

Chapter 26

1. (a) The charge that passes through any cross section is the product of the current and time. Since $t = 4.0 \text{ min} = (4.0 \text{ min})(60 \text{ s/min}) = 240 \text{ s}$,

$$q = it = (5.0 \text{ A})(240 \text{ s}) = 1.2 \times 10^3 \text{ C}.$$

(b) The number of electrons N is given by $q = Ne$, where e is the magnitude of the charge on an electron. Thus,

$$N = q/e = (1200 \text{ C})/(1.60 \times 10^{-19} \text{ C}) = 7.5 \times 10^{21}.$$

2. Suppose the charge on the sphere increases by Δq in time Δt . Then, in that time its potential increases by

$$\Delta V = \frac{\Delta q}{4\pi\epsilon_0 r},$$

where r is the radius of the sphere. This means $\Delta q = 4\pi\epsilon_0 r \Delta V$. Now, $\Delta q = (i_{\text{in}} - i_{\text{out}}) \Delta t$, where i_{in} is the current entering the sphere and i_{out} is the current leaving. Thus,

$$\begin{aligned} \Delta t &= \frac{\Delta q}{i_{\text{in}} - i_{\text{out}}} = \frac{4\pi\epsilon_0 r \Delta V}{i_{\text{in}} - i_{\text{out}}} = \frac{(0.10 \text{ m})(1000 \text{ V})}{(8.99 \times 10^9 \text{ F/m})(1.0000020 \text{ A} - 1.0000000 \text{ A})} \\ &= 5.6 \times 10^{-3} \text{ s}. \end{aligned}$$

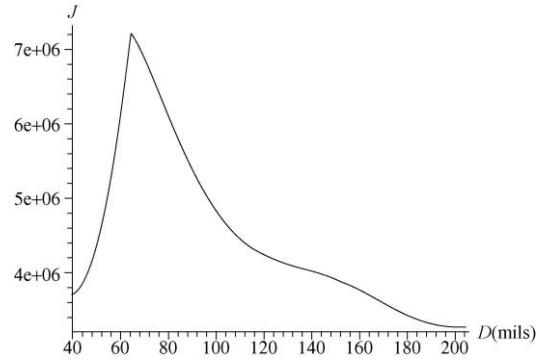
3. We adapt the discussion in the text to a moving two-dimensional collection of charges. Using σ for the charge per unit area and w for the belt width, we can see that the transport of charge is expressed in the relationship $i = \sigma v w$, which leads to

$$\sigma = \frac{i}{vw} = \frac{100 \times 10^{-6} \text{ A}}{(30 \text{ m/s})(50 \times 10^{-2} \text{ m})} = 6.7 \times 10^{-6} \text{ C/m}^2.$$

4. We express the magnitude of the current density vector in SI units by converting the diameter values in mils to inches (by dividing by 1000) and then converting to meters (by multiplying by 0.0254) and finally using

$$J = \frac{i}{A} = \frac{i}{\pi R^2} = \frac{4i}{\pi D^2}.$$

For example, the gauge 14 wire with $D = 64 \text{ mil} = 0.0016 \text{ m}$ is found to have a (maximum safe) current density of $J = 7.2 \times 10^6 \text{ A/m}^2$. In fact, this is the wire with the largest value of J allowed by the given data. The values of J in SI units are plotted below as a function of their diameters in mils.



5. THINK The magnitude of the current density is given by $J = nqv_d$, where n is the number of particles per unit volume, q is the charge on each particle, and v_d is the drift speed of the particles.

EXPRESS In vector form, we have (see Eq. 26-7) $\vec{J} = nq\vec{v}_d$. Current density \vec{J} is related to the current i by (see Eq. 26-4): $i = \int \vec{J} \cdot d\vec{A}$.

ANALYZE (a) The particle concentration is $n = 2.0 \times 10^8/\text{cm}^3 = 2.0 \times 10^{14} \text{ m}^{-3}$, the charge is

$$q = 2e = 2(1.60 \times 10^{-19} \text{ C}) = 3.20 \times 10^{-19} \text{ C},$$

and the drift speed is $1.0 \times 10^5 \text{ m/s}$. Thus, we find the current density to be

$$J = (2 \times 10^{14} / \text{m})(3.2 \times 10^{-19} \text{ C})(1.0 \times 10^5 \text{ m/s}) = 6.4 \text{ A/m}^2.$$

(b) Since the particles are positively charged the current density is in the same direction as their motion, to the north.

(c) The current cannot be calculated unless the cross-sectional area of the beam is known. Then $i = JA$ can be used.

LEARN That the current density is in the direction of the motion of the *positive* charge carriers means that it is in the opposite direction of the motion of the negatively charged electrons.

6. (a) Circular area depends, of course, on r^2 , so the horizontal axis of the graph in Fig. 26-24(b) is effectively the same as the area (enclosed at variable radius values), except for a factor of π . The fact that the current increases linearly in the graph means that $i/A = J = \text{constant}$. Thus, the answer is “yes, the current density is uniform.”

(b) We find $i/(\pi r^2) = (0.005 \text{ A})/(\pi \times 4 \times 10^{-6} \text{ m}^2) = 398 \approx 4.0 \times 10^2 \text{ A/m}^2$.

7. The cross-sectional area of wire is given by $A = \pi r^2$, where r is its radius (half its thickness). The magnitude of the current density vector is

$$J = i/A = i/\pi r^2,$$

so

$$r = \sqrt{\frac{i}{\pi J}} = \sqrt{\frac{0.50 \text{ A}}{\pi(440 \times 10^4 \text{ A/m}^2)}} = 1.9 \times 10^{-4} \text{ m}.$$

The diameter of the wire is therefore $d = 2r = 2(1.9 \times 10^{-4} \text{ m}) = 3.8 \times 10^{-4} \text{ m}$.

8. (a) The magnitude of the current density vector is

$$J = \frac{i}{A} = \frac{i}{\pi d^2/4} = \frac{4(1.2 \times 10^{-10} \text{ A})}{\pi(2.5 \times 10^{-3} \text{ m})^2} = 2.4 \times 10^{-5} \text{ A/m}^2.$$

(b) The drift speed of the current-carrying electrons is

$$v_d = \frac{J}{ne} = \frac{2.4 \times 10^{-5} \text{ A/m}^2}{(8.47 \times 10^{28} / \text{m}^3)(1.60 \times 10^{-19} \text{ C})} = 1.8 \times 10^{-15} \text{ m/s}.$$

9. We note that the radial width $\Delta r = 10 \text{ } \mu\text{m}$ is small enough (compared to $r = 1.20 \text{ mm}$) that we can make the approximation

$$\int Br2\pi r dr \approx Br2\pi r \Delta r$$

Thus, the enclosed current is $2\pi Br^2 \Delta r = 18.1 \text{ } \mu\text{A}$. Performing the integral gives the same answer.

10. Assuming \vec{J} is directed along the wire (with no radial flow) we integrate, starting with Eq. 26-4,

$$i = \int |\vec{J}| dA = \int_{9R/10}^R (kr^2) 2\pi r dr = \frac{1}{2} k\pi (R^4 - 0.656R^4)$$

where $k = 3.0 \times 10^8$ and SI units are understood. Therefore, if $R = 0.00200 \text{ m}$, we obtain $i = 2.59 \times 10^{-3} \text{ A}$.

11. (a) The current resulting from this non-uniform current density is

$$i = \int_{\text{cylinder}} J_a dA = \frac{J_0}{R} \int_0^R r \cdot 2\pi r dr = \frac{2}{3} \pi R^2 J_0 = \frac{2}{3} \pi (3.40 \times 10^{-3} \text{ m})^2 (5.50 \times 10^4 \text{ A/m}^2) \\ = 1.33 \text{ A}.$$

(b) In this case,

$$i = \int_{\text{cylinder}} J_b dA = \int_0^R J_0 \left(1 - \frac{r}{R}\right) 2\pi r dr = \frac{1}{3} \pi R^2 J_0 = \frac{1}{3} \pi (3.40 \times 10^{-3} \text{ m})^2 (5.50 \times 10^4 \text{ A/m}^2) \\ = 0.666 \text{ A}.$$

(c) The result is different from that in part (a) because J_b is higher near the center of the cylinder (where the area is smaller for the same radial interval) and lower outward, resulting in a lower average current density over the cross section and consequently a lower current than that in part (a). So, J_a has its maximum value near the surface of the wire.

