Asset Allocation & Portfolio Construction: A Practical Case

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Abstract

[To Be Added]

1 Introduction

Whether for the sake of trying to make a fortune or for the sake of knowledge, both practitioners and academicians have had interests in studying the behavior of financial time series data since the existence of financial markets. Not only their motives were different, but also their practices. On one hand, financial practitioners *believed* that financial time series can be forecasted. Driven by their financial motives, they were set to exploit profit opportunities by forecasting and predicting the behavior of financial time series in capital markets. Academicians, on the other hand, were occupied with answering the question of whether or not it is possible to forecast financial time series. Despite their different motives, they both enhanced our understanding of the price formation process in financial markets.

Academicians contributed equilibrium models that aim to describe the process of price formation in the financial market. Over time, two schools of thoughts were established. Proponents of the first school of thought believed that resources are efficiently allocated among participants in capital markets. In an efficient setup, capital markets provide accurate signals for firms and investors that enable them to make efficient investment decisions. In other words, the proponents of this school of though entertained the Efficient Markets Hypothesis (EMH), which posits that, at any point in time, security prices fully reflect all available information in the market. Empirical evidence, however, shows otherwise. The EMH does not hold all the time. Recent evidence from behavioral finance and neurosciences shows that investors (especially retail traders) exhibit irrational behavior, which can explain this violation of the EMH. This led to the formation of the second school of thought,-the behavioral finance school.

Practitioners are not interested in developing models of price formation; rather they are interested in developing techniques to analyze and predict the price movements. Same as academicians, practitioners are also divided into two schools of thought: the fundamental analysis school and the technical analysis school. Although both schools of thought share the same objective, which is to give advise on what and when to buy and sell assets for the sake of making profit, they differ in their ways of analysis. The proponents of the fundamental analysis believe that any asset has a foundation value or **intrinsic** value. Due to market conditions, the actual price of the asset fluctuates continuously around this intrinsic value; it could fall below or rise above this value. This fluctuation implies that the actual market price of the asset will eventually reach its intrinsic value

but will rarely remain at it. This, in turn, creates buying and selling opportunities when the asset is undervalued or overvalued respectively. Finding the intrinsic value of the asset under consideration is the main objective of fundamental analysts. The proponents of technical analysis, on the other hand, believe that the study of past price movements helps in predicting its future movements. The general consensus among technical analysts is that fundamentals are irrelevant because all market information are reflected in the price process, and thus, studying the past behavior of the price series is the best way to predict its future movements.

The purpose of this paper is to describe empirically the process of constructing efficient portfolios of assets in a way similar to that adopted by practitioners in the finance industry, e.g., financial planners and fund managers. The portfolio construction process begins by an assessment of the investor's attitude towards risk, or the so-called investor risk profile, his expected rate of return, and the time horizon of his financial plan. After careful assessment, a portfolio of assets that suits the investor's needs and goals is then suggested. The **type** of the suggested portfolio is a function of all the information gathered on the investor. The **assets allocated** in the suggested portfolio depend on the planner's vision and his analysis of the market. The portfolio type could be extremely conservative, moderate, balanced, risky, or extremely risky. In practice, the suggested portfolios are classified into 7 categories: all income (All Y), income (Y), income and growth (Y & G), balanced, growth and income (G & Y), growth (G), and all equity, as shown in Table 1, where the portfolios are ranked from the least risk (All Y) to the highest risk (all equity). As for the types of assets, they are grouped into 10 categories from the most liquid (cash) to the least liquid type (real estate) as shown on the left column of the table. The previous classification is considered the standard practice in the industry. The weights of each portfolio that are displayed in the table are Markowitz's optimal weights, which, according to the source, are computed based on 40 years of market data. The table also shows the expected rate of return μ, the risk σ, and the Sharpe ratio corresponding to each portfolio. The optimal weights for each portfolio are considered the mandates for the financial planner.

After figuring out the investor's risk profile, which, in turn, determines the portfolio type, the financial planner or the portfolio manager then populates the asset classes corresponding to the selected portfolio by allocating the investor's resources according to the mandates associated with the selected portfolio.

In this paper, we will demonstrate how Table 1 is constructed. In particular, we will attempt to replicate the balanced portfolio displayed in the fifth column of the table. Once the balanced portfolio is constructed, the next task is to populate each asset class. Due to space limitation, it suffices for the purpose of this paper to describe the population process of one asset class, namely, the Canadian Equities (CE) asset class. The other classes could be populated using a similar approach.

Assets/Portfolios Types	All Y†	Y	Y & G‡	Balanced	G & Y	G	Equity
Cash	25%	20%	15%	5%	5%	5%	
ST Fixed Income*	75%	25%	16%	20%	15%	5%	
Fixed Income		30%	29%	25%	15%	10%	
CDN Equities**		6%	8%	10%	11%	15%	15%
CDN Small Cap Equities		0%	3%	5%	7%	10%	12%
US Equities		9%	10%	10%	12%	14%	18%
US Small Cap Equities		0%	7%	8%	10%	15%	25%
International Equities		5%	7%	7%	10%	8%	10%
Emerging Markets		0%	0%	5%	10%	15%	20%
Real Estate		5%	5%	5%	5%	3%	
Return on Assets (%)	3.81	5.67	6.46	7.10	7.79	8.65	9.69
Standard Deviation (%)	4.46	5.45	7.06	8.67	10.97	13.7	17.23
Sharpe Ratio	0.10	0.42	0.44	0.43	0.40	0.38	0.36
*ST = Short-Term †Y= Income ‡G= Growth **CDN = Canadian							

Table 1: Different assets and portfolios types. Assets are classified top to bottom according to their liquidity. Portfolios are classified from left to right according to their riskiness. The weights in percentages of each portfolio are Markowitz's optimal weights computed based on 40-year market data. The last three lines of the table show the return, the risk, and Sharpe ratio corresponding to each portfolio. Source: Plan Plus Web Advisor, Plan Plus Inc

2 Data Description & Portfolio Construction

Following the classification of asset classes in Table 1, our universe consists of 10 asset classes. In particular, Short-Term Fixed Income (STFI), Fixed Income (FI), Canadian Equity (CE), Canadian Small Capital Equity (CSCE), U.S. Equity (USE), U.S. Small Capital Equity (USSCE), International Equity (IE), Real Estate (RE), and Cash (CASH). The rates of return on the first 9 asset classes are approximated by rates of return computed from monthly closing **prices** of 9 indexes from Bloomberg. As for the rate of return on Cash, it is approximated by the yield on one year Canadian government bond. A description of the proxy indexes and the one year Canadian government bond series, which are extracted from Bloomberg terminal, is found in a form of table in Appendix A. The first and second columns of the table display, respectively, the name and the abbreviation of each asset class. The third and fourth columns display, respectively, the tickers and the names of the proxy indexes as they appear on Bloomberg. Column five gives a brief description of each index demonstrating the rationale behind using it as a proxy for the corresponding asset class. Finally, the last column displays the type of proxy used. The data set consists of monthly observations on the previously mentioned indexes over the period between December 2000 and May 2014. In particular, end-of-month closing prices of the first 9 indexes and end-of-month annualized rates of return on the one year Canadian government bond between December 2000 and May 2014 are considered.

