WZÓR CAŁKOWY FOURIERA

$$f(t)=\int_0^{+\infty}[a(\omega)\cos\omega t+b(\omega)\sin\omega t]d\omega \quad \text{(CAŁKA FOURIERA)}$$

$$a(\omega)=\frac{1}{\pi}\int_{-\infty}^{+\infty} f(\tau)\cos\omega\tau d\tau \qquad b(\omega)=\frac{1}{\pi}\int_{-\infty}^{+\infty} f(\tau)\sin\omega\tau d\tau$$

PRZEKSZTAŁCENIE $\mathfrak{F}$

$$\mathfrak{F}[f(t)]\underset{df}{=}\int_{-\infty}^{+\infty} e^{-j\omega t} f(t)dt \qquad \left(\underset{ozn}{=} F(j\omega)\right)$$

$$\mathfrak{F}^{-1}[F(j\omega)]=\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{j\omega t}F(j\omega)d\omega \qquad \left(\underset{ozn}{=} f(t)\right)$$

$$F(j\omega)=|F(j\omega)|e^{j\theta(\omega)}, \quad -\pi\le\theta(\omega)\le+\pi$$

$$F(j\omega)=\pi[a(\omega)-jb(\omega)]$$

WIDMO AMPLITUDOWE

$$|F(j\omega)|=\pi\sqrt{a^2(\omega)+b^2(\omega)} \quad \text{(funkcja parzysta)}$$

WIDMO FAZOWE

$$\cos\theta(\omega)=\frac{a(\omega)}{\sqrt{a^2(\omega)+b^2(\omega)}}, \quad \sin\theta(\omega)=\frac{-b(\omega)}{\sqrt{a^2(\omega)+b^2(\omega)}}$$

Jeżeli $\cos\theta(\omega)=-1$ i $\sin\theta(\omega)=0$, to $\theta(\omega)=\pi\,\mathrm{sgn}\,\omega$ dla $\omega\ne 0$

θ - funkcja nieparzysta na $D\theta$ lub $D\theta-\{0\}$

PRZEKSZTAŁCENIE $\mathcal{L}$

$$\mathcal{L}[f(t)]\underset{df}{=}\int_0^{+\infty} e^{-st} f(t)dt \quad \left(\underset{ozn}{=}\overline{f}(s)\right) \quad ; \quad f - \text{oryginał}$$

$f(t)$	$\overline{f}(s)$	$f(t)$	$\overline{f}(s)$
$\mathbf{1}(t)$	$\frac{1}{s}$	$\sin\omega t,\ \omega\in\mathbf{R}$	$\frac{\omega}{s^2+\omega^2}$
$t^n,\ n\in\mathbf{N}$	$\frac{n!}{s^{n+1}}$	$\cos\omega t,\ \omega\in\mathbf{R}$	$\frac{s}{s^2+\omega^2}$
$e^{-\alpha t},\ \alpha\in\mathbf{C}$	$\frac{1}{s+\alpha}$	$e^{-\alpha t}\cdot\sin\omega t$, $\alpha\in\mathbf{C},\ \omega\in\mathbf{R}$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
$sh\beta t,\ \beta\in\mathbf{R}$	$\frac{\beta}{s^2-\beta^2}$	$ch\beta t,\ \beta\in\mathbf{R}$	$\frac{s}{s^2-\beta^2}$

$$\mathcal{L}[f^{(n)}(t)]=s^n\cdot\overline{f}(s)-\sum_{k=1}^{n}s^{n-k}\cdot f^{(k-1)}(0+)$$

$$\mathcal{L}[f'(t)]=s\overline{f}(s)-f(0+), \qquad \mathcal{L}[f''(t)]=s^2\overline{f}(s)-sf(0+)-f'(0+)$$

$$\mathcal{L}(f(at))=\frac{1}{a}\cdot\overline{f}\left(\frac{s}{a}\right), \quad \text{gdy } a>0$$

$$\mathcal{L}[f(t-t_0)]=e^{-st_0}\cdot\overline{f}(s), \quad t_0\ge 0$$

$$\mathcal{L}\left[e^{-\alpha t}\cdot f(t)\right]=\overline{f}(s+\alpha), \ \alpha\in\mathbf{C}$$

$$\mathcal{L}\left[\int_0^t f(\tau)d\tau\right]=\frac{\overline{f}(s)}{s}, \quad \mathcal{L}[t^n f(t)]=(-1)^n\frac{d^n\overline{f}(s)}{ds^n}, \quad n\in\mathbf{N}$$