12. (a) Since $1 \text{ cm}^3 = 10^{-6} \text{ m}^3$, the magnitude of the current density vector is

$$J = nev = \left(\frac{8.70}{10^{-6} \text{ m}^3} \right) (1.60 \times 10^{-19} \text{ C}) (470 \times 10^3 \text{ m/s}) = 6.54 \times 10^{-7} \text{ A/m}^2.$$

(b) Although the total surface area of Earth is $4\pi R_E^2$ (that of a sphere), the area to be used in a computation of how many protons in an approximately unidirectional beam (the solar wind) will be captured by Earth is its projected area. In other words, for the beam, the encounter is with a “target” of circular area πR_E^2 . The rate of charge transport implied by the influx of protons is

$$i = AJ = \pi R_E^2 J = \pi (6.37 \times 10^6 \text{ m})^2 (6.54 \times 10^{-7} \text{ A/m}^2) = 8.34 \times 10^7 \text{ A}.$$

13. We use $v_d = J/ne = i/Ane$. Thus,

$$t = \frac{L}{v_d} = \frac{L}{i/Ane} = \frac{LANe}{i} = \frac{(0.85 \text{ m}) (0.21 \times 10^{-14} \text{ m}^2) (8.47 \times 10^{28} / \text{m}^3) (1.60 \times 10^{-19} \text{ C})}{300 \text{ A}} \\ = 8.1 \times 10^2 \text{ s} = 13 \text{ min}.$$

14. Since the potential difference V and current i are related by $V = iR$, where R is the resistance of the electrician, the fatal voltage is $V = (50 \times 10^{-3} \text{ A})(2000 \Omega) = 100 \text{ V}$.

15. **THINK** The resistance of the coil is given by $R = \rho L/A$, where L is the length of the wire, ρ is the resistivity of copper, and A is the cross-sectional area of the wire.

EXPRESS Since each turn of wire has length $2\pi r$, where r is the radius of the coil, then

$$L = (250)2\pi r = (250)(2\pi)(0.12 \text{ m}) = 188.5 \text{ m}.$$

If r_w is the radius of the wire itself, then its cross-sectional area is

$$A = \pi r_w^2 = \pi(0.65 \times 10^{-3} \text{ m})^2 = 1.33 \times 10^{-6} \text{ m}^2.$$

According to Table 26-1, the resistivity of copper is $\rho = 1.69 \times 10^{-8} \Omega \cdot \text{m}$.

ANALYZE Thus, the resistance of the copper coil is

$$R = \frac{\rho L}{A} = \frac{(1.69 \times 10^{-8} \Omega \cdot \text{m})(188.5 \text{ m})}{1.33 \times 10^{-6} \text{ m}^2} = 2.4 \Omega.$$

LEARN Resistance R is the property of an object (depending on quantities such as L and A), while resistivity is a property of the material.

16. We use $R/L = \rho/A = 0.150 \Omega/\text{km}$.

(a) For copper $J = i/A = (60.0 \text{ A})(0.150 \Omega/\text{km})/(1.69 \times 10^{-8} \Omega \cdot \text{m}) = 5.32 \times 10^5 \text{ A/m}^2$.

(b) We denote the mass densities as ρ_m . For copper,

$$(m/L)_c = (\rho_m A)_c = (8960 \text{ kg/m}^3)(1.69 \times 10^{-8} \Omega \cdot \text{m})/(0.150 \Omega/\text{km}) = 1.01 \text{ kg/m}.$$

(c) For aluminum $J = (60.0 \text{ A})(0.150 \Omega/\text{km})/(2.75 \times 10^{-8} \Omega \cdot \text{m}) = 3.27 \times 10^5 \text{ A/m}^2$.

(d) The mass density of aluminum is

$$(m/L)_a = (\rho_m A)_a = (2700 \text{ kg/m}^3)(2.75 \times 10^{-8} \Omega \cdot \text{m})/(0.150 \Omega/\text{km}) = 0.495 \text{ kg/m}.$$

17. We find the conductivity of Nichrome (the reciprocal of its resistivity) as follows:

$$\sigma = \frac{1}{\rho} = \frac{L}{RA} = \frac{L}{(V/i)A} = \frac{Li}{VA} = \frac{(1.0 \text{ m})(4.0 \text{ A})}{(2.0 \text{ V})(1.0 \times 10^{-6} \text{ m}^2)} = 2.0 \times 10^6 / \Omega \cdot \text{m}.$$

18. (a) $i = V/R = 23.0 \text{ V}/15.0 \times 10^{-3} \Omega = 1.53 \times 10^3 \text{ A}$.

(b) The cross-sectional area is $A = \pi r^2 = \frac{1}{4} \pi D^2$. Thus, the magnitude of the current density vector is

$$J = \frac{i}{A} = \frac{4i}{\pi D^2} = \frac{4(1.53 \times 10^3 \text{ A})}{\pi(6.00 \times 10^{-3} \text{ m})^2} = 5.41 \times 10^7 \text{ A/m}^2.$$

(c) The resistivity is

$$\rho = \frac{RA}{L} = \frac{(15.0 \times 10^{-3} \Omega) \pi (6.00 \times 10^{-3} \text{ m})^2}{4(4.00 \text{ m})} = 10.6 \times 10^{-8} \Omega \cdot \text{m}.$$

(d) The material is platinum.

19. **THINK** The resistance of the wire is given by $R = \rho L / A$, where ρ is the resistivity of the material, L is the length of the wire, and A is its cross-sectional area.

EXPRESS In this case, the cross-sectional area is

$$A = \pi r^2 = \pi (0.50 \times 10^{-3} \text{ m})^2 = 7.85 \times 10^{-7} \text{ m}^2.$$

ANALYZE Thus, the resistivity of the wire is

$$\rho = \frac{RA}{L} = \frac{(50 \times 10^{-3} \Omega) (7.85 \times 10^{-7} \text{ m}^2)}{2.0 \text{ m}} = 2.0 \times 10^{-8} \Omega \cdot \text{m}.$$

LEARN Resistance R is the property of an object (depending on quantities such as L and A), while resistivity is a property of the material itself. The equation $R = \rho L / A$ implies that the larger the cross-sectional area A , the smaller the resistance R .

20. The thickness (diameter) of the wire is denoted by D . We use $R \propto L/A$ (Eq. 26-16) and note that $A = \frac{1}{4} \pi D^2 \propto D^2$. The resistance of the second wire is given by

$$R_2 = R \left(\frac{A_1}{A_2} \right) \left(\frac{L_2}{L_1} \right) = R \left(\frac{D_1}{D_2} \right)^2 \left(\frac{L_2}{L_1} \right) = R(2)^2 \left(\frac{1}{2} \right) = 2R.$$

21. The resistance at operating temperature T is $R = V/i = 2.9 \text{ V}/0.30 \text{ A} = 9.67 \Omega$. Thus, from $R - R_0 = R_0 \alpha (T - T_0)$, we find

$$T = T_0 + \frac{1}{\alpha} \left(\frac{R}{R_0} - 1 \right) = 20^\circ \text{C} + \left(\frac{1}{4.5 \times 10^{-3} / \text{K}} \right) \left(\frac{9.67 \Omega}{1.1 \Omega} - 1 \right) = 1.8 \times 10^3 \text{ }^\circ \text{C}.$$

Since a change in Celsius is equivalent to a change on the Kelvin temperature scale, the value of α used in this calculation is not inconsistent with the other units involved. Table 26-1 has been used.

22. Let $r = 2.00 \text{ mm}$ be the radius of the kite string and $t = 0.50 \text{ mm}$ be the thickness of the water layer. The cross-sectional area of the layer of water is

$$A = \pi[(r+t)^2 - r^2] = \pi[(2.50 \times 10^{-3} \text{ m})^2 - (2.00 \times 10^{-3} \text{ m})^2] = 7.07 \times 10^{-6} \text{ m}^2.$$

Using Eq. 26-16, the resistance of the wet string is

$$R = \frac{\rho L}{A} = \frac{(150 \Omega \cdot \text{m})(800 \text{ m})}{7.07 \times 10^{-6} \text{ m}^2} = 1.698 \times 10^{10} \Omega.$$

The current through the water layer is

$$i = \frac{V}{R} = \frac{1.60 \times 10^8 \text{ V}}{1.698 \times 10^{10} \Omega} = 9.42 \times 10^{-3} \text{ A}.$$

23. We use $J = E/\rho$, where E is the magnitude of the (uniform) electric field in the wire, J is the magnitude of the current density, and ρ is the resistivity of the material. The electric field is given by $E = V/L$, where V is the potential difference along the wire and L is the length of the wire. Thus $J = V/L\rho$ and

$$\rho = \frac{V}{LJ} = \frac{115 \text{ V}}{(10 \text{ m})(1.4 \times 10^8 \text{ A/m}^2)} = 8.2 \times 10^{-8} \Omega \cdot \text{m}.$$

24. (a) Since the material is the same, the resistivity ρ is the same, which implies (by Eq. 26-11) that the electric fields (in the various rods) are directly proportional to their current-densities. Thus, $J_1: J_2: J_3$ are in the ratio 2.5/4/1.5 (see Fig. 26-25). Now the currents in the rods must be the same (they are “in series”) so

$$J_1 A_1 = J_3 A_3, \quad J_2 A_2 = J_3 A_3.$$

Since $A = \pi r^2$, this leads (in view of the aforementioned ratios) to

$$4r_2^2 = 1.5r_3^2, \quad 2.5r_1^2 = 1.5r_3^2.$$

Thus, with $r_3 = 2 \text{ mm}$, the latter relation leads to $r_1 = 1.55 \text{ mm}$.