The construction of the balanced portfolio is based on the following mandates:

- The target rate of return on the portfolio is taken to be 7%
- The minimum weight of any asset class in the portfolio is 5%.
- The maximum weight of any asset class in the portfolio is 20%.
- The maximum weight of cash in the portfolio 5%.

Based on the previous mandates, the Markowitz problem is solved and the resulting efficient frontier is constructed. On the efficient frontier, two portfolios are identified: the minimum variance portfolio (MVP) and the tangency (T) portfolio. The constructed balanced portfolio, however, is the portfolio on the efficient frontier that yields the target rate of return of 7%. The computations are performed in R using the package "fPortfolio."[1]

The weight of the Canadian Equities asset class based on this optimization is 5%. This is to be expected, as the equities portion of the portfolio will most likely be the most volatile, with the highest returns.

3 Asset Allocation

Once the balanced portfolio is constructed, the next step is to populate the CE asset class in the constructed portfolio. To this end, 25-35 stocks from the top 60 stocks listed on the Standard & Poor Toronto Stock Exchange (SPTSX) are selected and the expected rate of return and risk of the resulting portfolio are computed. Therefore, the universe in Part 2 consists of the top 60 stocks listed SPTSX. The market index is the SPTSX top 60 index, which we will denote by SPTSX60. Data on the monthly closing prices of all stocks in the SPTSX60 index is found on the Thompson Reuters terminal. The following are the mandates of populating the CE asset class.

[1] See Wuertz et al (2009).

- The number of stocks selected from the top 60 in SPTSX60 is between 25 and 35 stocks inclusive.
- The historical daily data used to compute the expected returns, variances, and covariances of the selected stocks covers the period between the 30th of May, 2012 and the 30th of May 2016 including both months, i.e., a total of 1003 trading days.
- Fundamental data such as earning per share, profit margin, return on equity, and price-to-book ratio, could be used to select stocks.[2]
- Minimum variance optimization should be used to come up with maximum risk-adjusted return optimal portfolio, i.e., maximum Sharpe optimal portfolio.
- Weights must be non-negative.

Essentially, two strategies are pursued to construct the previous portfolio. Either to construct a portfolio that minimizes tracking error with fewer securities or to construct a portfolio that attempts to outperform the benchmark. A hybrid approach could also be used, but the previous two approaches are essentially the two strategies followed by practitioners in the industry. The former strategy is known as the **Beta Approach** to portfolio selection and the latter is known as the **Alpha Approach**.

The **Alpha Approach** to portfolio selection is pursued by portfolio managers who do not believe in the efficient markets hypothesis (EMH). They believe that through analysis (usually fundamental), they are able to identify miss-pricing opportunities and use it to achieve a rate of return on their portfolio that is in excess of the market rate of return by buying and selling these miss-priced securities. The **Alpha** of the portfolio is the difference between the **average rate of return** on the portfolio that is **actually** achieved by the portfolio manager and the **required rate of return** as suggested by the Capital Asset Pricing Model (CAPM).[3] According to the CAPM, alpha of any portfolio should be zero because the model is based on the assumption that all markets are efficient. In practice, however, this is not the case. A positive alpha is indicative of an outstanding performance of the fund manager. That's why, the proponents of this approach believe that it is possible to achieve a positive alpha on their portfolio through active management. Unfortunately, only a few portfolio managers who have been actually able to consistently outperform the market.

The **Beta Approach** to portfolio selection is adopted by portfolio managers who believe in markets efficiency. Their plan in general is to replicate the performance of the market portfolio in the least costly way.[4] They favour to retain exposure to various industries in the same proportion as in the index. The performance measure of this approach is known as the **tracking error** of the portfolio, which is the standard deviation of the difference between the actual portfolio return and the market return. The smaller the tracking error, the closer the portfolio's performance to that of the market, i.e., the close the portfolio is in tracking the market.

In practice, many portfolio managers use a hybrid approach that consists of features from both approaches. For example, if the portfolio manager is less familiar with a particular sector, he or she may choose to follow the Beta approach and choose stocks in that sector in the same proportion

[2]Some useful Bloomberg fields are: CUR_MKT_CAP, TRAIL_12_M_EPS, TRAIL_12M_PROF_MARGIN, NORMALIZED_ROE, PE_RATIO, and PX_TO_BOOK_RATIO.

[3]See Fahmy, H. (2014) Chapter 4, Section 4.2 for more details.

[4]Investment management fees and trading costs are examples of portfolio costs.

as the market portfolio. If, however, the manager is more familiar with that sector, he or she may choose to overweight or underweight the allocation of that sector relative to its weight in the broad market index to express his or her view on the future performance of the sector.

4 Methodology of Asset Selection

For this paper, we will be using a hybrid approach (some **Alpha**, some **Beta**) for asset selection, attempting to track the index; in this case the SPTSX60. The approach is as follows:

1. We separated the SPTSX60 securities into their respective GICS sectors; *Consumer Discretionary, Consumer Staples, Energy, Health Care, Industrials, Information Technology, Materials, Utilities, and Telecommunication Services.* We purposely excluded the heavily regulated Financial Sector to avoid any biased results. Detailed descriptions of these sectors can be found in Appendix A.

2. We then performed fundamental analysis on each security by comparing various ratios (Price to Earning, Earnings Per Share, Profit Margin, Return on Equity, and Price to Book Ratio) to their respective sector average. We then selected the securities that beat the average more often than others. While doing this, we attempted to avoid bias in sectors that had few stocks within them. For example, the healthcare sector was comprised of only one security, Valeant. Therefore we performed a more in depth analysis on Valeant and concluded that it was not a stable security to hold in our portfolio. This concluded the **Alpha** portion of the approach.

3. For the **Beta** portion of the approach, we looked at how well the returns of each equity tracked the returns of the index (SPTSX60) over the last four years[5]. That is, given the two vectors of returns $\vec{r_i} = (r_i^{(1)}, r_i^{(2)}, \ldots, r_i^{(n)})$ for the index, and $\vec{r_a} = (r_a^{(1)}, r_a^{(2)}, \ldots, r_a^{(n)})$ for the asset; we calculated the tracking error as follows, where n is the number of observations:

$$\text{Tracking Error} = \|\vec{r_i} - \vec{r_a}\|_2 = \sqrt{\sum_{k=1}^{n} |\vec{r_i}^{(k)} - \vec{r_a}^{(k)}|^2}$$

4. The items with the smallest tracking error from part 3 above were then cross-referenced with those from the **Alpha** approach above in order to pick the best assets to populate the equities class portion of the portfolio. In addition to this; equities with a negative daily average return over the period (Potash Corp of Saskatchewan, Cameo Corp., and Cenovus Energy Inc.) were removed.

In the end, 27 equities were selected.

5 Optimization of Canadian Equities

Some additional considerations were to be taken into account before proceeding to the optimization which would give the optimal weights of the 27 equities selected in the previous section. These considerations are:

- What is the target rate of return on the equities portfolio?

