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Quantum Entanglement in a Two-Electron Quantum Dot in Magnetic Field

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Abstract—The properties of quantum entanglement of the ground state in an exactly solvable model of a two-electron QD have been investigated. It is shown that the degree of entanglement increases with enhancement of interaction between electrons, irrespective of the shape of electron confining potential in a QD. A magnetic field destroys electron entanglement. However, the entanglement in deformed QDs is more stable against magnetic field.

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INTRODUCTION

It is well known that miniaturization of field-effect transistors and bipolar transistors with p–n junctions is physically limited (see, for example, [1]). Modern nanotechnologies based on electron and X-ray lithography provide structures with a cross-sectional size of 10–100 nm and a thickness of 1–10 nm. Thus, in principle, one can pass to a new element base, including atomic chains (quantum wires). The development of quantum traps (so-called artificial atoms or quantum dots (QDs)) is an important achievement in this field.

A QD is a potential well formed at the interface of several semiconductors with different configurations of conduction bands [2]. For example, a 10-nm thick GaAs layer, placed between insulating AlGaAs layers, forms a quantum channel, through which a two-dimensional electron gas flows. The two-dimensionality is due to the presence of quantum confinement along the z axis (thickness) and free electron motion in the xy plane. A voltage applied with the aid of external electrodes formed on the insulator surfaces confines this gas in a limited region (i.e., in a trap well). Since the electron mean free path exceeds the potential-well sizes, the energy levels in this well are quantized. Therefore, it is natural that the QD physical properties are determined by the quantum dynamics, which is due to the potential-well properties and electron–electron interaction [3]. Modern nanotechnologies make it possible to control the number of electrons populating the potential-well levels according to the Pauli principle. The QD physical characteristics (size and shape) can also be controlled, due to which the quantum distribution of electrons in a trap well can be varied [4]. The quantum nature of QDs is especially pronounced at low temperatures (~100 mK). At these temperatures, the conductivity of a system obeys quantum laws, changing stepwise with passage of each successive electron. The complete theory of QDs has not been formulated yet due to its complexity. Nevertheless, some fundamental concepts have been formed based on clear physical ideas; these concepts can explain some experimental data. By this, first of all, we mean the data of single-electron spectroscopy [4], which allow one to measure the QD conductivity in the quantum mode. In these experiments, a QD is weakly coupled to the environment and the data can be interpreted within models developed for closed systems. In this case, the physical observables are primarily determined by the electron properties of quasi-isolated QDs.

Obviously, the above-described characteristics and the possibility of their external control make it possible to investigate in detail the entanglement phenomenon. The results of this analysis may be very important for different applications in quantum information technology. In this paper, we report the results of studying the degree of entanglement in a two-electron QD, depending on the strength of electron–electron interaction and external magnetic field.

MODEL

The electron properties of a quasi-isolated QD with a small number of electrons can be determined in terms of relatively simple models. In these models, electrons are believed to move in an effective potential field, which is due to the joint dynamics of electron–electron interaction and the potential trap well. The crystal structure of the semiconductor material where the QD is located was taken into account through the effective mass m^* for conducting electrons. The well depth is several electronvolts for typical voltages

applied to the insulator. The typical mean spacing between quantum levels in a well does not exceed several millielectronvolts. If the number of electrons locked in a QD is small, the electron wave functions are localized near the potential-well minimum. According to quantum mechanics, the potential-well minimum can be approximated well by a harmonic oscillator potential for almost any functional form. Therefore, it is generally accepted that the model of a 3D harmonic oscillator where one frequency (ω_z) is much higher than two other frequencies (ω_x and ω_y) can be used to construct the effective potential field for a QD with a small number of electrons.

