Chapter 8

8.1 Inclusion / Exclusion

Given S a set

Conditions on S $c_1, c_2, c_3, \cdots c_k$

$$N(\bar{c}_1 \bar{c}_2 \cdots \bar{c}_k) = \#\text{elements of } S \text{ that satisfy none of } c_1, c_2, \cdots c_k$$
$$= |S| - [N(c_1) + N(c_2) + N(c_3) + \cdots N(c_k)] + [N(c_1c_2) + \cdots N(c_{k-1}c_k)] - \cdots + (-1)^k N(c_1c_2 \cdots c_k)$$
$$= S_0 - S_1 + S_2 \cdots S_k$$

Example: 8.5 – 8.7

8.2 Extensions of Inclusion and Exclusion

E_m = # elements of S that satisfy exactly m conditions

$$= S_m - \binom{m+1}{1} S_{m+1} + \binom{m+2}{2} S_{m+2} + \cdots + (-1)^{k-m}\binom{k}{k-m} S_k$$

L_m = # elements that satisfy at least m conditions

$$= S_m - \binom{m}{m-1} S_{m+1} + \binom{m+1}{m-1} S_{m+2} + \cdots (-1)^{k-m}\binom{k-1}{m-1} S_k.$$

8.3 Dearrangement

A dearrangement a permutation so that i is not sent to i

d_n = # dearrangement on $\{1, 2, 3, \cdots, n\}$

$$= n!\left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \cdots + (-1)^n \frac{1}{n!}\right]$$

☆ $r(C,x) = r(C_1, x)\, r(C_2, x)$

if C is comprised of C_1 & C_2 disjoint subboards

C_1 and C_2 share no rows or columns.

$r(C,x) = x \cdot r(C_s, x) + r(C_e, x)$

← board obtained by deleting the cell ∗.

board obtained by deleting row & column containing ∗.

Chapter 9 : Generating functions

$\boxed{9.1}$ Example : 9.1 – 9.3

$\boxed{9.2}$ (Regular) Generating function

$$f(x) := \sum_{k=0}^{\infty} a_n x^k$$

$\boxed{Ex}$ 1, 1, 1, 1, 1 .... $\Rightarrow 1 + x + x^2 + x^3 + \cdots$

$$= \frac{1}{1-x}$$

$\boxed{Ex}$ $\binom{n}{0}, \binom{n}{1}, \cdots \binom{n}{n}, 0, 0, \cdots \Rightarrow \binom{n}{0} + \binom{n}{1} x + \binom{n}{2} x^2 + \cdots + \binom{n}{n} x^n$

$$= (1+x)^n \quad n \geq 0$$

binomial thrm.

$\boxed{Ex}$ $\frac{1}{(1+x)^n} = \sum_{k=0}^{\infty} \binom{-n}{k} x^n$

$$\binom{-n}{k} := \binom{n+k-1}{k} (-1)^k$$

$\boxed{Ex}$ Partial Fraction Decomposition

$$\frac{1}{(x^2-4)} = \frac{A}{x-2} + \frac{B}{x+2}$$

Ex : 9.4, 9.5, 9.7, 9.8, 9.10, 9.12, 9.14, 9.16.

$\boxed{9.3}$ Partitions :

$\boxed{9\text{-}4}$ (Exponential) Generating Function

$$\{a_n\} \longrightarrow f(x) = \sum_{k=0}^{\infty} \frac{a_k}{k!} x^k$$

$\boxed{Ex}$ 9.26, 9.28.

Chapter 10

$\boxed{10.1 - 10.3}$ Linear RR. $C_k a_{n+k} + C_{k-1} a_{n+k-1} + \cdots + C_1 a_{n+1} + C_0 a_n = f(n)$

To solve $a_n = \underbrace{a_n^{(h)}}_{\text{solution}} + a_n^{(p)}$ — particular solution

$C_k a_{n+k} + \cdots + C_1 a_{n+1} + C_0 a_n = 0$

Homogeneous. $a_n^{(h)}$

Find characteristic Equation

$C_k r^k + C_{k-1} r^{k-1} + \cdots + C_1 r + C_0 = 0$.

Moral solution: ($r_1, r_2 \ldots r_k$ distinctive)

$a_n^{(n)} = C_1 r_1^n + C_2 r_2^n + \cdots + C_k r_k^n$

$\boxed{\deg 2}$ $C_2 r^2 + C_1 r + C_0 = 0$

$\boxed{r_1 \neq r_2 \text{ real}}$ $a_n^{(n)} = C_1 r_1^n + C_2 r_2^n$

$\boxed{r_1 = r_2 \text{ repeat}}$ $a_n^{(n)} = C_1 r_1^n + C_2 n r_2^n$

$\boxed{r_1, r_2 = a \pm bi}$ $a_n^{(n)} = r^n (C_1 \cos(n\theta) + C_2 \sin(n\theta))$

$\boxed{\text{Particular}}$ $a_n^{(p)}$

Method of Undetermined coefficient

i.e. guess $a_n^{(p)}$ looks like $f(n)$

$\boxed{\text{Ex}}$ $f(n) = 5 \Rightarrow a_n^{(p)} = A_0$

$f(n) = \cancel{5n} \; 5n+1 \Rightarrow a_n^{(p)} = A_1 n + A_0$.

To find coefficient $A_0, A_1, \ldots$

plug $a_n^{(p)}$ in RR.