EE141, Spring 2017 — Project

Due Thursday, June 8 in class

In this project, we will study control of a *chemostat*, a type of bioreactor used to culture microorganisms. Chemostats are used in a range of domains including ecology and evolutionary biology. The basic setup of a chemostat consists of a microorganism growing in culture liquid within a container. Nutrients are continuously added and excess nutrients and microorganisms are continuously removed. By controlling the flow rate of the added nutrient, we control the growth of the microorganism.

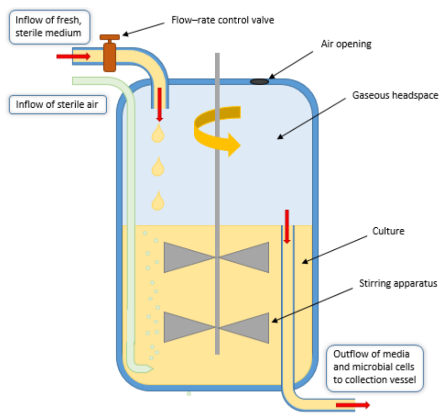

Figure 1: A chemostat. Source: wikimedia.org

Define the following variables:

- $x_N(t)$ = microorganism concentration within the chemostat at time t, units of mg/L
- $x_C(t)$ = nutrient concentration within the chemostat at time t, units of mg/L
- $u(t)$ = rate of nutrient supply inflow, also equal to constant outflow of culture chamber contents, units of L/h

Define the following constants:

- V = volume of the chemostat, units of L
- C = concentration of nutrient supply flowing into chemostat at time t, units of mg/L

Microorganism growth is proportional to the total microorganism concentration with proportionality $K(x_C)$ that depends on the concentration of nutrients within the chemostat. Also, due to the chemostat outflow, a fraction of the microorganism concentration is removed per time unit. This fraction is $u(t)/V$ so that we have the following dynamics:

$$\dot{x}_N = K(x_C)x_N - \frac{u}{V}x_N. \tag{1}$$

The proportionality term $K(x_C)$ is assumed to obey the *Michaelis-Menten kinetics* model:

$$K(x_C) = \frac{Qx_C}{P + x_C}, \tag{2}$$

where Q has units h^{-1} and P has units mg/L. The growing microorganisms deplete nutrient concentration at a rate of $\alpha K(x_C)x_N$ for some $\alpha > 0$, and nutrients flow into the chemostat with concentration C and flow out with concentration $x_C(t)$ so that

$$\dot{x}_C = -\alpha K(x_C)x_N + \frac{u}{V}(C - x_C). \tag{3}$$

The combined model is:

$$\dot{x}_N = \frac{Qx_C}{P + x_C}x_N - \frac{u}{V}x_N \tag{4}$$

$$\dot{x}_C = -\alpha\frac{Qx_C}{P + x_C}x_N + \frac{u}{V}(C - x_C). \tag{5}$$

Problem I (modeling)

Assume that $u(t) = F$ for all t for some $F > 0$.

1. The system (4)–(5) has two equilibria (each equilibrium is a pair (x_N^*, x_C^*)). Find these equilibria in terms of the constants α, C, F, P, Q, and V.
2. Because we are modeling a physical system for which we must have $x_C(t) \geq 0$ and $x_N(t) \geq 0$ for all time t, an equilibrium (x_N^*, x_C^*) is only meaningful if $x_N^* \geq 0$ and $x_C^* \geq 0$. Find conditions on the relationship between the variables α, C, F, Q, P, and V to ensure that each equilibrium pair found above satisfies $x_N^* \geq 0$ and $x_C^* \geq 0$.

For one of the equilibria, we have $x_N^* = 0$ for which there are no microorganisms present, an uninteresting case. In the remainder of this project, we focus on the <u>other</u> equilibrium and denote it by (x_N^*, x_C^*). Also in the remainder of this project, assume

$$V = 0.4, \qquad C = 11, \qquad P = 4, \qquad Q = 1.35, \qquad \alpha = 4.35.$$

3. Determine the constant input $u(t) = F$ corresponding to $x_N^* = 2$ mg/L.
4. Linearize the system around this equilibrium (x_N^*, x_C^*).
5. Determine the transfer function from u to x_N of the linearized system.

Problem II (analysis)

1. Is the system described by the transfer function stable?
2. Plot the step response of this transfer function and interpret in words what the plot is showing (*e.g.*, "The step response of the linear model shows that a change in X by Y amount results in ...")
3. Create a Simulink model for the original nonlinear model of the chemostat. Compare the step response of the linearized model with the step response of the original model. By how much do you need to increase the step input to see an appreciable mismatch between the original and the linearized models? (Hint: try starting with step inputs of magnitude less than 1).

Problem III (design)

1. Design a controller for the linearized system modeled by the transfer function to track step inputs with zero steady state error while producing an overshoot no greater than 10% and a settling time less than 5 hours.
2. Simulate your controller with the original nonlinear model. If needed, redesign your controller until the specifications are met starting from the equilibrium pair and using step inputs with magnitudes ranging from 0.05 to 0.5. Remember to try step inputs with a negative scaling factor.
3. By how much do you need to increase the step input to see an appreciable mismatch between the controlled original and the controlled linearized model?

In addition to answering the questions above, your project report should support any claims or statements with appropriate plots and should also include the block diagram of all components of your Simulink model.