(b) The $4r_2^2 = 1.5r_3^2$ relation leads to $r_2 = 1.22 \text{ mm}$.

25. **THINK** The resistance of an object depends on its length and the cross-sectional area.

EXPRESS Since the mass and density of the material do not change, the volume remains the same. If L_0 is the original length, L is the new length, A_0 is the original cross-sectional area, and A is the new cross-sectional area, then $L_0 A_0 = LA$ and

$$A = L_0 A_0 / L = L_0 A_0 / 3L_0 = A_0 / 3.$$

ANALYZE The new resistance is

$$R = \frac{\rho L}{A} = \frac{\rho 3L_0}{A_0/3} = 9 \frac{\rho L_0}{A_0} = 9R_0,$$

where R_0 is the original resistance. Thus, $R = 9(6.0 \, \Omega) = 54 \, \Omega$.

LEARN In general, the resistances of two objects made of the same material but different cross-sectional areas and lengths may be related by

$$R_2 = R_1 \left(\frac{A_1}{A_2} \right) \left(\frac{L_2}{L_1} \right).$$

26. The absolute values of the slopes (for the straight-line segments shown in the graph of Fig. 26-25(b)) are equal to the respective electric field magnitudes. Thus, applying Eq. 26-5 and Eq. 26-13 to the three sections of the resistive strip, we have

$$J_1 = \frac{i}{A} = \sigma_1 E_1 = \sigma_1 (0.50 \times 10^3 \, \text{V/m})$$

$$J_2 = \frac{i}{A} = \sigma_2 E_2 = \sigma_2 (4.0 \times 10^3 \, \text{V/m})$$

$$J_3 = \frac{i}{A} = \sigma_3 E_3 = \sigma_3 (1.0 \times 10^3 \, \text{V/m}) .$$

We note that the current densities are the same since the values of i and A are the same (see the problem statement) in the three sections, so $J_1 = J_2 = J_3$.

(a) Thus we see that $\sigma_1 = 2\sigma_3 = 2(3.00 \times 10^7 (\Omega \cdot \text{m})^{-1}) = 6.00 \times 10^7 (\Omega \cdot \text{m})^{-1}$.

(b) Similarly, $\sigma_2 = \sigma_3/4 = (3.00 \times 10^7 (\Omega \cdot \text{m})^{-1})/4 = 7.50 \times 10^6 (\Omega \cdot \text{m})^{-1}$.

27. **THINK** In this problem we compare the resistances of two conductors that are made of the same materials.

EXPRESS The resistance of conductor A is given by

$$R_A = \frac{\rho L}{\pi r_A^2},$$

where r_A is the radius of the conductor. If r_o is the outside diameter of conductor B and r_i is its inside diameter, then its cross-sectional area is $\pi(r_o^2 - r_i^2)$, and its resistance is

$$R_B = \frac{\rho L}{\pi(r_o^2 - r_i^2)}.$$

ANALYZE The ratio of the resistances is

$$\frac{R_A}{R_B} = \frac{r_o^2 - r_i^2}{r_A^2} = \frac{(1.0 \text{ mm})^2 - (0.50 \text{ mm})^2}{(0.50 \text{ mm})^2} = 3.0.$$

LEARN The resistance R of an object depends on how the electric potential is applied to the object. Also, R depends on the ratio L/A , according to $R = \rho L/A$.

28. The cross-sectional area is $A = \pi r^2 = \pi(0.002 \text{ m})^2$. The resistivity from Table 26-1 is $\rho = 1.69 \times 10^{-8} \Omega \cdot \text{m}$. Thus, with $L = 3 \text{ m}$, Ohm's Law leads to $V = iR = i\rho L/A$, or

$$12 \times 10^{-6} \text{ V} = i(1.69 \times 10^{-8} \Omega \cdot \text{m})(3.0 \text{ m}) / \pi(0.002 \text{ m})^2$$

which yields $i = 0.00297 \text{ A}$ or roughly 3.0 mA .

29. First we find the resistance of the copper wire to be

$$R = \frac{\rho L}{A} = \frac{(1.69 \times 10^{-8} \Omega \cdot \text{m})(0.020 \text{ m})}{\pi(2.0 \times 10^{-3} \text{ m})^2} = 2.69 \times 10^{-5} \Omega.$$

With potential difference $V = 3.00 \text{ nV}$, the current flowing through the wire is

$$i = \frac{V}{R} = \frac{3.00 \times 10^{-9} \text{ V}}{2.69 \times 10^{-5} \Omega} = 1.115 \times 10^{-4} \text{ A}.$$

Therefore, in 3.00 ms , the amount of charge drifting through a cross section is

$$\Delta Q = i\Delta t = (1.115 \times 10^{-4} \text{ A})(3.00 \times 10^{-3} \text{ s}) = 3.35 \times 10^{-7} \text{ C}.$$

30. We use $R \propto L/A$. The diameter of a 22-gauge wire is $1/4$ that of a 10-gauge wire. Thus from $R = \rho L/A$ we find the resistance of 25 ft of 22-gauge copper wire to be

$$R = (1.00 \Omega)(25 \text{ ft}/1000 \text{ ft})(4)^2 = 0.40 \Omega.$$

31. (a) The current in each strand is $i = 0.750 \text{ A}/125 = 6.00 \times 10^{-3} \text{ A}$.

(b) The potential difference is $V = iR = (6.00 \times 10^{-3} \text{ A})(2.65 \times 10^{-6} \Omega) = 1.59 \times 10^{-8} \text{ V}$.

(c) The resistance is $R_{\text{total}} = 2.65 \times 10^{-6} \Omega/125 = 2.12 \times 10^{-8} \Omega$.

32. We use $J = \sigma E = (n_+ + n_-)ev_d$, which combines Eq. 26-13 and Eq. 26-7.

(a) The magnitude of the current density is

$$J = \sigma E = (2.70 \times 10^{-14} / \Omega \cdot \text{m}) (120 \text{ V/m}) = 3.24 \times 10^{-12} \text{ A/m}^2.$$

(b) The drift velocity is

$$v_d = \frac{\sigma E}{(n_+ + n_-)e} = \frac{(2.70 \times 10^{-14} / \Omega \cdot \text{m})(120 \text{ V/m})}{[(620 + 550) / \text{cm}^3](1.60 \times 10^{-19} \text{ C})} = 1.73 \text{ cm/s}.$$

33. (a) The current in the block is $i = V/R = 35.8 \text{ V}/935 \Omega = 3.83 \times 10^{-2} \text{ A}$.

(b) The magnitude of current density is

$$J = i/A = (3.83 \times 10^{-2} \text{ A}) / (3.50 \times 10^{-4} \text{ m}^2) = 109 \text{ A/m}^2.$$

(c) $v_d = J/ne = (109 \text{ A/m}^2) / [(5.33 \times 10^{22} / \text{m}^3)(1.60 \times 10^{-19} \text{ C})] = 1.28 \times 10^{-2} \text{ m/s}$.

(d) $E = V/L = 35.8 \text{ V}/0.158 \text{ m} = 227 \text{ V/m}$.

34. The number density of conduction electrons in copper is $n = 8.49 \times 10^{28} / \text{m}^3$. The electric field in section 2 is $(10.0 \mu\text{V}) / (2.00 \text{ m}) = 5.00 \mu\text{V/m}$. Since $\rho = 1.69 \times 10^{-8} \Omega \cdot \text{m}$ for copper (see Table 26-1) then Eq. 26-10 leads to a current density vector of magnitude

$$J_2 = (5.00 \mu\text{V/m}) / (1.69 \times 10^{-8} \Omega \cdot \text{m}) = 296 \text{ A/m}^2$$

in section 2. Conservation of electric current from section 1 into section 2 implies

$$J_1 A_1 = J_2 A_2 \quad \Rightarrow \quad J_1 (4\pi R^2) = J_2 (\pi R^2)$$

(see Eq. 26-5). This leads to $J_1 = 74 \text{ A/m}^2$. Now, for the drift speed of conduction-electrons in section 1, Eq. 26-7 immediately yields

$$v_d = \frac{J_1}{ne} = 5.44 \times 10^{-9} \text{ m/s}$$

35. (a) The current i is shown in Fig. 26-30 entering the truncated cone at the left end and leaving at the right. This is our choice of positive x direction. We make the assumption that the current density J at each value of x may be found by taking the ratio i/A where $A = \pi r^2$ is the cone's cross-section area at that particular value of x .

