HW 2.4 #1, 2, 4, 9, 10, 13, 14, 17, 18, 39, 40, 45, 46
53, 54, 58, 65, 66, 67, 69
one even & one odd 21-36

1) a T b T
c F d T
e T f T
g F h F
i F j F
k T l F

2) a T b F
c F d T
e T f T
g T h T
i T j F
k T

4)

a) $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x) = 1$, $f(2) = 2$

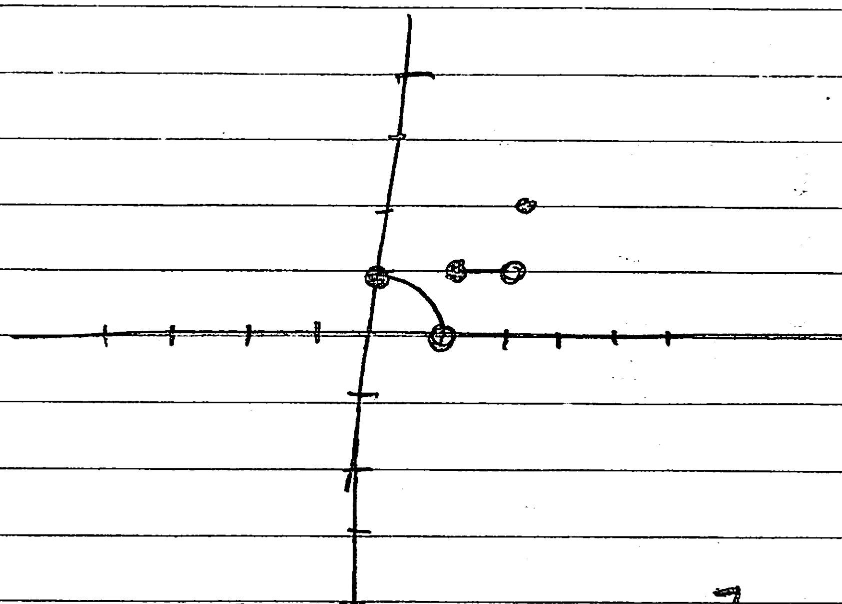
b) $\lim_{x\to 2} f(x)$ exists, $\lim_{x\to 2} f(x) = 1$

b/c $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x)$

c) $\lim_{x\to -1^-} f(x) = 4 = \lim_{x\to -1^+} f(x)$

d) $\lim_{x\to -1} f(x) = 4$

9) $f(x) = \begin{cases} \sqrt{1-x^2} & 0 \le x < 1 \\ 1, & 1 \le x < 2 \\ 2, & x = 2 \end{cases}$



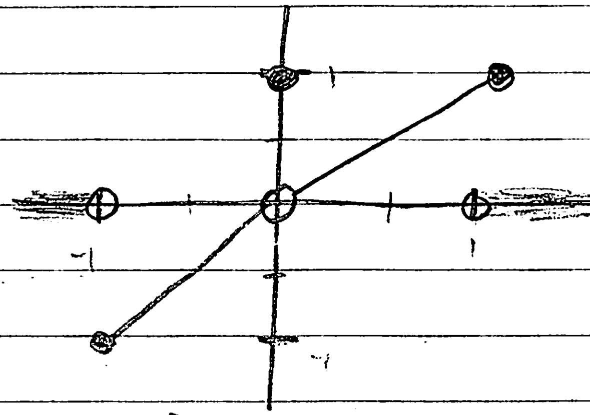
a) D: [1, 2]   R: (0, 1] ∪ {2}

b) limit exists on (0, 1) ∪ (1, 2)

c) left-hand limit exists on (0, 2]
RHL exists on [0, 2)
So point with LHL but no RHL is x=2

d) x=0 by above logic

10) $f(x) = \begin{cases} x, & -1 \le x < 0 \text{ or } 0 < x \le 1 \\ 1, & x = 0 \\ 0, & x < -1 \text{ or } x > 1 \end{cases}$



a) D: $(-\infty, \infty)$ aka $\mathbb{R}$

R: $[-1, 1]$

b) limit exists on $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

c) LHL exists on $(-\infty, \infty)$
RHL exists on $(-\infty, \infty)$
no points have a LHL but no RHL

