Let dragonite and snorlax battle. Let's define a few variables first:

Let "p" be the fraction of the time snorlax is using his fast move, so that (1-p) is the fraction of the time snorlax is using his charge move.

$\overline{dps}$ is snorlax's average dps over a cycle (the "–" stands for average)

Fdmg is the damage done by snorlax's fast move(to dragonite)

Fduration is the damage done by snorlax's fast move(to dragonite)

Cdmg is the damage done by snorlax's charge move(to dragonite), or "cinematic move" as titled in niantic's GAMEMASTER file.

similarly Cdmg is the damage done by dragonite's charge move(to snorlax).

The rest of the variables follow the same scheme.

$$\overline{dps} = p * \left(\frac{Fdmg}{Fduration}\right) + (1 - p) * \left(\frac{Cdmg}{Cduration}\right)$$

$$\overline{dps} = p * \left(\frac{Fdmg}{Fduration}\right) + (1 - p) * \left(\frac{Cdmg}{Cduration}\right)$$

so once we find out p and p we have formulas for their $\overline{dps}$. That is the hard part. Let's do it.

If $\overline{EPS}$ is the averge energy gain per second of dragonite (only including gains, so no losses from charge moves) then $\frac{1}{\overline{EPS}}$ is the number of seconds to gain one energy, so $\frac{Cenergycost}{\overline{EPS}}$ is the average number of seconds needed to gain enough energy to do a charge move.

Now notice that (for either or):

Suppose that $Cenergycost = 50$ (for example dragon claw, which takes 50 energy). And note that p, the fraction of the time dragonite spends using his fast move, is also the fraction of the time that he has less than 50 energy (in a weave):

$$p = \frac{(\overline{time} \text{ per cycle that dragonite spends with } < \ 50 \text{ energy})}{(\overline{time}\ per\ cycle\ spent\ with < 50\ energy) + (\overline{time}\ per\ cycle\ spent\ with \geq 50\ energy)}$$

(the denominator is also just equal to the total time of a cycle)

Next, notice that:

-The time spent with ≥ 50 energy is just (approximately) the time it takes to shoot off a cinematic move. In other words for dragonite:

$$(\overline{time}\text{ spent with} \geq\ 50\text{ energy}) = Cduration$$

^ elephant come back to this later, on average you will reach 50 enrgy during a fast move, so if on average you get through X percent of your fast move the actual formula is

$$Cduration + X * Fduration$$

-the time spent with less than 50 energy is equal to the avg time required to get 50 energy, which we just calculated half a page ago:

$$(\overline{time}\ per\ cycle\ spent\ with < 50\ energy) = \frac{Cenergycost}{\overline{EPS}}$$

so I now replace all those into the formula, using as an example again:

$$p = \frac{(Cenergycost/\overline{EPS})}{(Cenergycost/\overline{EPS}) + Cduration}$$

We can simplify this by multiplying by $\overline{EPS}$ on the top and bottom:

$$p = \frac{Cenergycost}{Cenergycost + Cduration * \overline{EPS}}$$

This formula works for as well.

Let's count how many equations and how many unkowns we will have:

If you think about it, $\overline{EPS}$ can be expressed as a function of p and p (if you don't see, don't worry I'll do it in a sec). We can also express $\overline{EPS}$ as a function of p and p. When combined with the above formula for p or p we will have two equations in two variables and then we can solve for p and p. Let's do it.

First, let $hurt\overline{EPS}$ be the average EPS dragonite gains just from being hurt by snorlax's fast/charge damage weave. Then

$$\overline{EPS} = p * \left(hurt\overline{EPS} + \frac{Fenergygain}{Fduration}\right) + (1 - p) * hurt\overline{EPS}$$

We can simplify this by distributing everything and canceling:

$$\overline{EPS} = p * \frac{Fenergygain}{Fduration} + hurt\overline{EPS}$$

Since Fenergygain and Fduration are in the GM file, we just need hurtEPS. Total hurt EPS is a combination of fast hurt EPS and cinematic hurt EPS:

$$hurt\overline{EPS} = p * \left(Fhurt\overline{EPS}\right) + \left(1 - p\right) * \left(Churt\overline{EPS}\right)$$

Yay, now we're getting to the dependence of all of this on p so that we can connect everything together.