The direction of \vec{J} is identical to that shown in the figure for i (our $+x$ direction). Using Eq. 26-11, we then find an expression for the electric field at each value of x , and next find the potential difference V by integrating the field along the x axis, in accordance with the ideas of Chapter 25. Finally, the resistance of the cone is given by $R = V/i$. Thus,

$$J = \frac{i}{\pi r^2} = \frac{E}{\rho}$$

where we must deduce how r depends on x in order to proceed. We note that the radius increases linearly with x , so (with c_1 and c_2 to be determined later) we may write $r = c_1 + c_2x$.

Choosing the origin at the left end of the truncated cone, the coefficient c_1 is chosen so that $r = a$ (when $x = 0$); therefore, $c_1 = a$. Also, the coefficient c_2 must be chosen so that (at the right end of the truncated cone) we have $r = b$ (when $x = L$); therefore, $c_2 = (b - a)/L$. Our expression, then, becomes

$$r = a + \left(\frac{b-a}{L}\right)x.$$

Substituting this into our previous statement and solving for the field, we find

$$E = \frac{i\rho}{\pi} \left(a + \frac{b-a}{L}x\right)^{-2}.$$

Consequently, the potential difference between the faces of the cone is

$$\begin{aligned} V &= -\int_0^L E dx = -\frac{i\rho}{\pi} \int_0^L \left(a + \frac{b-a}{L}x\right)^{-2} dx = \frac{i\rho}{\pi} \frac{L}{b-a} \left(a + \frac{b-a}{L}x\right)^{-1} \Bigg|_0^L \\ &= \frac{i\rho}{\pi} \frac{L}{b-a} \left(\frac{1}{a} - \frac{1}{b}\right) = \frac{i\rho}{\pi} \frac{L}{b-a} \frac{b-a}{ab} = \frac{i\rho L}{\pi ab}. \end{aligned}$$

The resistance is therefore

$$R = \frac{V}{i} = \frac{\rho L}{\pi ab} = \frac{(731 \Omega \cdot \text{m})(1.94 \times 10^{-2} \text{ m})}{\pi(2.00 \times 10^{-3} \text{ m})(2.30 \times 10^{-3} \text{ m})} = 9.81 \times 10^5 \Omega$$

Note that if $b = a$, then $R = \rho L / \pi a^2 = \rho L / A$, where $A = \pi a^2$ is the cross-sectional area of the cylinder.

36. Since the current spreads uniformly over the hemisphere, the current density at any given radius r from the striking point is $J = I / 2\pi r^2$. From Eq. 26-10, the magnitude of the electric field at a radial distance r is

$$E = \rho_w J = \frac{\rho_w I}{2\pi r^2},$$

where $\rho_w = 30 \Omega \cdot \text{m}$ is the resistivity of water. The potential difference between a point at radial distance D and a point at $D + \Delta r$ is

$$\Delta V = -\int_D^{D+\Delta r} E dr = -\int_D^{D+\Delta r} \frac{\rho_w I}{2\pi r^2} dr = \frac{\rho_w I}{2\pi} \left(\frac{1}{D+\Delta r} - \frac{1}{D} \right) = -\frac{\rho_w I}{2\pi} \frac{\Delta r}{D(D+\Delta r)},$$

which implies that the current across the swimmer is

$$i = \frac{|\Delta V|}{R} = \frac{\rho_w I}{2\pi R} \frac{\Delta r}{D(D+\Delta r)}.$$

Substituting the values given, we obtain

$$i = \frac{(30.0 \Omega \cdot \text{m})(7.80 \times 10^4 \text{ A})}{2\pi(4.00 \times 10^3 \Omega)} \frac{0.70 \text{ m}}{(35.0 \text{ m})(35.0 \text{ m} + 0.70 \text{ m})} = 5.22 \times 10^{-2} \text{ A}.$$

37. From Eq. 26-25, $\rho \propto \bar{\tau}^{-1} \propto v_{\text{eff}}$. The connection with v_{eff} is indicated in part (b) of Sample Problem 26.05 —“Mean free time and mean free distance,” which contains useful insight regarding the problem we are working now. According to Chapter 20, $v_{\text{eff}} \propto \sqrt{T}$. Thus, we may conclude that $\rho \propto \sqrt{T}$.

38. The slope of the graph is $P = 5.0 \times 10^{-4} \text{ W}$. Using this in the $P = V^2/R$ relation leads to $V = 0.10 \text{ Vs}$.

39. Eq. 26-26 gives the rate of thermal energy production:

$$P = iV = (10.0 \text{ A})(120 \text{ V}) = 1.20 \text{ kW}.$$

Dividing this into the 180 kJ necessary to cook the three hotdogs leads to the result $t = 150 \text{ s}$.

40. The resistance is $R = P/i^2 = (100 \text{ W})/(3.00 \text{ A})^2 = 11.1 \Omega$.

41. **THINK** In an electrical circuit, the electrical energy is dissipated through the resistor as heat.

EXPRESS Electrical energy is converted to heat at a rate given by $P = V^2/R$, where V is the potential difference across the heater and R is the resistance of the heater.

ANALYZE With $V = 120 \text{ V}$ and $R = 14 \Omega$, we have

$$P = \frac{(120 \text{ V})^2}{14 \Omega} = 1.0 \times 10^3 \text{ W} = 1.0 \text{ kW}.$$

(b) The cost is given by $(1.0\text{kW})(5.0\text{h})(5.0\text{cents/kW}\cdot\text{h}) = \text{US}\0.25 .

LEARN The energy transferred is lost because the process is irreversible. The thermal energy causes the temperature of the resistor to rise.

42. (a) Referring to Fig. 26-33, the electric field would point down (toward the bottom of the page) in the strip, which means the current density vector would point down, too (by Eq. 26-11). This implies (since electrons are negatively charged) that the conduction electrons would be “drifting” upward in the strip.

(b) Equation 24-6 immediately gives 12 eV, or (using $e = 1.60 \times 10^{-19} \text{ C}$) $1.9 \times 10^{-18} \text{ J}$ for the work done by the field (which equals, in magnitude, the potential energy change of the electron).

(c) Since the electrons don’t (on average) gain kinetic energy as a result of this work done, it is generally dissipated as heat. The answer is as in part (b): 12 eV or $1.9 \times 10^{-18} \text{ J}$.

43. The relation $P = V^2/R$ implies $P \propto V^2$. Consequently, the power dissipated in the second case is

$$P = \left(\frac{1.50 \text{ V}}{3.00 \text{ V}} \right)^2 (0.540 \text{ W}) = 0.135 \text{ W}.$$

44. Since $P = iV$, the charge is

$$q = it = Pt/V = (7.0 \text{ W})(5.0 \text{ h})(3600 \text{ s/h})/9.0 \text{ V} = 1.4 \times 10^4 \text{ C}.$$

45. **THINK** Let P be the power dissipated, i be the current in the heater, and V be the potential difference across the heater. The three quantities are related by $P = iV$.

EXPRESS The current is given by $i = P/V$. Using Ohm’s law $V = iR$, the resistance of the heater can be written as

$$R = \frac{V}{i} = \frac{V}{P/V} = \frac{V^2}{P}.$$

ANALYZE (a) Substituting the values given, we have $i = \frac{P}{V} = \frac{1250 \text{ W}}{115 \text{ V}} = 10.9 \text{ A}$.

(b) Similarly, the resistance is

$$R = \frac{V^2}{P} = \frac{(115 \text{ V})^2}{1250 \text{ W}} = 10.6 \Omega.$$

(c) The thermal energy E generated by the heater in time $t = 1.0 \text{ h} = 3600 \text{ s}$ is

$$E = Pt = (1250 \text{ W})(3600 \text{ s}) = 4.50 \times 10^6 \text{ J}.$$

LEARN Current in the heater produces a transfer of mechanical energy to thermal energy, with a rate of the transfer equal to $P = iV = V^2 / R$.

