Variance in betting

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Assume that we place n bets.
Let X_i be our payout from the i'th game.
Let p_i be our probability of winning bet i.
Let s_i be our stake at bet i.

For simplicity, assume that $p_i = \frac{1}{o_i}$, where o_i is the bookmaker odds (implying no edge)

Then, according to the definition of expected value,
$E(X_i) = s_i o_i p_i + (1 - p_i) \cdot 0$
$E(X_i) = s_i$

This was to be expected, as we have no edge due to my assumption above. We simply expect to get our stake in return.

Then we calculate the variance of X_i:
$Var(X_i) = p_i\left(s_i o_i - E(X_i)\right)^2 + (1 - p_i)\left(0 - E(X_i)\right)^2$
$Var(X_i) = p_i(s_i o_i - s_i)^2 + (1 - p_i)(0 - 1)^2$
$Var(X_i) = p_i\left(s_i(o_i - 1)\right)^2 + (1 - p_i)$
$Var(X_i) = p_i s_i^2 (o_i - 1)^2 + (1 - p_i)$
$Var(X_i) = \frac{s_i^2}{o_i}(o_i - 1)^2 + 1 - \frac{1}{o_i}$

Then we introduce a new variable, ROI, which is the sum of all returns divided by the sum of all stakes.

$$ROI = \frac{\sum_i X_i}{\sum_i s_i} = \frac{\sum_i X_i}{\sum_i s_i}$$

We expect intuitively that the expected ROI is 1, which also holds mathematically true because

$$E(ROI) = E\left(\frac{\sum_i X_i}{\sum_i s_i}\right) = \frac{E(\sum_i X_i)}{\sum_i s_i} = \frac{\sum_i E(X_i)}{\sum_i s_i} = \frac{\sum_i s_i}{\sum_i s_i} = 1$$

What is more interesting is the variance of ROI.

$$Var(ROI) = Var\left(\frac{\sum_i X_i}{\sum_i s_i}\right)$$
$$Var(ROI) = \frac{Var(\sum_i X_i)}{\left(\sum_i s_i\right)^2}$$
$$Var(ROI) = \frac{\sum_i Var(X_i)}{\left(\sum_i s_i\right)^2}$$
$$Var(ROI) = \frac{\sum_i\left(\frac{s_i^2}{o_i}(o_i - 1)^2 + 1 - \frac{1}{o_i}\right)}{\left(\sum_i s_i\right)^2}$$

The standard deviation is calculated as follows

$$Sd(ROI) = \sqrt{Var(ROI)}$$
$$Sd(ROI) = \sqrt{\frac{\sum_i\left(\frac{s_i^2}{o_i}(o_i - 1)^2 + 1 - \frac{1}{o_i}\right)}{\left(\sum_i s_i\right)^2}}$$

This expression gets simplified if we assume that every stake $s_i = 1$

$$Sd(ROI)_{s_i=1} = \sqrt{\frac{\sum_i \left(\frac{1}{o_i}(o_i - 1)^2 + 1 - \frac{1}{o_i}\right)}{\left(\sum_i 1\right)^2}} = \sqrt{\frac{\sum_i \left(\frac{1}{o_i}\left(o_i^2 - 2o_i + 1\right) + 1 - \frac{1}{o_i}\right)}{n^2}} = \frac{1}{n}\sqrt{\sum_i \left(o_i - 2 + \frac{1}{o_i} + 1 - \frac{1}{o_i}\right)} = \frac{1}{n}\sqrt{\sum_i (o_i - 1)}$$

If we assume equal (constant) odds for every bet such that $o_i = o$, we have that

$$SD(ROI)_{s_i=1,o_i=o} = \frac{1}{n}\sqrt{\sum_i (o-1)} = \frac{1}{n}\sqrt{n(o-1)} = \sqrt{\frac{o-1}{n}}$$

Assume that you are placing 20 bets all with an odds of 2.

$$SD(ROI)_{s_i=1,o_i=2,n=35} = \sqrt{\frac{2-1}{20}} = 22.36\ \%$$

This confirms that 20% ROI obtained from 20 bets is not a statistically significant result.