Let's get formulas for $Fhurt\overline{EPS}$ and $Churt\overline{EPS}$. First, according to testing by u/homu from pokemongo.gamepress.gg, the formula for energy gain based on damage taken is:

$$energygain = ceil\left(\frac{dmg}{2}\right)$$

...where the ceil() function just rounds the argument up towards +∞. So ceil(2.1) = 3, ceil(2) = 2, ceil(3.5)=4, etc.

That means that

$$Fhurt\overline{EPS} = \frac{ceil\left(Fdmg/2\right)}{Fduration}$$

$$Churt\overline{EPS} = \frac{ceil\left(Cdmg/2\right)}{Cduration}$$

Shit, that's a lot of equations. But now let's plug everything back in and just get one formula.

Working backwards now. We had

$$hurt\overline{EPS} = p * \left(Fhurt\overline{EPS}\right) + \left(1 - p\right) * \left(Churt\overline{EPS}\right)$$

So with plugging in, it's

$$hurt\overline{EPS} = p * \frac{ceil\left(Fdmg/2\right)}{Fduration} + \left(1 - p\right) * \frac{ceil\left(Cdmg/2\right)}{Cduration}$$

Cool. And before that, we had

$$\overline{EPS} = p * \frac{Fenergygain}{Fduration} + hurt\overline{EPS}$$

So now that turns into:

$$\overline{EPS} = p * \frac{Fenergygain}{Fduration} + p * \frac{ceil\left(Fdmg/2\right)}{Fduration} + \left(1 - p\right) * \frac{ceil\left(Cdmg/2\right)}{Cduration}$$

The next formula we'd have to plug into would be the ole'

$$p = \frac{Cenergycost}{Cenergycost + Cduration * \overline{EPS}}$$

But that's horrible, HORRIBLE so let's at least solve for $\overline{EPS}$ first.

Multiply by the denominator:

$$p * \left(Cenergycost + Cduration * \overline{EPS}\right) = Cenergycost$$

Divide by p, and then subtract $Cenergycost$:

$$Cduration * \overline{EPS} = (Cenergycost/p) - Cenergycost$$

Simplify the right hand side and divide by $Cduration$:

$$\overline{EPS} = \frac{Cenergycost}{Cduration} * \left(\frac{1}{p} - 1\right)$$

Now let's plug in $\overline{EPS}$ (oh god.....)

$$p * \frac{Fenergygain}{Fduration} + p * \frac{ceil\left(\frac{Fdmg}{2}\right)}{Fduration} + \left(1 - p\right) * \frac{ceil\left(\frac{Cdmg}{2}\right)}{Cduration} = \frac{Cenergycost}{Cduration} * \left(\frac{1}{p} - 1\right)$$

Okay, well at least we've made progress: notice that this equation only depends on p and p because everything else is just a number from the GAME_MASTER file. This is the equation we got from choosing to follow dragonite's EPS, but if we had chosen to calculate snorlax's EPS, we would have gotten the same formula except you'd have to replace all instances of with , and you'd have to replace all instances of with . That gives two formulas in two unknowns. CAN WE SOLVE FOR THEM?

Right now I'm praying that these two equations have a solution. But never in hell would I try to solve this system of equations by hand. Let's plug into Mathematica instead...

$$\text{Solve}[\{\text{dcduration} * (\text{dp} * \text{dfenergygain}/\text{dfduration} + \text{sp} * (0.5 * \text{sfdmg} + 0.25)/\text{sfduration} + (1 - \text{sp}) * (0.5 * \text{scdmg} + 0.25)/\text{scduration}) = \\ = \text{dcenergycost} * (1/\text{dp} - 1), \text{scduration} * (\text{sp} * \text{sfenergygain}/\text{sfduration} + \text{dp} * (0.5 * \text{dfdmg} + 0.25)/\text{dfduration} + (1 - \text{dp}) * (0.5 * \text{dcdmg} + 0.25)/\text{dcduration}) \\ == \text{scenergycost} * (1/\text{sp} - 1)\}, \{\text{dp}, \text{sp}\}]$$

...