46. (a) Using Table 26-1 and Eq. 26-10 (or Eq. 26-11), we have

$$|\vec{E}| = \rho |\vec{J}| = (1.69 \times 10^{-8} \Omega \cdot \text{m}) \left(\frac{2.00 \text{ A}}{2.00 \times 10^{-6} \text{ m}^2} \right) = 1.69 \times 10^{-2} \text{ V/m}.$$

(b) Using $L = 4.0 \text{ m}$, the resistance is found from Eq. 26-16:

$$R = \rho L / A = 0.0338 \Omega.$$

The rate of thermal energy generation is found from Eq. 26-27:

$$P = i^2 R = (2.00 \text{ A})^2 (0.0338 \Omega) = 0.135 \text{ W}.$$

Assuming a steady rate, the amount of thermal energy generated in 30 minutes is found to be $(0.135 \text{ J/s})(30 \times 60 \text{ s}) = 2.43 \times 10^2 \text{ J}$.

47. (a) From $P = V^2 / R = AV^2 / \rho L$, we solve for the length:

$$L = \frac{AV^2}{\rho P} = \frac{(2.60 \times 10^{-6} \text{ m}^2)(75.0 \text{ V})^2}{(5.00 \times 10^{-7} \Omega \cdot \text{m})(500 \text{ W})} = 5.85 \text{ m}.$$

(b) Since $L \propto V^2$ the new length should be $L' = L \left(\frac{V'}{V} \right)^2 = (5.85 \text{ m}) \left(\frac{100 \text{ V}}{75.0 \text{ V}} \right)^2 = 10.4 \text{ m}$.

48. The mass of the water over the length is

$$m = \rho AL = (1000 \text{ kg/m}^3)(15 \times 10^{-5} \text{ m}^2)(0.12 \text{ m}) = 0.018 \text{ kg},$$

and the energy required to vaporize the water is

$$Q = Lm = (2256 \text{ kJ/kg})(0.018 \text{ kg}) = 4.06 \times 10^4 \text{ J}.$$

The thermal energy is supplied by joule heating of the resistor:

$$Q = P\Delta t = I^2 R \Delta t.$$

Since the resistance over the length of water is

$$R = \frac{\rho_w L}{A} = \frac{(150 \Omega \cdot \text{m})(0.120 \text{ m})}{15 \times 10^{-5} \text{ m}^2} = 1.2 \times 10^5 \Omega,$$

the average current required to vaporize water is

$$I = \sqrt{\frac{Q}{R\Delta t}} = \sqrt{\frac{4.06 \times 10^4 \text{ J}}{(1.2 \times 10^5 \Omega)(2.0 \times 10^{-3} \text{ s})}} = 13.0 \text{ A}.$$

49. (a) Assuming a 31-day month, the monthly cost is

$$(100 \text{ W})(24 \text{ h/day})(31 \text{ days/month})(6 \text{ cents/kW} \cdot \text{h}) = 446 \text{ cents} = \text{US\$}4.46.$$

(b) $R = V^2/P = (120 \text{ V})^2/100 \text{ W} = 144 \Omega$.

(c) $i = P/V = 100 \text{ W}/120 \text{ V} = 0.833 \text{ A}$.

50. The slopes of the lines yield $P_1 = 8 \text{ mW}$ and $P_2 = 4 \text{ mW}$. Their sum (by energy conservation) must be equal to that supplied by the battery: $P_{\text{batt}} = (8 + 4) \text{ mW} = 12 \text{ mW}$.

51. **THINK** Our system is made up of two wires that are joined together. To calculate the electrical potential difference between two points, we first calculate their resistances.

EXPRESS The potential difference between points 1 and 2 is $\Delta V_{12} = iR_C$, where R_C is the resistance of wire C . Similarly, the potential difference between points 2 and 3 is $\Delta V_{23} = iR_D$, where R_D is the resistance of wire D . The corresponding rates of energy dissipation are $P_{12} = i^2 R_C$ and $P_{23} = i^2 R_D$, respectively.

ANALYZE (a) Using Eq. 26-16, we find the resistance of wire C to be

$$R_C = \rho_C \frac{L_C}{\pi r_C^2} = (2.0 \times 10^{-6} \Omega \cdot \text{m}) \frac{1.0 \text{ m}}{\pi (0.00050 \text{ m})^2} = 2.55 \Omega.$$

Thus, $\Delta V_{12} = iR_C = (2.0 \text{ A})(2.55 \Omega) = 5.1 \text{ V}$.

(b) Similarly, the resistance for wire D is

$$R_D = \rho_D \frac{L_D}{\pi r_D^2} = (1.0 \times 10^{-6} \Omega \cdot \text{m}) \frac{1.0 \text{ m}}{\pi (0.00025 \text{ m})^2} = 5.09 \Omega$$

and the potential difference is $\Delta V_{23} = iR_D = (2.0 \text{ A})(5.09 \Omega) = 10.2 \text{ V} \approx 10 \text{ V}$.

(c) The power dissipated between points 1 and 2 is $P_{12} = i^2 R_C = 10 \text{ W}$.

(d) Similarly, the power dissipated between points 2 and 3 is $P_{23} = i^2 R_D = 20 \text{ W}$.

LEARN The results may be summarized in terms of the following ratios:

$$\frac{P_{23}}{P_{12}} = \frac{\Delta V_{23}}{\Delta V_{12}} = \frac{R_D}{R_C} = \frac{\rho_D}{\rho_C} \cdot \frac{L_D}{L_C} \cdot \left(\frac{r_C}{r_D}\right)^2 = \frac{1}{2} \cdot 1 \cdot (2)^2 = 2.$$

52. Assuming the current is along the wire (not radial) we find the current from Eq. 26-4:

$$i = \int |\vec{J}| dA = \int_0^R kr^2 2\pi r dr = \frac{1}{2} k\pi R^4 = 3.50 \text{ A}$$

where $k = 2.75 \times 10^{10} \text{ A/m}^4$ and $R = 0.00300 \text{ m}$. The rate of thermal energy generation is found from Eq. 26-26: $P = iV = 210 \text{ W}$. Assuming a steady rate, the thermal energy generated in 40 s is $Q = P\Delta t = (210 \text{ J/s})(3600 \text{ s}) = 7.56 \times 10^5 \text{ J}$.

53. (a) From $P = V^2/R$ we find $R = V^2/P = (120 \text{ V})^2/500 \text{ W} = 28.8 \Omega$.

(b) Since $i = P/V$, the rate of electron transport is

$$\frac{i}{e} = \frac{P}{eV} = \frac{500 \text{ W}}{(1.60 \times 10^{-19} \text{ C})(120 \text{ V})} = 2.60 \times 10^{19} / \text{s}.$$

54. From $P = V^2/R$, we have

$$R = (5.0 \text{ V})^2/(200 \text{ W}) = 0.125 \Omega.$$

To meet the conditions of the problem statement, we must therefore set

$$\int_0^L 5.00x dx = 0.125 \Omega$$

Thus,

$$\frac{5}{2} L^2 = 0.125 \Rightarrow L = 0.224 \text{ m}.$$

55. **THINK** Since the resistivity of Nichrome varies with temperature, the power dissipated through the Nichrome wire will also depend on temperature.

EXPRESS Let R_H be the resistance at the higher temperature (800°C) and let R_L be the resistance at the lower temperature (200°C). Since the potential difference is the same for the two temperatures, the power dissipated at the lower temperature is $P_L = V^2/R_L$, and the power dissipated at the higher temperature is $P_H = V^2/R_H$, so $P_L = (R_H/R_L)P_H$. Now,

$$R_H = \frac{\rho_H L}{A} = \frac{\rho_0 L}{A} [1 + \alpha(T_H - T_0)]$$

$$R_L = \frac{\rho_L L}{A} = \frac{\rho_0 L}{A} [1 + \alpha(T_L - T_0)]$$

so that

$$R_L = R_H + \alpha R_H \Delta T,$$

where ΔT is the temperature difference: $T_L - T_H = -600 \text{ C}^\circ = -600 \text{ K}$.

ANALYZE Thus, the dissipation rate at 200°C is

$$P_L = \frac{R_H}{R_H + \alpha R_H \Delta T} P_H = \frac{P_H}{1 + \alpha \Delta T} = \frac{500 \text{ W}}{1 + (4.0 \times 10^{-4} / \text{K})(-600 \text{ K})} = 660 \text{ W}.$$

LEARN Since the power dissipated is inversely proportional to R , at lower temperature where $R_L < R_H$, we expect a higher rate of energy dissipation: $P_L > P_H$.

