The Martian

Suppose you are a NASA scientist involved in helping astronaut Mark Watney embark on his journey to Schiaparelli crater to the Ares 4 launch site, where he will use the MAV to dock with Hermes and return to Earth. You are in charge of instructing Mark on how to create a $10L$ air tank for EVA use after he has damaged his. The air tank should be composed of 79% nitrogen and 21% oxygen, to match that of Earth's atmosphere. The problem is, Mark only has access to tanks of pure oxygen, pure nitrogen, and a tank containing 5% oxygen and 95% nitrogen. In addition, your supervisor mandates that Mark must use 4 times as much of the mixed gases tank as the pure oxygen tank because he will need to conserve the oxygen for rocket fuel. How much gas from each air tank (in Liters) is needed to create the $10L$ tank of Earth-like air, satisfying the given constraints?

Solution:

Let
x = Liters of pure O_2
y = Liters of pure N_2
z = Liters of 95% N_2 and 5% O_2

We must create a combination of the three given tanks to create a $10L$ tank. That is,

$$x + y + z = 10$$

We must have 21% O_2, or $2.1L$ of O_2, in our mixture, which can come from tank x and tank z; we get 100% O_2 from tank x, and 5% O_2 from tank z. So,

$$x + 0.05z = 2.1$$

Lastly, the supervisor's condition gives us

$$z = 4x$$

which can be written as

$$-4x + z = 0$$

We thus have the following system of linear equations:

$$\begin{cases} x + y + z = 10 \\ x + 0.05z = 2.1 \\ -4x + z = 0 \end{cases}$$

Written in augmented matrix form, we have:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 10 \\ 1 & 0 & 0.05 & 2.1 \\ -4 & 0 & 1 & 0 \end{array}\right] \tag{1}$$

Simple Gaussian elimination produces:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1.75 \\ 0 & 1 & 0 & 1.25 \\ 0 & 0 & 1 & 7 \end{array}\right] \tag{2}$$

From this, we see that $x = 1.75L$, $y = 1.25L$, and $z = 7L$.