(after 3 hours mathematica returned a 98,899 page totally incomprehensible “solution”)

...

...SHIT, mathematica can’t solve it. OK what the hell I’ll do it by hand. Since everything is horrible I will make the following variable remanings:

$$a = \frac{d_{Cenergycost}}{d_{Cduration}} \quad b = \frac{d_{Fenergygain}}{d_{Fduration}} \quad c = \frac{ceil\left(s_{Fdmg/2}\right)}{s_{Fduration}} \quad d = \frac{ceil\left(s_{Cdmg/2}\right)}{s_{Cduration}}$$

Now we have:

$$\mathrm{d}p * b + \mathrm{s}p * c + \left(1 - \mathrm{s}p\right) * d = a * \left(\frac{1}{\mathrm{d}p} - 1\right)$$

It will help later on to realize that a, b, c, d are all positive numbers (in fact each of their numerators and denominators are also positive)

Multiply by $\mathrm{d}p$ and move the constants to the beginning of each term for neatness:

$$b * \left(\mathrm{d}p\right)^2 + c * \mathrm{d}p * \mathrm{s}p + d * \mathrm{d}p\left(1 - \mathrm{s}p\right) = a * \left(1 - \mathrm{d}p\right)$$

Distribute everything:

$$b * \left(\mathrm{d}p\right)^2 + c * \mathrm{d}p * \mathrm{s}p + d * \mathrm{d}p - d * \mathrm{d}p * \mathrm{s}p = a - a * \mathrm{d}p$$

Factor out $\mathrm{d}p * \mathrm{s}p$:

$$b * \left(\mathrm{d}p\right)^2 + (c - d) * \mathrm{d}p * \mathrm{s}p + d * \mathrm{d}p = a - a * \mathrm{d}p$$

Subtract the right hand side stuff over to the left hand side, and move the snorlax term to the RHS.

$$b * \left(\mathrm{d}p\right)^2 + d * \mathrm{d}p + a * \mathrm{d}p - a = (d - c) * \mathrm{d}p * \mathrm{s}p$$

To solve for $\mathrm{s}p$, divide by the other stuff on the RHS:

$$\frac{b*\left(\text{[dragonite]}p\right)^2 + d*\text{[dragonite]}p + a*\text{[dragonite]}p - a}{(d-c)*\text{[dragonite]}p} = \text{[snorlax]}p$$

And to neaten it up, group the $\text{[dragonite]}p$ terms on the numerator:

$$\frac{b*\left(\text{[dragonite]}p\right)^2 + (d+a)*\text{[dragonite]}p - a}{(d-c)*\text{[dragonite]}p} = \text{[snorlax]}p$$

We've now solved for $\text{[snorlax]}p$ in the dragonite equation, but we would need to solve for it in the snorlax equation too, which will be a bitch. Let's start at the 3rd equation above this line, but replace all snorlax with dragonite and vice versa to get the snorlax equation:

$$b*\left(\text{[snorlax]}p\right)^2 + d*\text{[snorlax]}p + a*\text{[snorlax]}p - a = (d-c)*\text{[dragonite]}p*\text{[snorlax]}p$$

..but wait, that equation is wrong because we also have to switch the snorlax and dragonites in the constants a, b, c, d. I will now make new variables:

$$e = \frac{\text{[snorlax]}_{Cenergycost}}{\text{[snorlax]}_{Cduration}} \quad f = \frac{\text{[snorlax]}_{Fenergygain}}{\text{[snorlax]}_{Fduration}} \quad g = \frac{ceil\left(\text{[dragonite]}_{Fdmg/2}\right)}{\text{[dragonite]}_{Fduration}} \quad h = \frac{ceil\left(\text{[dragonite]}_{Cdmg/2}\right)}{\text{[dragonite]}_{Cduration}}$$