This assumption is natural, because the thickness of the layer where a QD is located is several times smaller than the sizes of the QD-location plane. Therefore, the quantum motion of electron in the vertical direction z is more restricted in comparison with its dynamics in the xy plane. The results of IR optical experiments in magnetic fields support these models. In particular, these experiments revealed that the resonant frequencies are independent of the number of electrons in a QD [5] and can be associated with the eigenmodes of the Hamiltonian of the model describing the electron dynamics in an external magnetic field in the potential of a two-dimensional isotropic harmonic oscillator [6]. Taking into account this fact, one can trace the main effects of magnetic field on a QD with a small number of electrons within the shell model, which was proposed for the first time in [7–9]. Although rather simple, this model contains the main concepts that are typical for realistic approaches and allows one to trace the effects of spontaneous break of QD symmetry in magnetic fields.

With allowance for the possibility of using an external perpendicular magnetic field to control the QD properties, the Hamiltonian of a quasi-isolated two-electron QD can be written as

$$\hat{H} = \hat{H}_0 + \hat{V}(\mathbf{r}_1 - \mathbf{r}_2). \quad (1)$$

In this model, the Hamiltonian

$$\hat{H}_0 = \sum_{i=1}^{N} \hat{h}_i$$

characterizes the dynamics of two ($N = 2$) independent electrons, each of which is described by the single-particle Hamiltonian

$$\hat{h} = \frac{1}{2m^*}\left(\hat{\mathbf{p}} + \frac{e}{c}\hat{\mathbf{A}}\right)^2 + \frac{m^*}{2}(\omega_x^2 x^2 + \omega_y^2 y^2) + \mu^* \hat{\sigma}_z B.$$

Here, we used the symmetric gauge: $\mathbf{A} = \mathbf{B} \times \mathbf{r}/2$, $\mathbf{B} = (0, 0, B)$, and $\hat{\sigma}_z$ is the Pauli matrix. Note that this model includes the Fock model [6] as a particular case. The temperature effects are neglected. This approximation is in agreement with experiments at low temperatures (~100 mK), where $kT \ll \Delta$ (Δ is the mean spacing between quantum levels) [10]. For a QD with a small number of electrons, $\Delta = \hbar\sqrt{\omega_x \omega_y} \approx 3$ meV. As an illustration, we will take the GaAs parameters: $m^* = 0.067 m_e$, $\mu^* = g_L \mu_B$, $g_L = 0.44$, and $\mu_B = e\hbar/2m^*c$. It should be emphasized that the magnetic orbital effects greatly exceed the spin effects (Zeeman splitting, which is determined by the term $\mu^* \sigma_z B$); therefore, the Zeeman term is neglected here.

Obviously, the Coulomb interaction between electrons in free space determines their spatial behavior. The electron–electron interaction in a QD can be modified in a fairly nontrivial way. To consider the problem analytically, we will use the Johnson–Payne model [11] for the effective electron–electron interaction in a QD, which can be written as

$$V(\mathbf{r}_1 - \mathbf{r}_2) = V_0 - \lambda^2 m^* (\mathbf{r}_1 - \mathbf{r}_2)^2/2.$$

After introduction of the coordinates of relative motion and the center of mass (respectively, subscripts "rel" and "CM")

$$\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2, \quad \mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2,$$

total Hamiltonian (1) is transformed into

$$\hat{H} = \hat{H}_{\text{rel}} + \hat{H}_{\text{CM}} + V_0,$$

where

$$\hat{H}_{\text{rel}} = \hat{\mathbf{p}}^2/2\mu + \mu[\omega_1^2 \hat{x}^2 + \omega_2^2 \hat{y}^2]/2 + \omega_L \hat{l}_z,$$
$$\hat{H}_{\text{CM}} = \hat{\mathbf{P}}^2/2M + M[\Omega_1^2 \hat{X}^2 + \Omega_2^2 \hat{Y}^2]/2 + \omega_L \hat{L}_z,$$

and

$$\omega_{1,2}^2 = \omega_{x,y}^2 + \omega_L^2 - 2\lambda^2, \quad \Omega_{1,2}^2 = \omega_{x,y}^2 + \omega_L^2,$$
$$\mu = m^*/2, \quad M = 2m^*, \quad \omega_L = eB/2m^*c.$$