d) no points, for same reasoning as above

13) $\lim_{x \to -2^+} \left(\frac{x}{x+1}\right)\left(\frac{2x+5}{x^2+x}\right)$ can plug in -2 because don't get division by zero

$$= \left(\frac{-2}{-2+1}\right)\left(\frac{2(-2)+5}{(-2)^2+(-2)}\right) = \left(\frac{-2}{-1}\right)\left(\frac{-4+5}{4-2}\right)$$

$$= 2 \cdot \left(\frac{1}{2}\right) = \underline{1}$$

14) $\lim_{x \to 1^-} \left(\frac{1}{x+1}\right)\left(\frac{x+6}{x}\right)\left(\frac{3-x}{7}\right)$

can plug in 1, because don't divide by zero

$= \left(\frac{1}{2}\right)\left(\frac{7}{1}\right)\left(\frac{2}{7}\right) = 1$

17) a) $\lim_{x \to -2^+} (x+3)\frac{|x+2|}{x+2}$

on $(-2,\infty)$, $|x+2| = x+2$

so $\lim_{x \to -2^+} (x+3)\frac{|x+2|}{x+2} = \lim_{x \to -2^+} (x+3)\frac{x+2}{x+2}$

$= \lim_{x \to -2^+} x+3 = 1$

b) $\lim_{x \to -2^-} (x+3)\frac{|x+2|}{x+2}$

on $(-\infty,-2)$, $|x+2| = -(x+2)$

so $\lim_{x \to -2^-} (x+3)\frac{|x+2|}{x+2} = \lim_{x \to -2^-} (x+3) \cdot \frac{-(x+2)}{x+2}$

$= -1$

18) a) $\lim_{x\to 1^+} \frac{\sqrt{2x}\,(x-1)}{|x-1|}$

on $(1,\infty)$, $|x-1| = x-1$

so $\lim_{x\to 1^+} \frac{\sqrt{2x}\,(x-1)}{|x-1|} = \lim_{x\to 1^+} \sqrt{2}\, x^{1/2} \cancel{\frac{x-1}{x-1}}^{\,1}$

$= \sqrt{2}\left(\lim_{x\to 1^+} x\right)^{1/2} = \sqrt{2}$

b) $\lim_{x\to 1^-} \frac{\sqrt{2x}\,(x-1)}{|x-1|}$

on $(-\infty,1)$, $|x-1| = -(x-1)$

so $\lim_{x\to 1^-} \frac{\sqrt{2x}\,(x-1)}{|x-1|} = \lim_{x\to 1^-} \frac{\sqrt{2}\, x^{1/2}\cancel{(x-1)}}{-\cancel{(x-1)}}$

$= -\sqrt{2}\left(\lim_{x\to 1^-} x\right)^{1/2} = -\sqrt{2}$

39) a) $\lim_{x\to\infty} \frac{1}{2+\frac{1}{x}} = \frac{1}{2+\lim_{x\to\infty}\frac{1}{x}} = \frac{1}{2}$

b) $\lim_{x\to -\infty} \frac{1}{2+\frac{1}{x}} = \frac{1}{2+\lim_{x\to -\infty}\frac{1}{x}} = \frac{1}{2}$

40) a) $$\lim_{x\to\infty} \frac{1}{8-\frac{5}{x^2}} = \frac{1}{8-5\left(\lim\limits_{x\to\infty}\frac{1}{x}\right)^2} = \frac{1}{8-5(0)^2} = \frac{1}{8}$$

b) $$\lim_{x\to-\infty} \frac{1}{8-\frac{5}{x^2}} = \frac{1}{8-5\left(\lim\limits_{x\to-\infty}\frac{1}{x}\right)^2} = \frac{1}{8-5(0)^2} = \frac{1}{8}$$

45) $$\lim_{t\to-\infty} \frac{2-t+\sin t}{t+\cos t} = \lim_{t\to-\infty} \frac{2-t+\sin t}{t+\cos t}\cdot\frac{\frac{1}{t}}{\frac{1}{t}}$$

$$= \lim_{t\to-\infty} \frac{\cancelto{0}{\frac{2}{t}} - 1 + \cancelto{0}{\frac{\sin t}{t}}}{1+\cancelto{0}{\frac{\cos t}{t}}}$$

$$= \frac{0-1+0}{1+0} = -1$$

Note:

$-1 \le \sin t \le 1$

$\Rightarrow -\frac{1}{t} \le \frac{\sin t}{t} \le \frac{1}{t}$

$\Rightarrow \lim\limits_{t\to\infty} \frac{\sin t}{t} = 0$ by sandwich theorem

Likewise with $\frac{\cos t}{t}$

46) $$\lim_{r\to\infty} \frac{r+\sin r}{2r+7-5\sin r} = \lim_{r\to\infty} \frac{r+\sin r}{2r+7-5\sin r}\cdot\frac{\frac{1}{r}}{\frac{1}{r}}$$

$$= \lim_{r\to\infty} \frac{1+\cancelto{0}{\frac{\sin r}{r}}}{2+\cancelto{0}{\frac{7}{r}}-5\cancelto{0}{\frac{\sin r}{r}}} = \frac{1}{2}$$

53) a) $\lim\limits_{x\to\infty} \dfrac{10x^5+x^4+31}{x^6}$

$= \lim\limits_{x\to\infty} \dfrac{10x^5+x^4+31}{x^6} \quad \dfrac{\frac{1}{x^6}}{\frac{1}{x^6}}$

$= \lim\limits_{x\to\infty} \dfrac{\cancel{\frac{10}{x}}^{\to 0} + \cancel{\frac{1}{x^2}}^{\to 0} + \cancel{\frac{31}{x^6}}^{\to 0}}{1} = \dfrac{0}{1} = 0$

b) $\lim\limits_{x\to-\infty} \dfrac{\cancel{\frac{10}{x}}^{\to 0} + \cancel{\frac{1}{x^2}}^{\to 0} + \cancel{\frac{31}{x^6}}^{\to 0}}{1} = 0$

54) note $h(x) = \dfrac{9x^4+x}{2x^4+5x^2-x+6}$

$= \dfrac{9x^4+x}{2x^4+5x^2-x+6} \quad \dfrac{\frac{1}{x^4}}{\frac{1}{x^4}} = \dfrac{9+\frac{1}{x^3}}{2+5\frac{1}{x^2}-\frac{1}{x^3}+\frac{6}{x^4}}$

a) $\lim\limits_{x\to\infty} h(x) = \lim\limits_{x\to\infty} \dfrac{9+\cancel{\frac{1}{x^3}}^{\to 0}}{2+\cancel{\frac{5}{x^2}}^{\to 0}-\cancel{\frac{1}{x^3}}^{\to 0}+\cancel{\frac{6}{x^4}}^{\to 0}} = \dfrac{9}{2}$

b) $\lim\limits_{x\to-\infty} h(x) = \lim\limits_{x\to-\infty} \dfrac{9+\cancel{\frac{1}{x^3}}^{\to 0}}{2+\cancel{\frac{5}{x^2}}^{\to 0}-\cancel{\frac{1}{x^3}}^{\to 0}+\cancel{\frac{6}{x^4}}^{\to 0}} = \dfrac{9}{2}$

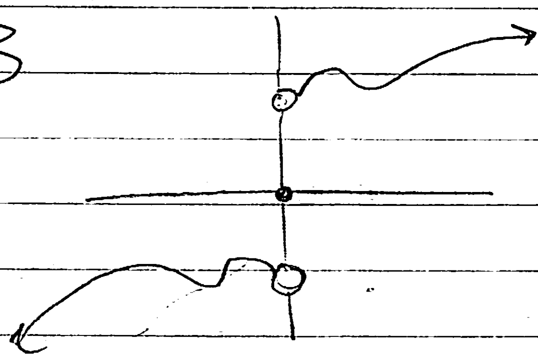
58) $\displaystyle\lim_{x\to\infty} \frac{2+\sqrt{x}}{2-\sqrt{x}} = \lim_{x\to\infty} \frac{2+x^{1/2}}{2-x^{1/2}}$

$\displaystyle = \lim_{x\to\infty} \frac{2+x^{1/2}}{2-x^{1/2}} \cdot \frac{\frac{1}{x^{1/2}}}{\frac{1}{x^{1/2}}} = \lim_{x\to\infty} \frac{\frac{2}{x^{1/2}}+1}{\frac{2}{x^{1/2}}-1}$