[5]Note that for the assets for which data was not available - no more than 3 of them - we simply used the most stable index return available in that time period that later contained the asset in question

- What are the minimum and maximum weights for each asset in the equities portfolio?

Let us first address the issue of selecting minimum and maximum weights for the assets. Since we want all of our 27 selected stocks to be in the portfolio, we set the minimum weight at **1%**.

The maximum weight should be chosen in a way such that the portfolio will still be **diversified**†. That is; there is no one stock which overshadows the others, because this would subject expose the portfolio to more of the risk that that specific security faces. We call this type of risk *non-systematic risk* or *firm specific risk*, as opposed to *systematic risk* which affects the entire market. Some examples of systematic risk are inflation, GDP fluctuations, advances in technology etc.

Therefore we will allow a maximum weight of **7.8%** in order to allow for some assets to occupy approximately double the weight they could in an equally weighted portfolio.

† **Aside**: Diversification of a portfolio refers to increasing the number of assets and reducing the weight attributed to each asset. In doing so, we reduce firm-specific risk exposure of the portfolio. Theoretically, we can diversify adequately to achieve *zero* firm specific risk.

Proof:

Let the expected return of the portfolio be defined as: $E[\tilde{r}_p] = \frac{(r_1+\ldots+r_n)}{n}$ where $r_1, \ldots, r_n$ are the returns of each asset in the portfolio. Note that the division by n implies this is an equally weighted portfolio.

Now let us look at the variance of this portfolio:

$$\text{Var}[\tilde{r}_p] = \text{Var}\Big(\frac{r_1 + \ldots + r_n}{n}\Big) = \frac{1}{n^2}\text{Var}\big(r_1 + \ldots + r_n\big)$$

Now if we take the limit as $n \to \infty$, we get:

$$\lim_{n\to\infty} \frac{1}{n^2}\text{Var}\big(r_1 + \ldots + r_n\big) = 0 \text{ by Hopital's Rule}$$

QED □

Now what should the annual target rate of return be?

The optimization uses historical daily return data to calculate the optimal weights. The averages of the daily returns on the assets are quite high; ranging from 4% to 48% when converted to annual returns[6][7]. If we were to take equal weighting in all 27 assets, the return would be 18.41%, therefore we can could create a portfolio with very high return (but high risk).

In order to select the proper rate of return, we ran the optimization on many different target annual rates ranging from 12% to 20%, and selected the return with the lowest variance since we are risk averse.

We selected an annual target rate of return of **12%** for our portfolio, but there were complications in this, as we will see later.

In summary: the Markowitz optimization for the Canadian Equities portfolio will be executed with the following parameters:

- 27 equities selected from the SPTSX60, as outlined in Section 4.

[6] where the conversion is done with returns ϕ in the formula: $\phi_{\text{annual}} = 250 * \phi_{\text{daily}}$

[7] details in Appendix B

- 12% annual return.
- 1% minimum weight in all 27 equities.
- 7.8% maximum weight in all 27 equities.

6 Performance Evaluation & Concluding Remarks

The results of the Markowitz optimization are as follows:

Equity	Weight	Equity	Weight
Canadian Tire	1%	Canadian Natural Resources Ltd	1%
Magna International	1%	Canadian National Railway Co	1%
Gildan Activewear Inc	1%	SNC Lavalin Group	6.08%
Dollarama Inc	1%	Canadian Pacific Railway Ltd	1%
Saputo Inc	1%	CGI Group Inc	1%
George Weston Inc	7.11%	Constellation Software Inc Canada	1%
Metro Inc	1%	Agrium Inc	7.8%
Alimentation Couche Tard Inc	1%	Franco Nevada Corp	1%
TransCanada Corp	7.8%	Silver Wheaton Corp	7.12%
Enbridge Inc	7.8%	BCE Inc	7.8%
Imperial Oil Ltd	7.8%	TELUS Corp	7.8%
Pembina Pipeline Corp	2.29%	Fortis Inc Canada	7.8%
Suncor Energy Inc	1%	Emera Inc	7.8%
Inter Pipeline Ltd	1%	*(see pie chart on next page)*	

Given these weights we achieve the target rate of return of 12% annually while still staying within the constraints of the minimum and maximum weights of 1% and 7.8% respectively.

Volatility of the Equities Portfolio:

In order to calculate the volatility of the portfolio we must convert the daily standard deviation on returns given by the optimization to an annual standard deviation. In practice, the convention[8] is to use the following formula:

$$\sigma_{\text{annual}} = \sigma_{\text{daily}} * \sqrt{\#\text{ of trading days}}$$

For our portfolio, we have $\sigma_{\text{daily}} = 0.007082578$, and 250 trading days.

Therefore $\sigma_{\text{annual}} = 0.1119854$.

So the annual standard deviation of our equities portfolio is $\approx 11.2\%$[9]; we are very bullish.[10]

Our Sharpe Ratio is therefore $\frac{0.12}{0.1119854} = 1.071568$.

The Sharpe Ratio quantifies the relationship between return and risk; that is, if we were to accept an increase of 1 unit in σ_{annual}, we would get an additional return of 1.071568 units.

This is a low Sharpe Ratio, and in the next section we will look at why the optimization came out this way.

[8] see `http://www.investopedia.com/articles/04/021804.asp`

[9] Note that the standard deviation here refers to a Normal Distribution; therefore this means that there is a 68% chance that our return will be between 0.8% and 23.2%, but there is also a significant (15%) chance we will make a negative return

[10] Aggressive, high-risk investors are bullish; whereas cautious, risk-averse investors are bearish.