Using the standard creation and annihilation operators for a harmonic oscillator, we can write the Hamiltonians $\hat{H}_{\text{rel}}$ and $\hat{H}_{\text{CM}}$ in the form

$$\hat{H}_{\text{rel}} = \hbar\omega_1(\hat{c}_1^\dagger \hat{c}_1 + 1/2) + \hbar\omega_2(\hat{c}_2^\dagger \hat{c}_2 + 1/2) - i\hbar g_1(\hat{c}_1^\dagger \hat{c}_2 - \hat{c}_2^\dagger \hat{c}_1) - i\hbar g_2(\hat{c}_1^\dagger \hat{c}_2^\dagger - \hat{c}_2 \hat{c}_1), \quad (2)$$

$$\hat{H}_{\text{CM}} = \hbar\Omega_1(\hat{C}_1^\dagger \hat{C}_1 + 1/2) + \hbar\Omega_2(\hat{C}_2^\dagger \hat{C}_2 + 1/2) - i\hbar G_1(\hat{C}_1^\dagger \hat{C}_2 - \hat{C}_2^\dagger \hat{C}_1) - i\hbar G_2(\hat{C}_1^\dagger \hat{C}_2^\dagger - \hat{C}_2 \hat{C}_1), \quad (3)$$

where the interaction parameters are as follows:

$$g_1 = \omega_L(\omega_1 + \omega_2)/2\sqrt{\omega_1\omega_2},$$
$$g_2 = \omega_L(\omega_1 - \omega_2)/2\sqrt{\omega_1\omega_2},$$

$$G_1 = \omega_L(\Omega_1 + \Omega_2)/2\sqrt{\Omega_1\Omega_2},$$
$$G_2 = \omega_L(\Omega_1 - \Omega_2)/2\sqrt{\Omega_1\Omega_2}.$$

The Bogoliubov transformations

$$\hat{a}_\pm = \sum_{m=1}^{2}\left(A_m^\pm \hat{c}_m + B_m^\pm \hat{c}_m^\dagger\right), \quad \hat{b}_\pm = \sum_{m=1}^{2}\left(F_m^\pm \hat{C}_m + D_m^\pm \hat{C}_m^\dagger\right)$$

allow one to reduce Hamiltonians (2) and (3) to the diagonal form (see, for example, [12])

$$\hat{H}_{\text{rel}} = \sum_{\pm} \hbar\omega_\pm(\hat{a}_\pm^\dagger \hat{a}_\pm + 1/2),$$

$$\hat{H}_{\text{CM}} = \sum_{\pm} \hbar\Omega_\pm(\hat{b}_\pm^\dagger \hat{b}_\pm + 1/2).$$

The eigenmodes of the Hamiltonians $\hat{H}_{\text{rel}}$ and $\hat{H}_{\text{CM}}$ can be written as

$$\omega_\pm^2 = \Big[\omega_x^2 + \omega_y^2 + 4(\omega_L^2 - \lambda^2) \pm \sqrt{(\omega_x^2 - \omega_y^2)^2 + 8\omega_L^2(\omega_x^2 + \omega_y^2 + 2\omega_L^2 - 4\lambda^2)}\Big]\Big/2,$$

$$\Omega_\pm^2 = \Big[\omega_x^2 + \omega_y^2 + 4\omega_L^2 \pm \sqrt{(\omega_x^2 - \omega_y^2)^2 + 8\omega_L^2(\omega_x^2 + \omega_y^2 + 2\omega_L^2)}\Big]\Big/2.$$