These constants are in the same order as before but with snorlax and dragonite switched, and with abcd renamed to efgh. So the snorlax formula actually reads

$$f*\left(\text{[snorlax]}p\right)^2 + h*\text{[snorlax]}p + e*\text{[snorlax]}p - e = (h-g)*\text{[dragonite]}p*\text{[snorlax]}p$$

subtract the RHS away:

$$f*\left(\text{[snorlax]}p\right)^2 + h*\text{[snorlax]}p + e*\text{[snorlax]}p + (g-h)*\text{[dragonite]}p*\text{[snorlax]}p - e = 0$$

combine the $\text{[snorlax]}p$ terms on the LHS:

$$f*\left(\text{[snorlax]}p\right)^2 + \left(h + e + (g-h)*\text{[dragonite]}p\right)*\text{[snorlax]}p - e = 0$$

This is a quadratic equation. If you're up to this point you must know the quadratic equation:

$$\frac{-B \pm \sqrt{B^2 - 4*A*C}}{2*A}$$

For us that translates to:

$$p = \frac{-\left(h + e + (g - h) * p\right) \pm \sqrt{\left(h + e + (g - h) * p\right)^2 + 4 * f * e}}{2 * f}$$

Great! Let's set it equal to the other p from before:

$$\frac{b * \left(p\right)^2 + (d + a) * p - a}{(d - c) * p} = \frac{-\left(h + e + (g - h) * p\right) \pm \sqrt{\left(h + e + (g - h) * p\right)^2 + 4 * f * e}}{2 * f}$$

Goddamn I never thought pokemon could be such a mess. Okay folks, you know the deal, get rid of those denominators:

$$2 * f * \left(b * \left(p\right)^2 + (d + a) * p - a\right) = -(d - c)\left(h + e + (g - h) * p\right) * p \pm (d - c) * p\sqrt{\left(h + e + (g - h) * p\right)^2 + 4 * f * e}$$

Move over the first term on the RHS and reorder that term a bit:

$$2 * f * \left(b * \left(p\right)^2 + (d + a) * p - a\right) + (d - c) * p * \left(h + e + (g - h) * p\right) = \pm(d - c) * p\sqrt{\left(h + e + (g - h) * p\right)^2 + 4 * f * e}$$

Let's factor the LHS by powers of p. But first distribute. Each line is one of the terms:

$$LHS = 2 * f * b * \left(p\right)^2 + 2 * f * (d + a) * p - 2 * f * a$$

$$+(d - c) * (h + e) * p + (d - c) * (g - h) * \left(p\right)^2$$

$$= \left(2 * f * b + (d - c) * (g - h)\right) * \left(p\right)^2$$

$$+(2 * f * (d + a) + (d - c) * (h + e)) * p$$

$$-2 * f * a$$

I hate to be the bringer of bad news but now we must square both sides of the equation to get rid of the root. OASIOAISJFOAA! Here's the result:

$$\left[\left(2*f*b+(d-c)*(g-h)\right)*\left(\text{🐉}p\right)^2+\left(2*f*(d+a)+(d-c)*(h+e)\right)*\text{🐉}p-2*f*a\right]^2$$
$$=(d-c)^2*\left(\text{🐉}p\right)^2*\left(h+e+(g-h)*\text{🐉}p\right)^2+4*f*e$$

Let’s simplify the RHS. Note that

$$\left(h+e+(g-h)*\text{🐉}p\right)^2=(h+e)^2+2*(h+e)*(g-h)*\text{🐉}p+(g-h)^2*\left(\text{🐉}p\right)^2$$

