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Chapter 4:

Problems

4–1. A thin dielectric rod of cross section A extends along the x-axis from $x = 0$ to $x = L$. The polarization of the rod is along its length, and is given by $P_x = ax^2 + b$. Find the volume density of polarization charge and the surface polarization charge on each end. Show *explicitly* that the total bound charge vanishes in this case.

4–2. A dielectric cube of side L has a radial polarization given by $\mathbf{P} = A\mathbf{r}$, where A is a constant, and $\mathbf{r} = \mathbf{i}x + \mathbf{j}y + \mathbf{k}z$. The origin of coordinates is at the center of the cube. Find all bound charge densities, and show *explicitly* that the total bound charge vanishes.

4–3. A dielectric rod in the shape of a right circular cylinder of length L and radius R is polarized in the direction of its length. If the polarization is uniform and of magnitude P, calculate the electric field resulting from this polarization at a point on the axis of the rod.

4–4. Prove the following relationship between the polarization, $\mathbf{P}$, and the bound charge densities ρ_P and σ_P, for a dielectric specimen of volume V and surface S.

$$\int_V \mathbf{P}\, dv = \int_V \rho_P \mathbf{r}\, dv + \int_S \sigma_P \mathbf{r}\, da.$$

Here, $\mathbf{r} = \mathbf{i}x + \mathbf{j}y + \mathbf{k}z$ is the position vector from any fixed origin. [*Hint:* Expand div $(x\mathbf{P})$ according to Eq. (4–10).]

4–5. Two semi-infinite blocks of dielectric are placed almost in contact so that there exists a narrow gap of constant separation between them. The polarization $\mathbf{P}$ is constant throughout all of the dielectric material, and it makes the angle γ with the normal to the planes bounding the gap. Determine the electric field in the gap.