The spatial component of the total wave function of the system is characterized by a set of four quantum numbers, $n_\pm$ and $N_\pm$:

$$|n_+ n_- N_+ N_-\rangle = \frac{1}{\sqrt{n_+! n_-! N_+! N_-!}}(\hat{a}_+^\dagger)^{n_+}(\hat{a}_-^\dagger)^{n_-}(\hat{b}_+^\dagger)^{N_+}(\hat{b}_-^\dagger)^{N_-}|0000\rangle,$$

which determine the total QD energy,

$$E = \hbar\omega_+(n_+ + 1/2) + \hbar\omega_-(n_- + 1/2) + \hbar\Omega_+(N_+ + 1/2) + \hbar\Omega_-(N_- + 1/2).$$

The total wave function includes the spin component and has the form

$$\Phi = |n_+ n_- N_+ N_-\rangle\chi_s.$$

QUANTUM ENTANGLEMENT

Low-lying states for interacting electrons in weak magnetic fields are generally determined by the quantum numbers of relative motion $n_\pm$, because the quantum numbers of the center-of-mass motion are zero: $N_+ = N_- = 0$ [13].

Here, we investigate quantum entanglement caused by electron orbital motion. The analysis is performed by the example of QD ground state with the quantum numbers $n_+ = n_- = 0$, which corresponds to a spin singlet with an antisymmetric spin wave function χ_s. The degree of this entanglement will be determined using the logarithmic negativity [14], which is related to the existence of negative eigenvalues in the partially transposed (with respect to one of the subsystems) density matrix of the entangled state. In the case of Gaussian states (to which the state under consideration belongs), the logarithmic negativity is completely determined by the covariance matrix. For the two-electron two-dimensional state, the covariance matrix has a dimension of 8×8 and is determined by the relation

$$\gamma_{jk} = \frac{1}{\hbar}\text{Tr}(\hat{\varrho}\hat{R}_j\hat{R}_k) - \frac{i}{2}\sigma_{jk}, \tag{4}$$

where the eight-dimensional column operator

$$\hat{R} = (\hat{x}_1, \hat{p}_{x1}, \hat{y}_1, \hat{p}_{y1}, \hat{x}_2, \hat{p}_{x2}, \hat{y}_2, \hat{p}_{y2})^T$$

consists of the corresponding projections of the coordinate and momentum operators of electrons, $\hat{\varrho}$ is the density matrix of electron orbital motion, and the antisymmetric matrix σ has a block structure:

$$\sigma = \bigoplus_{j=1}^{4}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

Then, the symplectic spectrum of partially transposed covariance matrix γ^{T_1} (obtained from matrix (4) by the replacement $\hat{\mathbf{p}}_1 \to -\hat{\mathbf{p}}_1$), which consists of the values $(\xi_1, \xi_2, \xi_3, \xi_4)$ of block-diagonal matrix,

$$-\sigma\gamma^{T_1}\sigma\gamma^{T_1} = \text{diag}(\xi_1, \xi_1, \xi_2, \xi_2, \xi_3, \xi_3, \xi_4, \xi_4),$$

determines the logarithmic negativity according to the formula

$$E_{\mathcal{N}} = -\sum_{j=1}^{4}\log_2\left(\min(1, 2\xi_j)\right). \tag{5}$$

To analyze the quantum entanglement, we varied the magnetic induction B, degree of deformation ω_y/ω_x, and strength of electron–electron interaction λ. The maximum magnetic induction in the calculations did not exceed 1 T, a value corresponding to the maximum Larmor frequency $\omega_L \approx \omega_x$ for the electron confinement frequencies under consideration. Obviously, when using the Johnson–Payne interaction, the electron–electron interaction plays a role of perturbation. Therefore, the strength of interaction was determined by the condition $\lambda/\omega_x < 1$ in our calculations. As can be seen in Fig. 1, the enhancement of electron–electron interaction at small deformation of the system in the absence of magnetic field increases significantly the entanglement. However, an increase in the magnetic field breaks the entanglement; the confining potential is efficiently increased to reduce the Coulomb interaction (see, for example, [15, 16]). Thus, the higher the confining potential at a fixed strength of electron–electron interaction, the weaker the correlation effects. At a fixed interaction (Fig. 2), the QD deformation reduces the entanglement. Note that, in the absence of magnetic field, our results are in qualitative agreement with the calculations of the degree of entanglement based on the von Neumann

