

Theorem: Provided $|T'| \neq 0$ and $|r''| \neq 0$, then $|r'' \times T'| = 0$ if and only if $\frac{d}{dt}|r'| = 0$

By use of the division rule one can find:

$$T' = \frac{d}{dt} \frac{r'}{|r'|} = \frac{r''}{|r'|} - \frac{r' \frac{d}{dt}|r'|}{|r'|^2}$$

Cross product is distributive over addition, thus

$$r'' \times T' = \frac{1}{|r'|} \cdot (r'' \times r'') - \frac{\frac{d}{dt}|r'|}{|r'|^2} \cdot (r'' \times r')$$

Simplifying further we reach

$$r'' \times T' = -\frac{\frac{d}{dt}|r'|}{|r'|^2} \cdot (r'' \times r')$$

If $\frac{d}{dt}|r'| = 0$, then $|r'' \times T'| = 0$. Conversely if $|r'' \times T'| = 0$, then $|r'' \times r'| \neq 0$ as $|T' \times r'| \neq 0$, one can conclude $-\frac{\frac{d}{dt}|r'|}{|r'|^2} = 0$ and $\frac{d}{dt}|r'| = 0$.