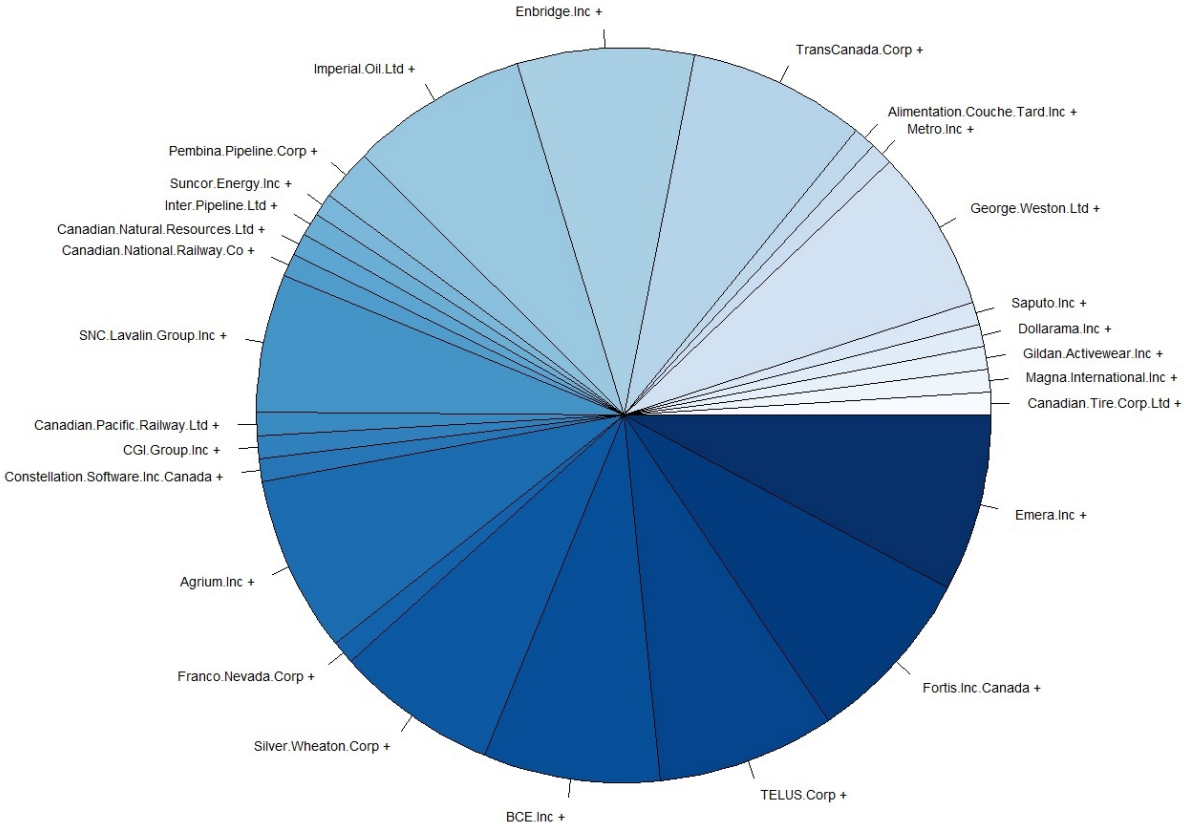

Figure 1: Equity Weights

7 Some limitations of Markowitz Portfolio Theory

It is well known that Markowitz Portfolio Theory is not perfect, and in the creation of our portfolio we encountered some peculiarities of Markowitz which were not included in the paper.

As briefly explained in Section 1, Markowitz theory is based on some fundamental assumptions about the way in which the market behaves, which are collectively named the **Efficient Market Hypothesis**. These assumptions are not always consistent with the markets, nor do they account for some other peculiarities which we have encountered here.

In particular, with the strategy outlined in Section 4: Methodology of Asset Selection, we have chosen assets with very high returns[11] due to selecting the assets whose fundamentals beat their sector averages. The average of the returns on our 27 selected assets was approximately 18%, which is much higher than the rate of return we'd chosen of 12%.

So what does that mean? Well, let us look at the *Markowitz Efficient Frontier* to explain further:

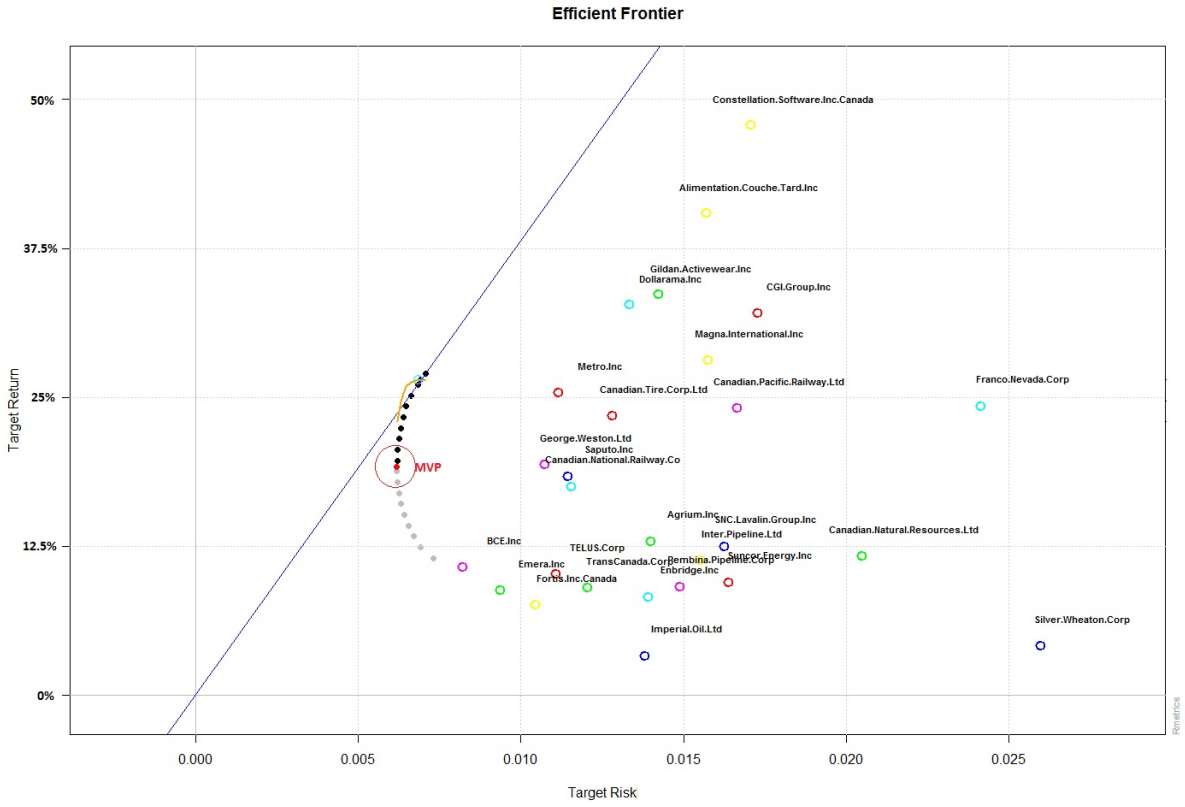

Figure 2: The Markowitz Efficient Frontier

[11] See Appendix B

Note that our target rate of return of 12% falls below the circled Minimum Variance Portfolio (MVP) with respect to return (y-axis). This is the portfolio with the least risk which we could assemble under the weight constraints that we chose. So wouldn't we want to build this portfolio, since its return is higher than 12%? Indeed we would. So why didn't we?

Upon closer inspection, we can see that the portfolios (the gray and black dots) are very close together with respect to horizontal distance. That is, they all have very similar variances.

So, what does this all mean? It means that further analysis on these stocks is needed before investing, or perhaps a different strategy is needed. It is rare that equity portfolios give 19% return[12] (the return on MVP); and the only reason our portfolio has given such a high return is that we chose some stocks which performed extremely well over the last 4 years. This is easy to do in hindsight, while avoiding stocks like Valeant, mentioned earlier, which lost nearly 90% of its value in the sample period.

These are some peculiarities of Markowitz, and portfolio theory in general: the assumptions that one will avoid extremely volatile stocks such as Valeant who plummet suddenly, and that the stocks selected will continue to perform in the same manner they have been in the sample period. This second assumption can be tested with professional financial analysts' forecasts and through one's own fundamental & technical analysis; but this is a costly and time-consuming process.