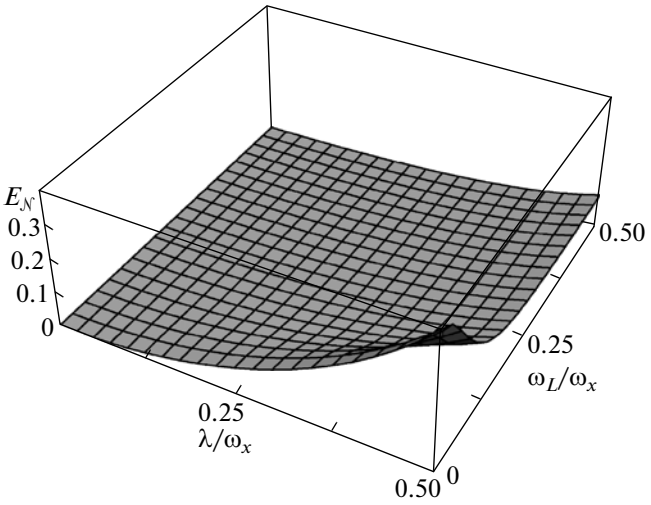

Fig. 1. Dependence of the measure of quantum entanglement (logarithmic negativity (5)) of electron orbital motion in the QD ground state on the magnetic induction B (Larmor frequency ω_L in ω_x units) and the strength of interaction λ/ω_x at $\omega_y/\omega_x = 1.2$.

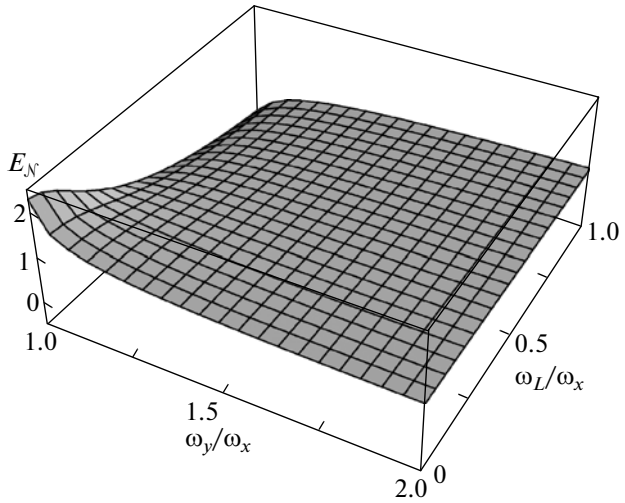

Fig. 2. Dependence of the measure of quantum entanglement on the magnetic induction B and the degree of QD deformation (ω_y/ω_x) at $\lambda/\omega_x = 0.7$.

entropy for a two-electron QD with Coulomb interaction (see, for example, [17]). A magnetic field breaks the entanglement to a great extent in a circular QD. Although the entanglement in deformed QDs is weakened, it is more stable against magnetic field.

CONCLUSIONS

The degree of quantum entanglement for the ground state of two-electron QDs, depending on the intensity of perpendicular magnetic field and electron–electron interaction strength, was analyzed within a simple, analytically solvable, model. We used logarithmic negativity as a measure of entanglement [14]. The results of our analysis showed that the degree of entanglement depends on the confining-potential shape: it is maximum for a circular QD at a fixed strength of electron–electron interaction and decreases with an increase in deformation. A magnetic field breaks entanglement. Nevertheless, the degree of entanglement in deformed QDs is more stable to magnetic field than in circular QDs.

Note that the model under consideration can adequately describe the physical characteristics of self-assembled QDs (see, for example, [2]). The potential energy of a trap well in these systems dominates in all physical processes, while the Coulomb interaction is a small perturbation. Thus, intense technological research aimed at finding a relatively inexpensive way to fabricate these QDs and use their properties at room temperature is currently underway.

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