56. (a) The current is

$$i = \frac{V}{R} = \frac{V}{\rho L / A} = \frac{\pi V d^2}{4 \rho L} = \frac{\pi(1.20 \text{ V})[(0.0400 \text{ in.})(2.54 \times 10^{-2} \text{ m/in.})]^2}{4(1.69 \times 10^{-8} \Omega \cdot \text{m})(33.0 \text{ m})} = 1.74 \text{ A}.$$

(b) The magnitude of the current density vector is

$$|\vec{J}| = \frac{i}{A} = \frac{4i}{\pi d^2} = \frac{4(1.74 \text{ A})}{\pi[(0.0400 \text{ in.})(2.54 \times 10^{-2} \text{ m/in.})]^2} = 2.15 \times 10^6 \text{ A/m}^2.$$

(c) $E = V/L = 1.20 \text{ V}/33.0 \text{ m} = 3.63 \times 10^{-2} \text{ V/m}$.

(d) $P = Vi = (1.20 \text{ V})(1.74 \text{ A}) = 2.09 \text{ W}$.

57. We find the current from Eq. 26-26: $i = P/V = 2.00 \text{ A}$. Then, from Eq. 26-1 (with constant current), we obtain

$$\Delta q = i \Delta t = 2.88 \times 10^4 \text{ C}.$$

58. We denote the copper rod with subscript c and the aluminum rod with subscript a .

(a) The resistance of the aluminum rod is

$$R = \rho_a \frac{L}{A} = \frac{(2.75 \times 10^{-8} \Omega \cdot \text{m})(1.3 \text{ m})}{(5.2 \times 10^{-3} \text{ m})^2} = 1.3 \times 10^{-3} \Omega.$$

(b) Let $R = \rho_c L / (\pi d^2 / 4)$ and solve for the diameter d of the copper rod:

$$d = \sqrt{\frac{4\rho_c L}{\pi R}} = \sqrt{\frac{4(1.69 \times 10^{-8} \Omega \cdot \text{m})(1.3 \text{ m})}{\pi(1.3 \times 10^{-3} \Omega)}} = 4.6 \times 10^{-3} \text{ m}.$$

59. (a) Since

$$\rho = \frac{RA}{L} = \frac{R(\pi d^2 / 4)}{L} = \frac{(1.09 \times 10^{-3} \Omega)\pi(5.50 \times 10^{-3} \text{ m})^2 / 4}{1.60 \text{ m}} = 1.62 \times 10^{-8} \Omega \cdot \text{m},$$

the material is silver.

(b) The resistance of the round disk is

$$R = \rho \frac{L}{A} = \frac{4\rho L}{\pi d^2} = \frac{4(1.62 \times 10^{-8} \Omega \cdot \text{m})(1.00 \times 10^{-3} \text{ m})}{\pi(2.00 \times 10^{-2} \text{ m})^2} = 5.16 \times 10^{-8} \Omega.$$

60. (a) Current is the transport of charge; here it is being transported “in bulk” due to the volume rate of flow of the powder. From Chapter 14, we recall that the volume rate of flow is the product of the cross-sectional area (of the stream) and the (average) stream velocity. Thus, $i = \rho Av$ where ρ is the charge per unit volume. If the cross section is that of a circle, then $i = \rho \pi R^2 v$.

(b) Recalling that a coulomb per second is an ampere, we obtain

$$i = (1.1 \times 10^{-3} \text{ C/m}^3) \pi (0.050 \text{ m})^2 (2.0 \text{ m/s}) = 1.7 \times 10^{-5} \text{ A}.$$

(c) The motion of charge is not in the same direction as the potential difference computed in problem 70 of Chapter 24. It might be useful to think of (by analogy) Eq. 7-48; there, the scalar (dot) product in $P = \vec{F} \cdot \vec{v}$ makes it clear that $P = 0$ if $\vec{F} \perp \vec{v}$. This suggests that a radial potential difference and an axial flow of charge will not together produce the needed transfer of energy (into the form of a spark).

(d) With the assumption that there is (at least) a voltage equal to that computed in problem 70 of Chapter 24, in the proper direction to enable the transference of energy (into a spark), then we use our result from that problem in Eq. 26-26:

$$P = iV = (1.7 \times 10^{-5} \text{ A})(7.8 \times 10^4 \text{ V}) = 1.3 \text{ W}.$$

(e) Recalling that a joule per second is a watt, we obtain $(1.3 \text{ W})(0.20 \text{ s}) = 0.27 \text{ J}$ for the energy that can be transferred at the exit of the pipe.

(f) This result is greater than the 0.15 J needed for a spark, so we conclude that the spark was likely to have occurred at the exit of the pipe, going into the silo.

61. **THINK** The amount of charge that strikes the surface in time Δt is given by $\Delta q = i \Delta t$, where i is the current.

EXPRESS Since each alpha particle carries charge $q = +2e$, the number of particles that strike the surface is

$$N = \frac{\Delta q}{2e} = \frac{i \Delta t}{2e}.$$

For part (b), let N' be the number of particles in a length L of the beam. They will all pass through the beam cross section at one end in time $t = L/v$, where v is the particle speed. The current is the charge that moves through the cross section per unit time. That is,

$$i = \frac{2eN'}{t} = \frac{2eN'v}{L}.$$

Thus $N' = iL/2ev$.

ANALYZE (a) Substituting the values given, we have

$$N = \frac{\Delta q}{2e} = \frac{i \Delta t}{2e} = \frac{(0.25 \times 10^{-6} \text{ A})(3.0 \text{ s})}{2(1.6 \times 10^{-19} \text{ C})} = 2.34 \times 10^{12}.$$

(b) To find the particle speed, we note the kinetic energy of a particle is

$$K = 20 \text{ MeV} = (20 \times 10^6 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV}) = 3.2 \times 10^{-12} \text{ J}.$$

Since $K = \frac{1}{2}mv^2$, the speed is $v = \sqrt{2K/m}$. The mass of an alpha particle is (very nearly) 4 times the mass of a proton, or $m = 4(1.67 \times 10^{-27} \text{ kg}) = 6.68 \times 10^{-27} \text{ kg}$, so

$$v = \sqrt{\frac{2(3.2 \times 10^{-12} \text{ J})}{6.68 \times 10^{-27} \text{ kg}}} = 3.1 \times 10^7 \text{ m/s}.$$

Therefore, the number of particles in a length $L = 20 \text{ cm}$ of the beam is

$$N' = \frac{iL}{2ev} = \frac{(0.25 \times 10^{-6})(20 \times 10^{-2} \text{ m})}{2(1.60 \times 10^{-19} \text{ C})(3.1 \times 10^7 \text{ m/s})} = 5.0 \times 10^3.$$

(c) We use conservation of energy, where the initial kinetic energy is zero and the final kinetic energy is $20 \text{ MeV} = 3.2 \times 10^{-12} \text{ J}$. We note too, that the initial potential energy is

$$U_i = qV = 2eV,$$

and the final potential energy is zero. Here V is the electric potential through which the particles are accelerated. Consequently, $K_f = U_i = 2eV$, which gives

$$V = \frac{K_f}{2e} = \frac{3.2 \times 10^{-12} \text{ J}}{2(1.60 \times 10^{-19} \text{ C})} = 1.0 \times 10^7 \text{ V}.$$

LEARN By the work-kinetic energy theorem, the work done on 2.34×10^{12} such alpha particles is

$$W = (2.34 \times 10^{12})(20 \text{ MeV}) = (2.34 \times 10^{12})(3.2 \times 10^{-12} \text{ J}) = 7.5 \text{ J}.$$

The same result can also be obtained from

$$W = q\Delta V = (i\Delta t)\Delta V = (0.25 \times 10^{-6} \text{ A})(3.0 \text{ s})(1.0 \times 10^7 \text{ V}) = 7.5 \text{ J}.$$

62. We use Eq. 26-28: $R = \frac{V^2}{P} = \frac{(200 \text{ V})^2}{3000 \text{ W}} = 13.3 \Omega$.

63. Combining Eq. 26-28 with Eq. 26-16 demonstrates that the power is inversely proportional to the length (when the voltage is held constant, as in this case). Thus, a new length equal to $7/8$ of its original value leads to

$$P = \frac{8}{7} (2.0 \text{ kW}) = 2.4 \text{ kW}.$$

64. (a) Since $P = i^2 R = J^2 A^2 R$, the current density is

$$J = \frac{1}{A} \sqrt{\frac{P}{R}} = \frac{1}{A} \sqrt{\frac{P}{\rho L/A}} = \sqrt{\frac{P}{\rho LA}} = \sqrt{\frac{1.0 \text{ W}}{\pi(3.5 \times 10^{-5} \Omega \cdot \text{m})(2.0 \times 10^{-2} \text{ m})(5.0 \times 10^{-3} \text{ m})^2}} \\ = 1.3 \times 10^5 \text{ A/m}^2.$$

