Another proof that the harmonic series diverges

If the harmonic series converges, then its kth 'tail'

$$T(k) := \sum_{n=k+1}^{\infty} \frac{1}{n}$$

is well-defined, and is a strictly decreasing function of k. Now note that

$$\begin{aligned} T(2m) &= \frac{1}{2m+1} + \frac{1}{2m+2} + \frac{1}{2m+3} + \frac{1}{2m+4} + \cdots \\ &\geq \frac{1}{2m+2} + \frac{1}{2m+2} + \frac{1}{2m+4} + \frac{1}{2m+4} + \cdots \\ &= \frac{2}{2m+2} + \frac{2}{2m+4} + \cdots \\ &= \frac{1}{m+1} + \frac{1}{m+2} + \cdots \\ &= T(m), \end{aligned}$$

and if $m \geq 1$ then this contradicts that T is a strictly decreasing function.