It is also important to note that our optimization did not account for dividends, which are important in equity selection. The Financial Planning Standards Council of Canada, which oversees Certified Financial Planners notes in its 2016 Projection Assumption Guidelines:

Historically, from 25% *to* 50% *of overall equity returns has been made up of dividends. It therefore seems reasonable to assume that* 33% *of the overall equity return will be made up of dividencds and the rest will be capital gains*[13]

We should learn from this case study that one should not apply the results of Markowitz Portfolio Theory blindly, as it could lead to extremely costly and unforeseen consequences. We wanted a portfolio with 12% return, and the optimization did indeed give us that. But it wasn't actually the optimal choice; and in this scenario it's hard to know what the absolute best option really is. Some more tools will be needed, in addition to Markowitz, to make an informed decision.

[12]Equity Portfolios give about 5-10% on average. One such example is the BMO Private Canadian Income Equity Portfolio, which contains some of the assets in this optimization. It has an average return of 7.1% over the last 10 years `http://fundfacts.bmo.com/HarrisEnglish/BMO_Private_Canadian_Income_Equity_Portfolio-EN-Series_All.pdf`

[13]`http://www.fpsc.ca/docs/default-source/FPSC/projection-assumption-guidelines-2016.pdf`

8 Appendix A: SPTSX60 Sectors

Consumer Discretionary

The Consumer Discretionary industry is the third most important sector of the SPTSX60 index (excluding the financials sector) by market capitalization, following the energy sector (20.95%) and the materials sector (10.8%) with a weight of 8.61%. The consumer discretionary industry is composed of companies that produce goods that are not essential but desirable when the income permits it. Specifically, these companies are in the areas of auto manufacturing, media, casino and gaming, movies and entertainment, internet and catalog retail, diversified consumer services, hotels restaurants and leisure. The consumer discretionary industry based on the SPTSX60 Index consists of Canadian Tire Corp Ltd, Gildan Activewear Inc, Restaurant Brands International Inc, Thomson Reuters Corp, Dollarama Inc, Magna International Inc and Shaw Communications Inc.

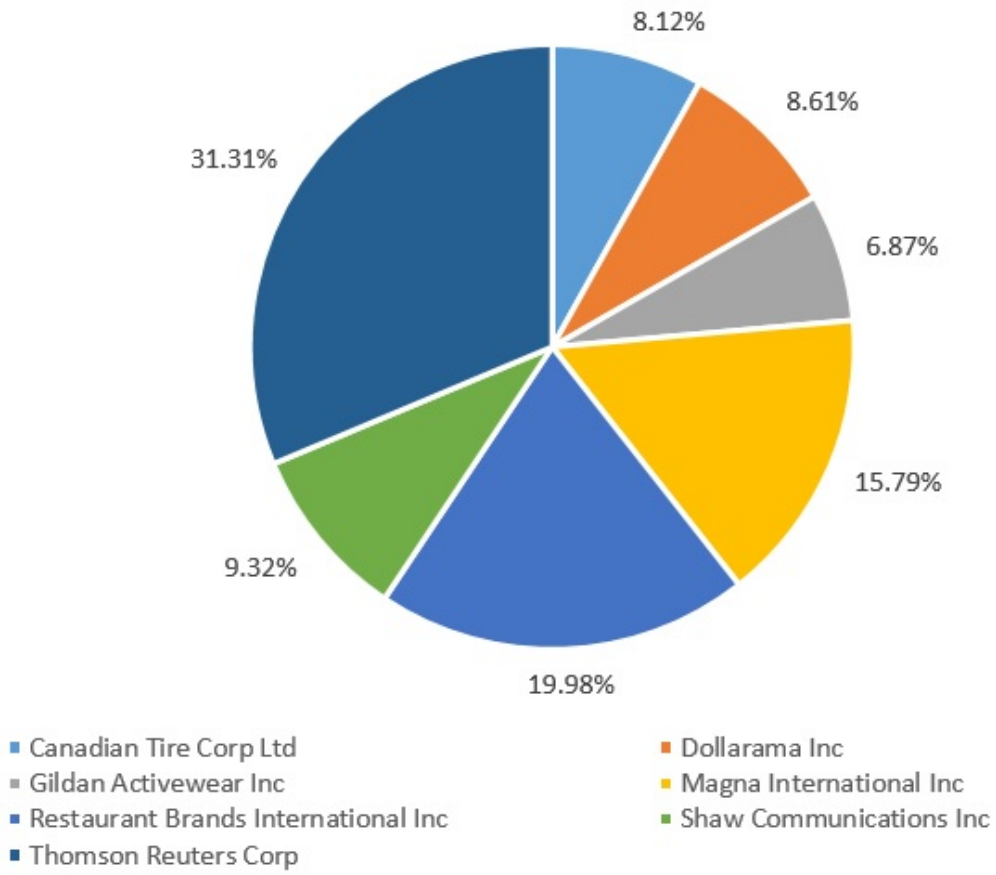

Figure 3: Market Capitalization of CD Sector

Consumer Staples

The consumer staples industry is the fifth most important sector of the SPTSX60 index (excluding the financials) by market capitalization, following the energy sector (20.95%), the materials sector (10.8%), the consumer discretionary sector (8.61%) and the telecommunication services sector (6.89%) with a weight of 6.63%. The consumer staples sector comprises companies that produce essential products that people are unable or unwilling to cut out of their budgets regardless of their financial situation. Specifically, the consumer staples sector is composed of food, beverages and tobacco companies and producers of non-durable household goods and personal products. The consumer staples industry based on the SPTSX60 Index consists of Alimentation Couche-Tard Inc, George Weston Ltd, Loblaw Cos Ltd, Metro Inc and Saputo Inc.

