar{V}_{R/c.v.} = \bar{V} - \bar{W}_{c.v.}$

$\bar{a}_{c.v.} = \frac{d\,\bar{W}_{c.v.}}{dt}$

$$\underbrace{\frac{\partial}{\partial t}\int_{C.V.} \rho\, d\forall}_{\Delta \text{mass in C.V.}} = \underbrace{-\oint_{C.S.} \rho \bar{U} \cdot d\bar{A}}_{\text{flux of mass thru C.S.}}$$

Steady Flow

$$\frac{\partial}{\partial t}\int_{C.V.} \rho\, d\forall = 0$$

Incompressible Flow

$$0 = \oint_{C.S.} \bar{U} \cdot d\bar{A}$$

Both $\bar{V}$ and $\bar{U}$ are used for velocity.

$$\left.\frac{\partial N}{\partial t}\right)_{sys} = \underbrace{\frac{\partial}{\partial t}\int_{C.V.} \eta \rho\, d\forall}_{\text{Temporal change of P inside C.V.}} + \underbrace{\oint_{C.S.} \eta \rho (\bar{V} \cdot d\bar{A})}_{\text{Net flux of P entering/leaving C.S.}}$$

$$\eta = \begin{cases} 1 & \text{Mass} & M \\ \bar{V} & \text{Momentum} & P \\ \bar{r} \times \bar{V} & \text{Ang. Mom.} & H \\ e & \text{Energy} & E \end{cases}$$

Accelerating C.V.

$$\sum \bar{F}_{ext} - \int_{C.V.} \bar{a}_{c.v.} \rho\, d\forall = \frac{\partial}{\partial t}\int_{C.V.} \rho \bar{V}_{R/c.v.}\, d\forall + \oint_{C.S.} \rho \bar{V}_{R/c.v.} (\bar{V}_{R/c.v.} \cdot d\bar{A})$$

Strain Rate Tensor

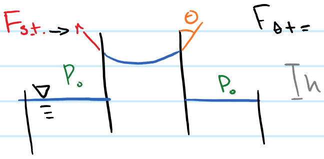
$\tau_{ij} = \mu \epsilon_{ij}$ & $\epsilon_{ij} = \frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i}$

$$\bar{\tau} = \begin{pmatrix} 0 & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & 0 & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & 0 \end{pmatrix}$$

1	2	3	
u	v	w	V_i
x	y	z	X_i

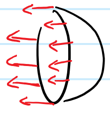
$\sum F = F_s + F_B = dm\bar{g} + F_B = -\hat{k}\rho g\, d\forall + F_s$

Capillary



$F_{st} = m_w \cdot g$

Surface Tension



$F_{ST} = L \cdot \sigma = (2\pi R) \cdot \sigma \cdot \cos\theta$

$\sigma \left(\frac{\text{Force}}{\text{Length}}\right)$

$$\bar{\bar{\sigma}}_s = \begin{vmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{vmatrix}$$

Gauss' Thrm

$$\int_{C.S.} \rho \bar{V} \cdot d\bar{A} = \int_{C.V.} \nabla \cdot (\rho \bar{V})\, d\forall$$

Stream Function

$U = \frac{\partial \psi}{\partial y}$ $\quad v = -\frac{\partial \psi}{\partial x}$

$V_r = \frac{1}{r} \cdot \frac{\partial \psi}{\partial \theta}$ $\quad V_\theta = -\frac{\partial \psi}{\partial r}$

Total Derivative

$$\bar{a}_p = \frac{D\bar{V}}{Dt} = \frac{\partial \bar{V}}{\partial t} + (\bar{V} \cdot \nabla)\bar{V}$$

→ Potential of vel. field

$$\bar{V} = \nabla \Phi = \hat{i}\frac{\partial \Phi}{\partial x} + \hat{j}\frac{\partial \Phi}{\partial y} + \hat{k}\frac{\partial \Phi}{\partial z}$$

$$\rho \frac{D\bar{V}}{Dt} = \rho \bar{g} + \nabla \cdot \bar{\bar{\sigma}}$$

Vorticity Vector

$$\omega_z = \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right) k$$

$\bar{\omega} = \nabla \times \bar{V}$

Rate of Rotation

$\bar{\eta} = \frac{1}{2}\omega = \frac{1}{2} \cdot \nabla \times \bar{V}$

Laplace EQ

$$\nabla^2 \Phi = \frac{d^2\Phi}{dx^2} + \frac{d^2\Phi}{dy^2} + \frac{d^2\Phi}{dz^2} = 0$$

$$\Phi = \nabla \cdot \bar{V} \rightarrow \begin{cases} > 0 & \text{Expansion} \\ < 0 & \text{Contraction} \\ = 0 & \text{Incompressible} \end{cases}$$

$$\nabla \cdot \bar{\bar{\sigma}} = \hat{i}\left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}\right) + \hat{j}\left(\frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z}\right) + \hat{k}\left(\frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y}\right)$$