Dla oryginału okresowego f: $\overline{f}(s)=\dfrac{\int_0^T f(t)\cdot e^{-st}dt}{1-e^{-sT}}$ (T – okres)

Dla $\overline{f}(s)=\dfrac{L(s)}{M(s)}$ (funkcja wymierna) : $\quad f(t)=\sum_i res_{s_i}[\overline{f}(s)\cdot e^{st}]$

SPLOT ORYGINAŁÓW: $F(t)=f_1(t)*f_2(t)$

$$F(t)\underset{df}{=}\left(\int_0^t f_1(\tau)f_2(t-\tau)d\tau\right)\cdot\mathbf{1}(t), \qquad \mathcal{L}[F(t)]=\overline{f_1}(s)\cdot\overline{f_2}(s)$$

PRZEKSZTAŁCENIE $\mathcal{Z}$

$$\mathcal{Z}[(x_n)] \underset{df}{=} \sum_{n=0}^{\infty} \frac{x_n}{z^n} \quad \left(\underset{ozn}{=} X(z) \right); \quad (x_n) = (x_0, x_1, \ldots)$$

x_n	$X(z)$	x_n	$X(z)$
1	$\frac{z}{z-1}$	$e^{\alpha n}, \alpha \in \mathbf{C}$	$\frac{z}{z-e^{\alpha}}$
n	$\frac{z}{(z-1)^2}$	$\sin \omega n, \omega \in \mathbf{C}$	$\frac{z \sin \omega}{z^2 - 2z\cos\omega + 1}$
n^2	$\frac{z(z+1)}{(z-1)^3}$	$\cos \omega n, \omega \in \mathbf{C}$	$\frac{z(z-\cos\omega)}{z^2 - 2z\cos\omega + 1}$
$a^n, a \in \mathbf{C} - \{0\}$	$\frac{z}{z-a}$	$\frac{1}{n!}$	$e^{\frac{1}{z}}$

$$\mathcal{Z}[(x_{n-k})] = z^{-k} \cdot X(z), \qquad k \in \mathbf{N}$$

$$\mathcal{Z}[(x_{n+k})] = z^k \cdot \left[X(z) - \sum_{\nu=0}^{k-1} \frac{x_\nu}{z^\nu} \right]$$

$$\mathcal{Z}[(nx_n)] = -z \frac{dX(z)}{dz}$$

Dla $X(z) = \frac{L(z)}{M(z)}$ (funkcja wymierna) : $x_n = \sum_i res_{z_i} [X(z) \cdot z^{n-1}]$

SPLOT CIĄGÓW: $(u_n) = (x_n) * (y_n)$

$$u_n = \sum_{\nu=0}^{n} x_\nu \cdot y_{n-\nu}, \quad n = 0,1,2,\ldots \quad ; \quad \mathcal{Z}[(u_n)] = X(z) \cdot Y(z)$$

$$\sin 2\alpha = 2 \sin\alpha \cos\alpha$$

$$\cos 2a = 2\cos^2\alpha - 1 = \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha$$

$$\sin(\alpha \pm \beta) = \sin\alpha \cdot \cos\beta \pm \cos\alpha \cdot \sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha \cdot \cos\beta \mp \sin\alpha \cdot \sin\beta$$

$$\sin\alpha \cdot \sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos\alpha \cdot \cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin\alpha \cdot \cos\beta = \frac{1}{2}[\sin(\alpha - \beta) + \sin(\alpha + \beta)]$$