And now let’s distribute the preceding factor over that term:

$$(d-c)^2*\left(\text{🐉}p\right)^2*\left((h+e)^2+2*(h+e)*(g-h)*\text{🐉}p+(g-h)^2*\left(\text{🐉}p\right)^2\right)$$

$$=(h+e)^2*(d-c)^2*\left(\text{🐉}p\right)^2$$

$$+2*(h+e)*(g-h)*(d-c)^2*\left(\text{🐉}p\right)^3$$

$$+(g-h)^2*(d-c)^2*\left(\text{🐉}p\right)^4$$

That makes the whole RHS be (in order of powers):

$$(g-h)^2*(d-c)^2*\left(\text{🐉}p\right)^4+2*(h+e)*(g-h)*(d-c)^2*\left(\text{🐉}p\right)^3$$

$$+(h+e)^2*(d-c)^2*\left(\text{🐉}p\right)^2+4*f*e$$

I think it’s about time to rename more stuff. I want the equation to look like this:

$$\left[L*\left(\text{🐉}p\right)^2+M*\text{🐉}p+N\right]^2=P*\left(\text{🐉}p\right)^4+Q*\left(\text{🐉}p\right)^3+R*\left(\text{🐉}p\right)^2+S$$

For that I need to assign:

$$L=2*f*b+(d-c)*(g-h)$$

$$M=2*f*(d+a)+(d-c)*(h+e)$$

$$N=-2*f*a$$

$$P=(g-h)^2*(d-c)^2$$

$$Q=2*(h+e)*(g-h)*(d-c)^2$$

$$R=(h+e)^2*(d-c)^2$$

$$S=4*f*e$$

These new variables have the following signs (1 if > 0, -1 if < 0):

$$sgn(L) = 2 * f * b + (d - c) * (g - h)$$

$$sgn(M) = 1\ if\ d > c, -1\ if\ M < 0$$

$$sgn(N) = -1$$

$$sgn(P) = 1$$

$$sgn(Q) = sgn(g - h)$$

$$sgn(R) = 1$$

$$sgn(S) = 1$$

OK, with our new variable names let's distribute the LHS. I will now use the fact that

$$(A + B + C)^2 = A^2 + B^2 + C^2 + 2AB + 2AC + 2BC$$

This makes

$$\left[L * \left(\text{🐉}p\right)^2 + M * \text{🐉}p + N\right]^2 = L^2 * \left(\text{🐉}p\right)^4 + M^2 * \left(\text{🐉}p\right)^2 + N^2$$

$$+2LM * \left(\text{🐉}p\right)^3 + 2LN * \left(\text{🐉}p\right)^2 + 2MN * \text{🐉}p$$

Combine like-powers:

$$= L^2 * \left(\text{🐉}p\right)^4 + 2LM * \left(\text{🐉}p\right)^3 + (2LN + M^2) * \left(\text{🐉}p\right)^2 + 2MN * \text{🐉}p + N^2$$

Now that it's expanded, let's plug it into what we had before, to give:

$$L^2 * \left(\text{🐉}p\right)^4 + 2LM * \left(\text{🐉}p\right)^3 + (2LN + M^2) * \left(\text{🐉}p\right)^2 + 2MN * \text{🐉}p + N^2$$
$$= P * \left(\text{🐉}p\right)^4 + Q * \left(\text{🐉}p\right)^3 + R * \left(\text{🐉}p\right)^2 + S$$

Move everything to the LHS:

$$L^2 * \left(\text{🐉}p\right)^4 + 2LM * \left(\text{🐉}p\right)^3 + (2LN + M^2) * \left(\text{🐉}p\right)^2 + 2MN * \text{🐉}p + N^2 - P * \left(\text{🐉}p\right)^4 - Q$$
$$* \left(\text{🐉}p\right)^3 - R * \left(\text{🐉}p\right)^2 - S = 0$$

Order by power:

$$L^2 * \left(\text{🐉}p\right)^4 - P * \left(\text{🐉}p\right)^4 + 2LM * \left(\text{🐉}p\right)^3 - Q * \left(\text{🐉}p\right)^3 + (2LN + M^2) * \left(\text{🐉}p\right)^2 - R * \left(\text{🐉}p\right)^2$$

$$+2MN * p + N^2 - S = 0$$

Factor by power:

$$(L^2 - P) * \left(p\right)^4 + (2LM - Q) * \left(p\right)^3 + (2LN + M^2 - R) * \left(p\right)^2 + 2MN * p + N^2 - S = 0$$

Holy crap we've done it. The solution to this equation is the average dps of dragonite against snorlax.

