

# THE HONG KONG POLYTECHNIC UNIVERSITY

# INVESTMENT SCIENCE

## DEPARTMENTAL SOCIETY OF APPLIED MATHEMATICS

Chau Chun Fai  
15059203.D  
AMA364 Assignment 2

$$1. f(x|\theta) = (1-\theta)^x \theta$$

$$L(\theta) = \prod_{i=1}^n [(1-\theta)^{x_i} \theta]$$

$$= (1-\theta)^{\sum_{i=1}^n x_i} \theta^n$$

$$= g(T(x_1, x_2, \dots, x_n), \theta) \times h(x_1, \dots, x_n)$$

where  $g(T(x_1, \dots, x_n), \theta) = (1-\theta)^{\sum_{i=1}^n x_i} \theta^n$

$$h(x_1, \dots, x_n) = 1$$

$$T(x_1, \dots, x_n) = \sum_{i=1}^n x_i$$

By Factorization Theorem,  $\sum_{i=1}^n x_i$  is a sufficient statistic for  $\theta$ .

$$2. f(x|\theta) = \frac{\Gamma(\theta+5)}{\Gamma(\theta)\Gamma(5)} x^{\theta-1} (1-x)^4$$

$$L(\theta) = \left[ \frac{\Gamma(\theta+5)}{\Gamma(\theta)\Gamma(5)} \right]^n \left( \sum_{i=1}^n x_i \right)^{\theta-1} \left[ \prod_{i=1}^n (1-x_i) \right]^4$$

$$= g(T(x_1, x_2, \dots, x_n), \theta) \times h(x_1, \dots, x_n)$$

where  $g(T(x_1, x_2, \dots, x_n), \theta) = \left[ \frac{\Gamma(\theta+5)}{\Gamma(\theta)\Gamma(5)} \right]^n \left( \sum_{i=1}^n x_i \right)^{\theta-1}$

$$h(x_1, \dots, x_n) = \left[ \prod_{i=1}^n (1-x_i) \right]^4$$

$$T(x_1, \dots, x_n) = \sum_{i=1}^n x_i$$

By Factorization Theorem,  $\sum_{i=1}^n x_i$  is a sufficient statistic for  $\theta$ .

$$3. f(x|\theta) = \frac{1}{6\theta^4} x^3 e^{-\frac{x}{\theta}} \sim \Gamma(4, \theta)$$

$$\ln f(x|\theta) = 3\ln x - \frac{x}{\theta} - \ln 6 - 4\ln \theta$$

$$f(x|\theta) = e^{(-\frac{x}{\theta} + 3\ln x - \ln 6 - 4\ln \theta)}$$

$$= e^{K(x)p(\theta) + S(x) + q(\theta)}$$

$$\text{where } K(x) = x, p(\theta) = \frac{1}{\theta}, S(x) = 3\ln x - \ln 6, q(\theta) = -4\ln \theta$$

$\therefore Y_1 = \sum_{i=1}^n x_i$  is a complete sufficient statistic for  $\theta$ .

$$\because f(x|\theta) \sim \Gamma(4, \theta)$$

$$\therefore E(X) = 4\theta$$

$$E(Y_1) = \sum_{i=1}^n E(x_i) = 4n\theta$$

$\therefore \phi(Y_1) = \frac{Y_1}{4n}$  is an unbiased estimator for  $\theta$ .

$\therefore \phi(Y_1)$  is the MVUE of  $\theta$ .

$\therefore \phi(Y_1) = \frac{Y_1}{4n}$  is one to one function.

$\therefore \phi(Y_1)$  is also a complete sufficient statistic for  $\theta$ .

4.  $f(x|\theta) = \frac{1}{\theta} e^{\frac{x}{\theta}}$  where  $x < 0, \theta > 0$

$\therefore x < 0, \theta > 0$

$\therefore f(x|\theta) \sim \exp(\frac{1}{\theta})$

$\ln f(x|\theta) = \frac{x}{\theta} - \ln \theta$

$$\begin{aligned}f(x|\theta) &= e^{\frac{x}{\theta} - \ln \theta} \\&= e^{K(x)p(\theta) + S(\theta) + g(\theta)}\end{aligned}$$

where  $K(x) = x$ ,  $p(\theta) = \frac{1}{\theta}$ ,  $S(\theta) = 0$ ,  $g(\theta) = -\ln \theta$

$\therefore Z = \sum_{i=1}^n x_i$  is a complete sufficient statistic for  $\theta$ .

$\because f(x|\theta) \sim \exp(\frac{1}{\theta})$

$\therefore E(x) = \frac{1}{\theta} = \theta$

$E(Z) = E\left(\sum_{i=1}^n x_i\right) = n\theta$

$\therefore \phi(Z) = \frac{Z}{n}$  is an unbiased estimator for  $\theta$

$\therefore \frac{Z}{n}$  is a MVUE of  $\theta$ ,