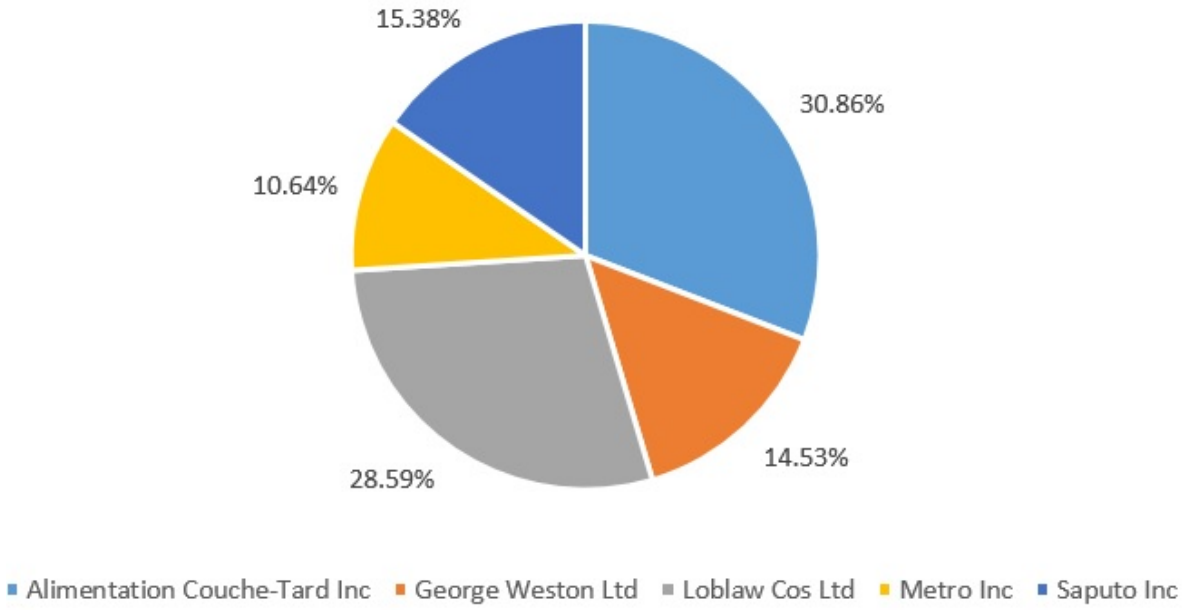

Figure 4: Market Capitalization of CS Sector

Energy

The energy industry is the most important sector of the SPTSX60 index (excluding the financials) by market capitalization with a weight of 20.95%. The energy sector is composed of companies that are involved in the production and sale of energy, including fuel extraction, manufacturing, refining and distribution. In particular, the energy industry comprises the petroleum industry, the gas industry, the electrical power industry, the coal industry, the nuclear power industry, the renewable energy industry and the traditional energy industry based on the collection and distribution of firewood. The energy industry based on the SPTSX60 Index consists of ARC Resources Ltd, Cenovus Energy Inc, Encana Corp, Inter Pipeline Ltd, TransCanada Corp, Cameco Corp, Crescent Point Energy Corp, Husky Energy Inc, Pembina Pipeline Corp, Canadian Natural Resources Ltd, Enbridge Inc, Imperial Oil Ltd and Suncor Energy Inc.

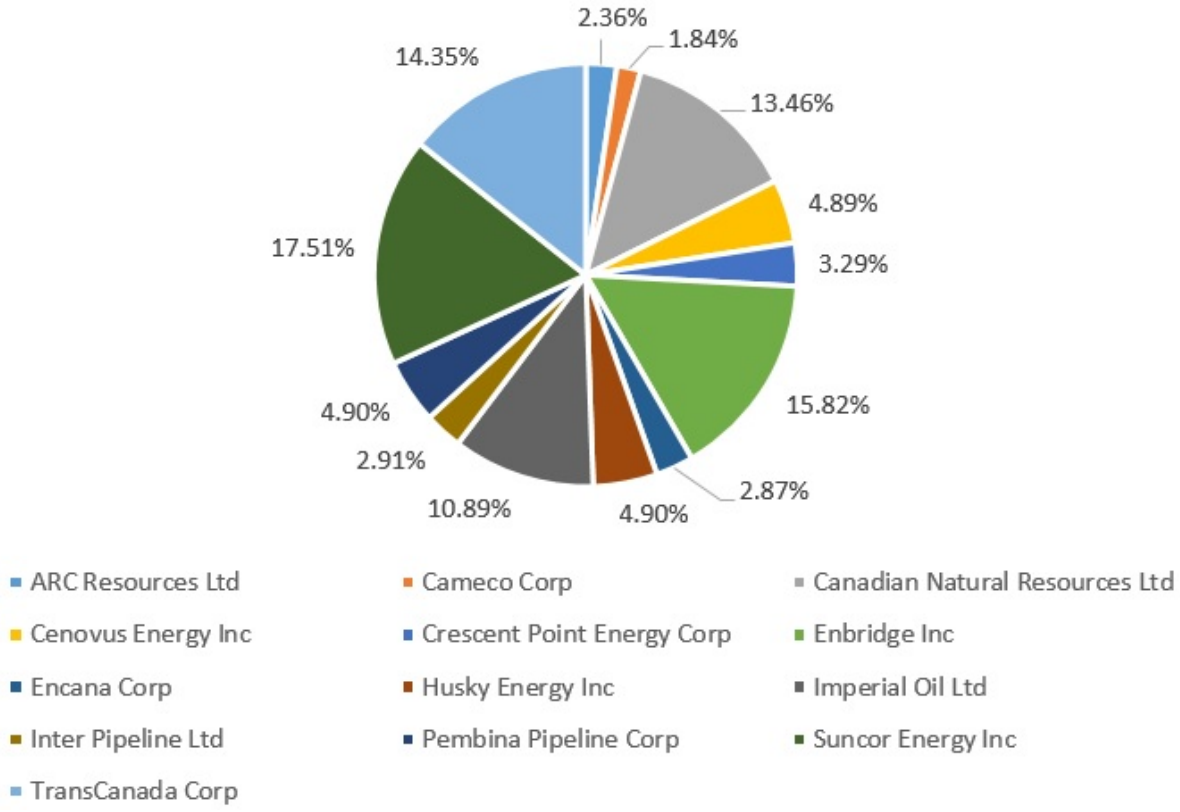

Figure 5: Market Capitalization of Energy Sector

Materials

The materials and mining industry is the second most important sector of the SPTSX60 index after energy (excluding financials), as it weighs 10.8% in terms of market capitalization. The industry incorporates companies that explore, develop and process raw materials such as rare metals, chemicals, and minerals. The sector consists of Agnico Eagle Mines Ltd, Agrium Inc, Barrick Gold Corp, Eldorado Gold Corp, First Quantum Minerals Ltd, Franco-Nevada Corp, Goldcorp Inc, Kinross Gold Corp, Potash Corp of Saskatchewan Inc, Silver Wheaton Corp, Teck Resources Ltd, and Yamana Gold Inc.