(b) From $P = iV = JAV$ we get

$$V = \frac{P}{AJ} = \frac{P}{\pi r^2 J} = \frac{1.0 \text{ W}}{\pi(5.0 \times 10^{-3} \text{ m})^2 (1.3 \times 10^5 \text{ A/m}^2)} = 9.4 \times 10^{-2} \text{ V}.$$

65. We use $P = i^2 R = i^2 \rho L/A$, or $L/A = P/i^2 \rho$.

(a) The new values of L and A satisfy

$$\left(\frac{L}{A}\right)_{\text{new}} = \left(\frac{P}{i^2 \rho}\right)_{\text{new}} = \frac{30}{4^2} \left(\frac{P}{i^2 \rho}\right)_{\text{old}} = \frac{30}{16} \left(\frac{L}{A}\right)_{\text{old}}.$$

Consequently, $(L/A)_{\text{new}} = 1.875(L/A)_{\text{old}}$, and

$$L_{\text{new}} = \sqrt{1.875} L_{\text{old}} = 1.37 L_{\text{old}} \Rightarrow \frac{L_{\text{new}}}{L_{\text{old}}} = 1.37.$$

(b) Similarly, we note that $(LA)_{\text{new}} = (LA)_{\text{old}}$, and

$$A_{\text{new}} = \sqrt{1/1.875} A_{\text{old}} = 0.730 A_{\text{old}} \Rightarrow \frac{A_{\text{new}}}{A_{\text{old}}} = 0.730.$$

66. The horsepower required is $P = \frac{iV}{0.80} = \frac{(10\text{A})(12\text{ V})}{(0.80)(746\text{ W/hp})} = 0.20\text{ hp}$.

67. (a) We use $P = V^2/R \propto V^2$, which gives $\Delta P \propto \Delta V^2 \approx 2V \Delta V$. The percentage change is roughly

$$\Delta P/P = 2\Delta V/V = 2(110 - 115)/115 = -8.6\%.$$

(b) A drop in V causes a drop in P , which in turn lowers the temperature of the resistor in the coil. At a lower temperature R is also decreased. Since $P \propto R^{-1}$ a decrease in R will result in an increase in P , which partially offsets the decrease in P due to the drop in V . Thus, the actual drop in P will be smaller when the temperature dependency of the resistance is taken into consideration.

68. We use Eq. 26-17: $\rho - \rho_0 = \rho\alpha(T - T_0)$, and solve for T :

$$T = T_0 + \frac{1}{\alpha} \left(\frac{\rho}{\rho_0} - 1 \right) = 20^\circ\text{C} + \frac{1}{4.3 \times 10^{-3} / \text{K}} \left(\frac{58\Omega}{50\Omega} - 1 \right) = 57^\circ\text{C}.$$

We are assuming that $\rho/\rho_0 = R/R_0$.

69. We find the rate of energy consumption from Eq. 26-28:

$$P = \frac{V^2}{R} = \frac{(90\text{ V})^2}{400\Omega} = 20.3\text{ W}$$

Assuming a steady rate, the energy consumed is $(20.3\text{ J/s})(2.00 \times 3600\text{ s}) = 1.46 \times 10^5\text{ J}$.

70. (a) The potential difference between the two ends of the caterpillar is

$$V = iR = i\rho \frac{L}{A} = \frac{(12 \text{ A})(1.69 \times 10^{-8} \Omega \cdot \text{m})(4.0 \times 10^{-2} \text{ m})}{\pi(5.2 \times 10^{-3} \text{ m}/2)^2} = 3.8 \times 10^{-4} \text{ V}.$$

(b) Since it moves in the direction of the electron drift, which is against the direction of the current, its tail is negative compared to its head.

(c) The time of travel relates to the drift speed:

$$t = \frac{L}{v_d} = \frac{lAne}{i} = \frac{\pi L d^2 n e}{4i} = \frac{\pi(1.0 \times 10^{-2} \text{ m})(5.2 \times 10^{-3} \text{ m})^2 (8.47 \times 10^{28} / \text{m}^3)(1.60 \times 10^{-19} \text{ C})}{4(12 \text{ A})}$$

$$= 238 \text{ s} = 3 \text{ min } 58 \text{ s}.$$

71. **THINK** The resistance of copper increases with temperature.

EXPRESS According to Eq. 26-17, the resistance of copper at temperature T can be written as

$$R = \frac{\rho L}{A} = \frac{\rho_0 L}{A} [1 + \alpha(T - T_0)]$$

where $T_0 = 20^\circ \text{C}$ is the reference temperature. Thus, the resistance is $R_0 = \rho_0 L / A$ at $T_0 = 20^\circ \text{C}$. The temperature at which $R = 2R_0$ (or equivalently, $\rho = 2\rho_0$) can be found by solving

$$2 = \frac{R}{R_0} = 1 + \alpha(T - T_0) \Rightarrow \alpha(T - T_0) = 1.$$

ANALYZE (a) From the above equation, we find the temperature to be

$$T = T_0 + \frac{1}{\alpha} = 20^\circ \text{C} + \frac{1}{4.3 \times 10^{-3} / \text{K}} \approx 250^\circ \text{C}.$$

(b) Since a change in Celsius is equivalent to a change on the Kelvin temperature scale, the value of α used in this calculation is not inconsistent with the other units involved.

LEARN It is worth noting that our result agrees well with Fig. 26-10.

72. Since $100 \text{ cm} = 1 \text{ m}$, then $10^4 \text{ cm}^2 = 1 \text{ m}^2$. Thus,

$$R = \frac{\rho L}{A} = \frac{(3.00 \times 10^{-7} \Omega \cdot \text{m})(10.0 \times 10^3 \text{ m})}{56.0 \times 10^{-4} \text{ m}^2} = 0.536 \Omega.$$

73. The rate at which heat is being supplied is

$$P = iV = (5.2 \text{ A})(12 \text{ V}) = 62.4 \text{ W}.$$

Considered on a one-second time frame, this means 62.4 J of heat are absorbed by the liquid each second. Using Eq. 18-16, we find the heat of transformation to be

$$L = \frac{Q}{m} = \frac{62.4 \text{ J}}{21 \times 10^{-6} \text{ kg}} = 3.0 \times 10^6 \text{ J/kg}.$$

74. We find the drift speed from Eq. 26-7:

$$v_d = \frac{|\vec{J}|}{ne} = \frac{2.0 \times 10^6 \text{ A/m}^2}{(8.49 \times 10^{28} / \text{m}^3)(1.6 \times 10^{-19} \text{ C})} = 1.47 \times 10^{-4} \text{ m/s}.$$

At this (average) rate, the time required to travel $L = 5.0 \text{ m}$ is

$$t = \frac{L}{v_d} = \frac{5.0 \text{ m}}{1.47 \times 10^{-4} \text{ m/s}} = 3.4 \times 10^4 \text{ s}.$$

75. The power dissipated is given by the product of the current and the potential difference:

$$P = iV = (7.0 \times 10^{-3} \text{ A})(80 \times 10^3 \text{ V}) = 560 \text{ W}.$$

76. (a) The current is $4.2 \times 10^{18} e$ divided by 1 second. Using $e = 1.60 \times 10^{-19} \text{ C}$ we obtain 0.67 A for the current.

(b) Since the electric field points away from the positive terminal (high potential) and toward the negative terminal (low potential), then the current density vector (by Eq. 26-11) must also point toward the negative terminal.

77. For the temperature of the gas to remain unchanged, the rate of the thermal energy dissipated through the resistor, $P_R = i^2 R$, must be equal to the rate of increase of mechanical energy of the piston, $P_m = mg(dh/dt) = mgv$. Thus,

$$i^2 R = mgv \Rightarrow v = \frac{i^2 R}{mg} = \frac{(0.240 \text{ A})^2 (550 \Omega)}{(12 \text{ kg})(9.8 \text{ m/s}^2)} = 0.27 \text{ m/s}.$$

78. We adapt the discussion in the text to a moving two-dimensional collection of charges. Using σ for the charge per unit area and w for the belt width, we can see that the transport of charge is expressed in the relationship $i = \sigma v w$, which leads to

$$\sigma = \frac{i}{vw} = \frac{100 \times 10^{-6} \text{ A}}{(30 \text{ m/s})(50 \times 10^{-2} \text{ m})} = 6.7 \times 10^{-6} \text{ C/m}^2.$$

79. (a) The total current density is equal to the sum of the contributions from the alpha particles and the electron. Using the general expression $J = nqv$, and noting that $n_e = 2n_\alpha$ (two electrons for each α particle), we have

$$\begin{aligned} J_{\text{total}} &= n_\alpha q_\alpha v_\alpha + n_e q_e v_e = n_\alpha (2e)v_\alpha + (2n_\alpha)(e)v_e = 2n_\alpha e(v_\alpha + v_e) \\ &= 2(2.80 \times 10^{21} / \text{m}^3)(1.6 \times 10^{-19} \text{ C})(88 \text{ m/s} + 25 \text{ m/s}) \\ &= 1.01 \times 10^5 \text{ A/m}^2 = 10.1 \text{ A/cm}^2 \end{aligned}$$

(b) The direction of the current is eastward (same as the motion of the alpha particles).