$\displaystyle = \frac{2\left(\lim_{x\to\infty} \frac{1}{x}\right)^{1/2}+1}{2\left(\lim_{x\to\infty} \frac{1}{x}\right)^{1/2}-1} = \frac{2\cdot 0^{1/2}+1}{2\cdot 0^{1/2}-1} = \frac{1}{-1} = -1$

65) f odd means $f(-x) = -f(x)$

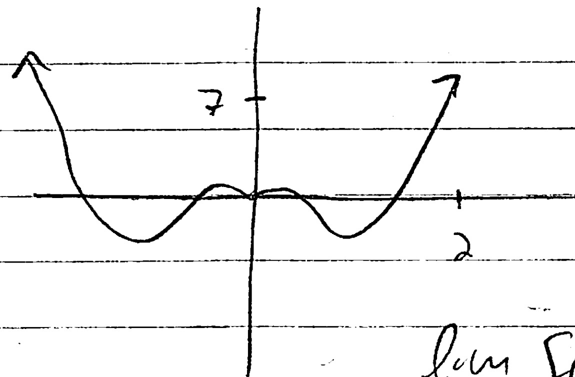
$\displaystyle\lim_{x\to 0^+} f(x) = 3$



Symmetry

So this forces $\displaystyle\lim_{x\to 0^-} f(x) = -3$

66) f even means $f(-x) = f(x)$



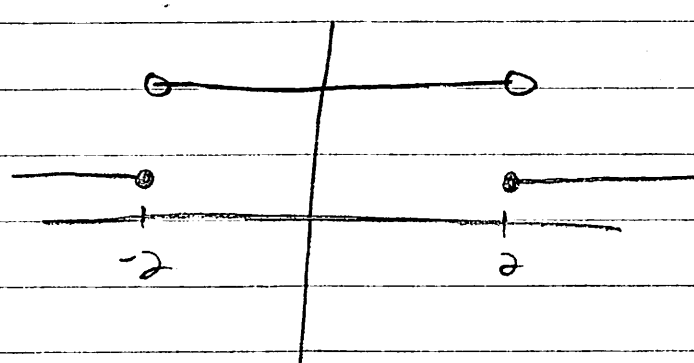
symmetry

$\lim_{x \to 2^-} f(x) = 7$

this forces $\lim_{x \to -2^+} f(x) = 7$

But no info about $\lim_{x \to -2^-} f(x)$.

example:



$$f(x) = \begin{cases} 7, & -2 < x < 2 \\ 1, & \text{else} \end{cases}$$

if we know $\lim_{x \to 2^-} f(x) = 7$, here we also have $\lim_{x \to 2^+} f(x) = 7$, but $\lim_{x \to -2^-} f(x) \neq 7$.

67) if $\lim_{x \to \infty} \frac{f(x)}{g(x)} = 2$,

Let $h(x) = \frac{f(x)}{g(x)}$

either
- f & g both even

$$h(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{g(x)} = h(x)$$

So h even.

- f & g both odd

$$h(-x) = \frac{f(-x)}{g(-x)} = \frac{-f(x)}{-g(x)} = \frac{f(x)}{g(x)} = h(x)$$

so h even

- f even, g odd

$$h(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -h(x)$$

so h odd

- f odd, g even

$$h(-x) = \frac{f(-x)}{g(-x)} = \frac{-f(x)}{g(x)} = -h(x),$$ so h odd

67 cont.

Now if $\lim\limits_{x \to \infty} \frac{f(x)}{g(x)} = J$, then

$\deg(f) = \deg(g)$, so either

f, g even or f, g odd.

by previous page, then h is even.

so $\lim\limits_{x \to \infty} \frac{f(x)}{g(x)} = \lim\limits_{x \to -\infty} \frac{f(x)}{g(x)} = J$. ■

69) a rational function $\frac{f(x)}{g(x)}$

can have either none or exactly one horizontal asymptote.

By 67, if $\lim\limits_{x \to \infty} \frac{f(x)}{g(x)} = L$, then

$\lim\limits_{x \to -\infty} \frac{f(x)}{g(x)} = L$ too.