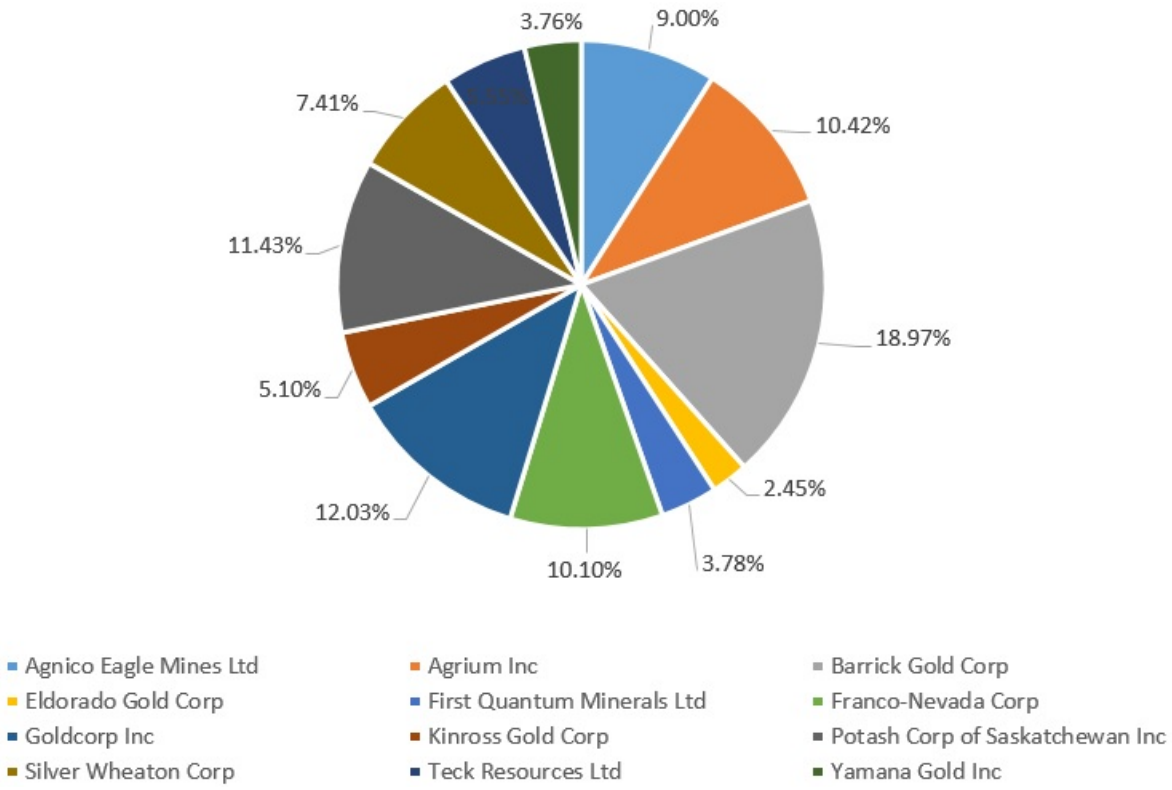

Figure 6: Market Capitalization of Materials Sector

Industrials

The industrials sector is the sixth industry in the SPTSX60 index, weighing 6.49% in terms of market capitalization. The industry incorporates companies that produces goods and services that are used in construction and manufacturing. More specifically, these companies produce vehicles for transportation, machinery for construction companies and offer freight services. The sector consists of Bombardier Inc., Canadian National Railway Co, Canadian Pacific Railway Ltd. and SNC-Lavalin Group Inc.

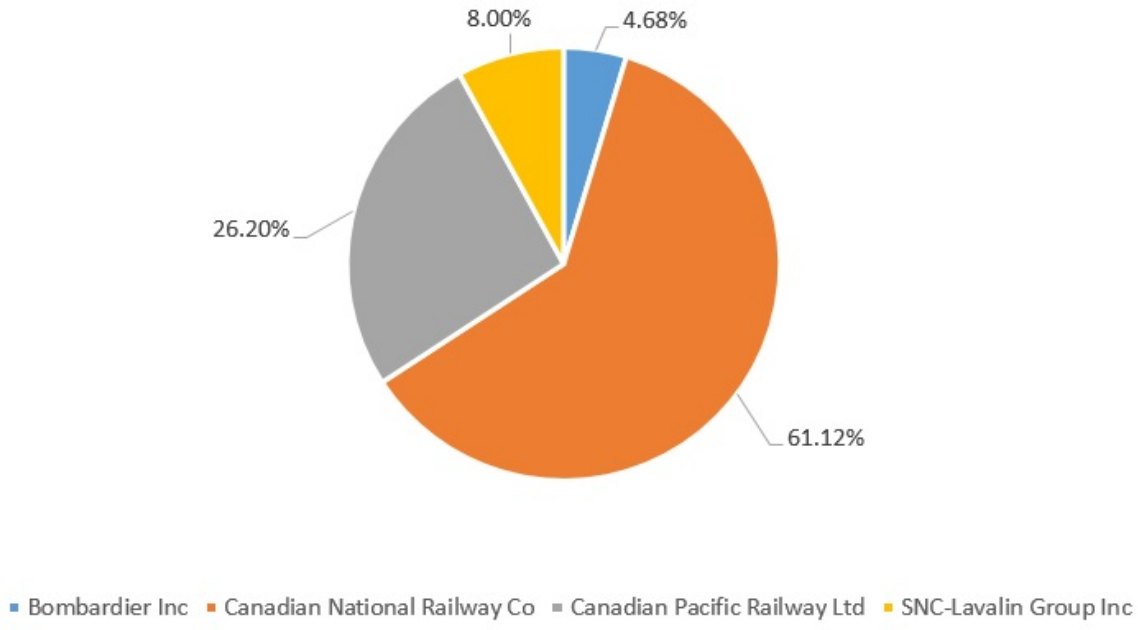

Figure 7: Market Capitalization of Industrials Sector

Information Technology

The information and technology sector is the seventh industry in the SPTSX60 index, weighing 2.14% in terms of market capitalization. The industry incorporates companies that research, develop and sell computational hardware and software. More specifically these companies produces software solutions to institutional clients and offer technological services. The sector consist of Blackberry Ltd, CGI Group Inc and Constellation Software Inc.

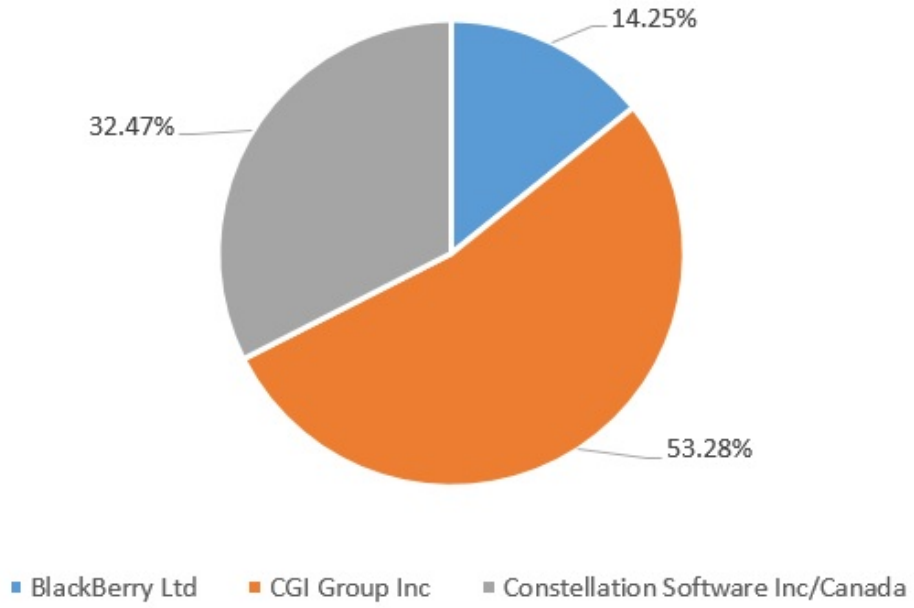

Figure 8: Market Capitalization of IT Sector

Utilities

The utility industry is the second to last industry in terms of market capitalization in the nine industries, taking up only 1.27% of the total market share. The utilities sector is a category of stocks for utilities such as gas and power. The sector contains companies such as electric, gas and water firms, and integrated providers. The utility industry based on the SPTSX60 Index consists of Emera Inc and Fortis Inc Canada.