80. (a) Let ΔT be the change in temperature and κ be the coefficient of linear expansion for copper. Then $\Delta L = \kappa L \Delta T$ and

$$\frac{\Delta L}{L} = \kappa \Delta T = (1.7 \times 10^{-5} / \text{K})(1.0^\circ \text{C}) = 1.7 \times 10^{-5}.$$

This is equivalent to 0.0017%. Since a change in Celsius is equivalent to a change on the Kelvin temperature scale, the value of κ used in this calculation is not inconsistent with the other units involved.

(b) Incorporating a factor of 2 for the two-dimensional nature of A , the fractional change in area is

$$\frac{\Delta A}{A} = 2\kappa \Delta T = 2(1.7 \times 10^{-5} / \text{K})(1.0^\circ \text{C}) = 3.4 \times 10^{-5}$$

which is 0.0034%.

(c) For small changes in the resistivity ρ , length L , and area A of a wire, the change in the resistance is given by

$$\Delta R = \frac{\partial R}{\partial \rho} \Delta \rho + \frac{\partial R}{\partial L} \Delta L + \frac{\partial R}{\partial A} \Delta A.$$

Since $R = \rho L/A$, $\partial R/\partial \rho = L/A = R/\rho$, $\partial R/\partial L = \rho/A = R/L$, and $\partial R/\partial A = -\rho L/A^2 = -R/A$. Furthermore, $\Delta \rho/\rho = \alpha \Delta T$, where α is the temperature coefficient of resistivity for copper ($4.3 \times 10^{-3}/\text{K} = 4.3 \times 10^{-3}/\text{C}^\circ$, according to Table 27-1). Thus,

$$\begin{aligned} \frac{\Delta R}{R} &= \frac{\Delta \rho}{\rho} + \frac{\Delta L}{L} - \frac{\Delta A}{A} = (\alpha + \kappa - 2\kappa)\Delta T = (\alpha - \kappa)\Delta T \\ &= (4.3 \times 10^{-3} / \text{C}^\circ - 1.7 \times 10^{-5} / \text{C}^\circ)(1.0 \text{ C}^\circ) = 4.3 \times 10^{-3}. \end{aligned}$$

This is 0.43%, which we note (for the purposes of the next part) is primarily determined by the $\Delta \rho/\rho$ term in the above calculation.

(d) The fractional change in resistivity is much larger than the fractional change in length and area. Changes in length and area affect the resistance much less than changes in resistivity.

81. (a) Using $i = dq/dt = e(dN/dt)$, we obtain

$$\frac{dN}{dt} = \frac{i}{e} = \frac{15 \times 10^{-6} \text{ A}}{1.6 \times 10^{-19} \text{ C}} = 9.4 \times 10^{13} / \text{s}.$$

(b) The rate of thermal energy production is

$$P = \frac{dU}{dt} = \left(\frac{dN}{dt} \right) U_1 = (9.4 \times 10^{13} / \text{s})(16 \text{ MeV}) \left(\frac{1.6 \times 10^{-13} \text{ J}}{1 \text{ MeV}} \right) = 240 \text{ W}.$$

82. (a) The charge q that flows past any cross section of the beam in time Δt is given by $q = i\Delta t$, and the number of electrons is $N = q/e = (i/e) \Delta t$. This is the number of electrons that are accelerated. Thus,

$$N = \frac{(0.50 \text{ A})(0.10 \times 10^{-6} \text{ s})}{1.60 \times 10^{-19} \text{ C}} = 3.1 \times 10^{11}.$$

(b) Over a long time t the total charge is $Q = nqt$, where n is the number of pulses per unit time and q is the charge in one pulse. The average current is given by $i_{\text{avg}} = Q/t = nq$. Now $q = i\Delta t = (0.50 \text{ A})(0.10 \times 10^{-6} \text{ s}) = 5.0 \times 10^{-8} \text{ C}$, so

$$i_{\text{avg}} = (500 / \text{s})(5.0 \times 10^{-8} \text{ C}) = 2.5 \times 10^{-5} \text{ A}.$$

(c) The accelerating potential difference is $V = K/e$, where K is the final kinetic energy of an electron. Since $K = 50 \text{ MeV}$, the accelerating potential is $V = 50 \text{ kV} = 5.0 \times 10^7 \text{ V}$. During a pulse the power output is

$$P = iV = (0.50 \text{ A})(5.0 \times 10^7 \text{ V}) = 2.5 \times 10^7 \text{ W}.$$

This is the peak power. The average power is

$$P_{\text{avg}} = i_{\text{avg}} V = (2.5 \times 10^{-5} \text{ A})(5.0 \times 10^7 \text{ V}) = 1.3 \times 10^3 \text{ W}.$$

83. With the voltage reduced by 6.00% while resistance remains unchanged, the current through the heating element also decreases by 6.00% ($i' = 0.94i$). The power delivered is now

$$P' = i'^2 R = (0.94i)^2 R = 0.884i^2 R = 0.884P,$$

where $P = i^2 R$ is the power delivered to the heating element under normal circumstance. Since the energy required to heat the water remains the same in both cases, $P\Delta t = P'\Delta t'$, the time required becomes

$$\Delta t' = \left(\frac{P}{P'} \right) \Delta t = \frac{100 \text{ min}}{0.884} = 113 \text{ min.}$$

84. (a) The mass of the water is $m = \rho V = (1000 \text{ kg/m}^3)(2.0 \text{ L})(10^{-3} \text{ m}^3/\text{L}) = 2.00 \text{ kg}$. The energy required to raise the water temperature to the boiling point is

$$Q_1 = mc\Delta T = (2.00 \text{ kg})(4187 \text{ J/kg} \cdot \text{C}^\circ)(100 \text{ }^\circ\text{C} - 20 \text{ }^\circ\text{C}) = 6.70 \times 10^5 \text{ J.}$$

With $P = 400 \text{ W}$ at 80% efficiency, we find the time needed to be

$$\Delta t_1 = \frac{Q_1}{P_{\text{eff}}} = \frac{6.70 \times 10^5 \text{ J}}{(0.80)(400 \text{ W})} = 2.09 \times 10^3 \text{ s} \approx 35 \text{ min.}$$

(b) The energy required to vaporize half of the water is

$$Q_2 = L_v (m/2) = (2.256 \times 10^6 \text{ J/kg})(2.00 \text{ kg}/2) = 2.256 \times 10^6 \text{ J.}$$

Thus, the additional time elapsed is

$$\Delta t_2 = \frac{Q_2}{P_{\text{eff}}} = \frac{2.256 \times 10^6 \text{ J}}{(0.80)(400 \text{ W})} = 7.05 \times 10^3 \text{ s} \approx 118 \text{ min,}$$

or about 1.96 h.

85. (a) At $t = 0.500 \text{ s}$, the charge on the capacitor is

$$\begin{aligned} q &= CV = C(6.00 + 4.00t - 2.00t^2) = (30 \times 10^{-6} \text{ F}) [6.00 + 4.00(0.500) - 2.00(0.500)^2] \\ &= 225 \times 10^{-6} \text{ C} = 225 \text{ } \mu\text{C.} \end{aligned}$$

(b) The current flowing into the capacitor is

$$\begin{aligned} i &= \frac{dq}{dt} = C \frac{dV}{dt} = C \frac{d}{dt} (6.00 + 4.00t - 2.00t^2) = C(4.00 - 4.00t) \\ &= (30 \times 10^{-6} \text{ F}) [4.00 - 4.00(0.500)] = 60.0 \times 10^{-6} \text{ A} = 60.0 \text{ } \mu\text{A.} \end{aligned}$$

(c) The corresponding power output is

$$P = iV = (60.0 \times 10^{-6} \text{ A}) [6.00 + 4.00(0.500) - 2.00(0.500)^2] = 4.50 \times 10^{-4} \text{ W.}$$