First I'm going to re-substitute in variables in case they simplify:

$$L = 2 * f * b + (d - c) * (g - h)$$

$$M = 2 * f * (d + a) + (d - c) * (h + e)$$

$$N = -2 * f * a$$

$$P = (g - h)^2 * (d - c)^2$$

$$Q = 2 * (h + e) * (g - h) * (d - c)^2$$

$$R = (h + e)^2 * (d - c)^2$$

$$S = 4 * f * e$$

Using the lazy approach this time, wolfram alpha says

$$L^2 - P = 4bf * (bf - cg + ch + dg - dh)$$

$2LM - Q =$4 f (2 a b f-a c g+a c h+a d g-a d h-b c e-b c h+b d e+2 b d f+b d h-c d g+c d h+d^2 g-d^2 h)

Now how the hell do we solve this?? It's a 4th order polynomial, and 4th order polynomials have a corresponding "quadratic formula" which won't fit on a single page. Not only that but there will be four roots. Which three will be false?

LIST OF MODIFICATIONS IN ORDER OF PREDICTED IMPORTANCE TO OUTCOME:

1) Low enemy HP strategy: weave or no weave?

When the enemy has very low hp, it doesn't make sense to weave anymore, you just fastattack.

An upper bound for when this comes into effect is when HP < Cdmg but it doesn't happen exactly at that point.

At this point, there are effectively new formulas for the $Cdmg$ and $Cduration$:

$$Cdmg = HP$$

(because you can't overkill)

$$Cduration = Cdws$$

(because you don't lose the downtime which normally comes after the damage is done)

..where $Cdws$ is the time it takes for the Cdmg to kick in after the attack is used. Cdws stands for "Cinematic move Damage Window Start", since the GAME MASTER file calls this time "damageWindowStartMs".

This clearly changes the dps of dragonite's cmove, which will be higher at first because of the reduced time (doesn't have to "pay" for the move by waiting afterwards anymore) and then shrinks toward 0 damage as the enemy nears 0HP.

The "phase change" happens at the moment when it's better to use the Fmove than the Cmove, either because

situationA=(the Fmove would kill before $Cdws$ ms have passed),

or

situationB=(because the energy gained for the next pokemon results in a higher overall damage done before dying)

The fundamental question: at what average HP or what average time does situationA or situationB occur in a battle?

Let's start with situationA. What is its "trigger"?

The trigger is if #secondsToKillWithFDPS < $Cdws$.

$Fdps$ is actually nontrivial here because 1. you have to consider snorlax's dmg output and 2. on the last hit, its dps is equal to snorlaxHP/Fdws instead. First let's suppose Fdps just stays the same, in other words

$$Fdps\_simple = \frac{Fdmg}{Fduration}$$

The number of seconds it takes to kill snorlax with dragonite's fmove is

$$\frac{HP}{Fdps\_simple} = \frac{HP * Fduration}{Fdmg}$$

So situationA happens approximately when

$$\frac{HP * Fduration}{Fdmg} < Cdws$$

In other words, when

$$HP < \frac{Cdws * Fdmg}{Fduration}$$

Let's see if we can improve on the formula $Fdps\_simple$ by

ANOTHER:

once HP < Fdmg, dragonite's fmove also gets a new damage and duration:

$$Fdmg = HP$$

$$Fduration = Fdws$$

but this is a small effect so later

APPENDIX:

If the ceil function gives trouble, there's a good approximation for it:

If Fdmg is even, then ceil(Fdmg/2) = 0.5*Fdmg (exactly)

If Fdmg is odd, then ceil(Fdmg/2) = 0.5*Fdmg+0.5 (exactly)

And if Fdmg is just as likely to be an even number as an odd number, then on average, we can use

$$ceil\left(\frac{Fdmg}{2}\right) \cong 0.5 * Fdmg + 0.25$$

That's a pretty reasonable estimation, and anyway we're talking about a difference of 0.25 damage here at most.