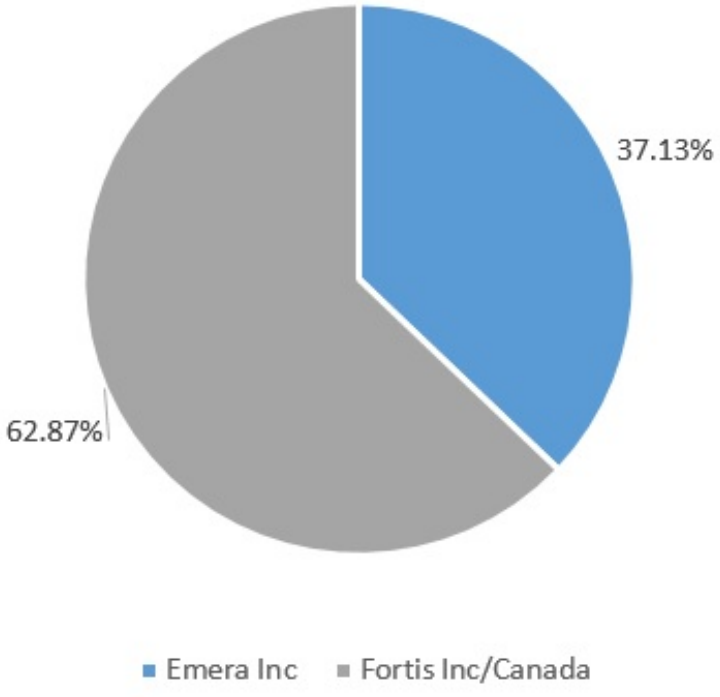

Figure 9: Market Capitalization of Utilities Sector

Telecommunication Services

The telecommunication industry ranks the 5th in the SPTSX60 index in terms of market share. This industry takes up 6.89% of the total market capitalization of the index. The telecommunications sector comprises companies that make communication possible on a global scale whether through the phone or Internet. These companies created the infrastructure that allows data to be sent anywhere in the world. The telecommunication industry based on the SPTSX60 Index consists of BCE Inc and Rogers Communications Inc and TELUS Corp.

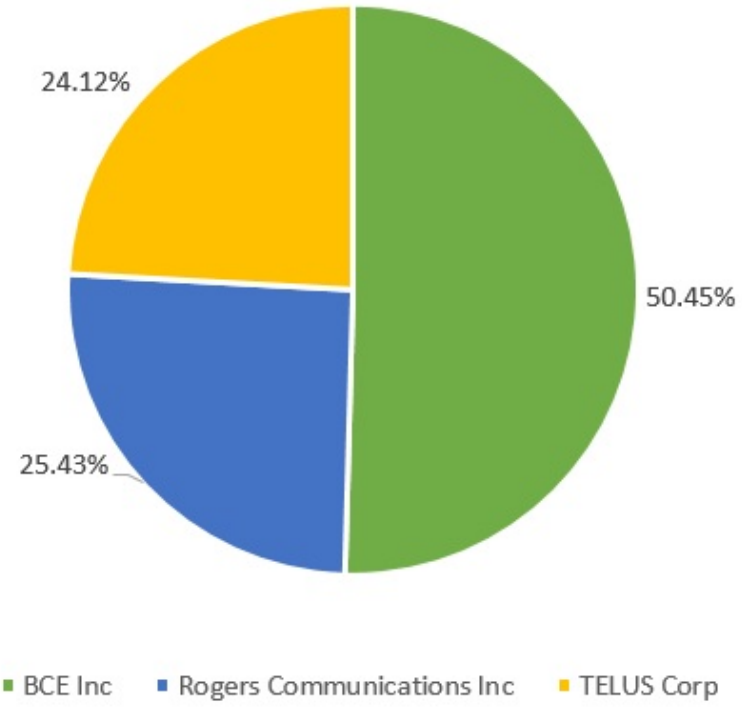

Figure 10: Market Capitalization of Telecom Sector

Healthcare

The healthcare industry is the most trivial in terms of market capitalization among the 9 industries, taking up only 0.65% of the index. The healthcare industry is an aggregation and integration of sectors within the economic system that provides goods and services to treat patients with curative, preventive, rehabilitative, and palliative care. It includes the generation and commercialization of goods and services lending themselves to maintaining and re-establishing health. The healthcare industry based on the SPTSX60 Index consists of Valeant Pharmaceuticals International In.

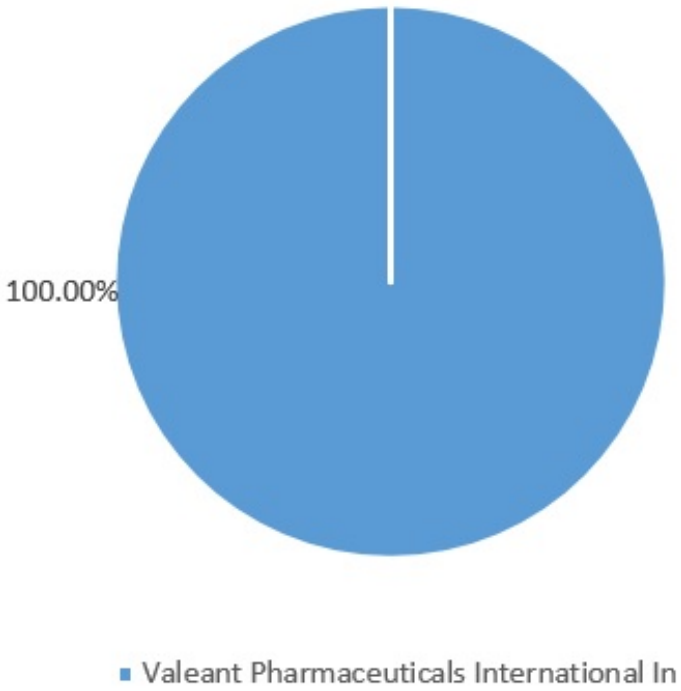

Figure 11: Market Capitalization of Health Sector

9 Appendix B: Asset Data

Canadian Tire Corp Ltd	Magna International Inc	Gildan Activewear Inc	Dollarama Inc	Saputo Inc
23.46%	28.15%	33.64%	32.79%	18.40%
George Weston Ltd	Metro Inc	Alimentation Couche-Tard Inc	TransCanada Corp	Enbridge Inc
19.37%	25.45%	40.46%	9.04%	8.27%
Imperial Oil Ltd	Pembina Pipeline Corp	Suncor Energy Inc	Inter Pipeline Ltd	Canadian Natural Resources Ltd
3.34%	9.15%	9.51%	11.34%	11.75%
Canadian National Railway Co	SNC-Lavalin Group Inc	Canadian Pacific Railway Ltd	CGI Group Inc	Constellation Software Inc/Canada
17.53%	12.51%	24.10%	32.08%	47.88%
Agrium Inc	Franco-Nevada Corp	Silver Wheaton Corp	BCE Inc	TELUS Corp
12.97%	24.30%	4.15%	10.82%	10.20%
Fortis Inc Canada	Emera Inc			
7.64%	8.86%			

Figure 12: Annual Returns of 27 selected Assets

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