Math 241 Exam 1 June 30, 2017

Name: Answer Key

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

No calculators or notes allowed.

Show all work clearly.

Use proper notation.

Question	Points	Score
1	3	
2	10	
3	7	
4	15	
5	5	
6	10	
7	5	
8	5	
Total	50	

1. (3 points) Let $f(x) = \sqrt{x+2}$, find the average rate of change over the following the intervals: [-1,2], [-1,7], [-1,14]

- $[-1,2]$ $\quad m = \dfrac{f(2)-f(-1)}{2-(-1)} = \dfrac{\sqrt{4}-\sqrt{1}}{3} = \dfrac{2-1}{3} = \dfrac{1}{3}$

- $[-1,7]$ $\quad m = \dfrac{f(7)-f(-1)}{7-(-1)} = \dfrac{\sqrt{9}-\sqrt{1}}{8} = \dfrac{2}{8} = \dfrac{1}{4}$

- $[-1,14]$ $\quad m = \dfrac{f(14)-f(-1)}{14-(-1)} = \dfrac{\sqrt{16}-\sqrt{1}}{15} = \dfrac{4-1}{15} = \dfrac{3}{15} = \dfrac{1}{5}$

2. (10 points) For the following limits, evaluate the limit, or explain why the limit does not exist. Remember to use correct notation.

(a) (5 points)

$$\lim_{x \to -1} \frac{x^2 - x - 2}{x^2 - 1}$$

$(-1)^2 - 1 = 0$, can't just plug -1 in.

$$= \lim_{x\to -1} \frac{x^2-x-2}{x^2-1} = \lim_{x\to -1} \frac{(x+1)(x-2)}{(x-1)(x+1)}$$

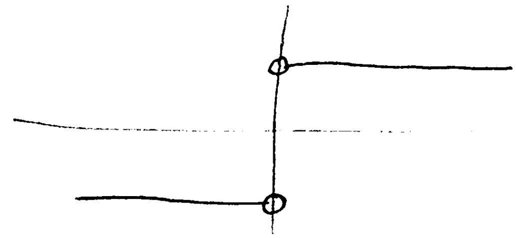
$$= \lim_{x\to -1} \frac{x-2}{x-1} = \frac{-3}{-2} = \frac{3}{2}$$

(b) (5 points)

$$\lim_{x\to 0}\frac{x}{|x|}$$

hint: try writing the function in the limit as a piecewise function, the two conditions being $x < 0$ and $x > 0$

$$\frac{x}{|x|} = \begin{cases} -1 & x < 0 \\ 1 & x > 0 \end{cases}$$



$$\lim_{x\to 0^-}\frac{x}{|x|} = -1 \neq 1 = \lim_{x\to 0^+}\frac{x}{|x|}$$

so $\lim_{x\to 0}\frac{x}{|x|}$ DNE

3. (7 points) Does

$$y = \frac{\sqrt[3]{x}-\sqrt{x}}{\sqrt[3]{x}+\sqrt{x}} = \frac{x^{\frac{1}{3}} - x^{\frac{1}{2}}}{x^{\frac{1}{3}} + x^{\frac{1}{2}}}$$

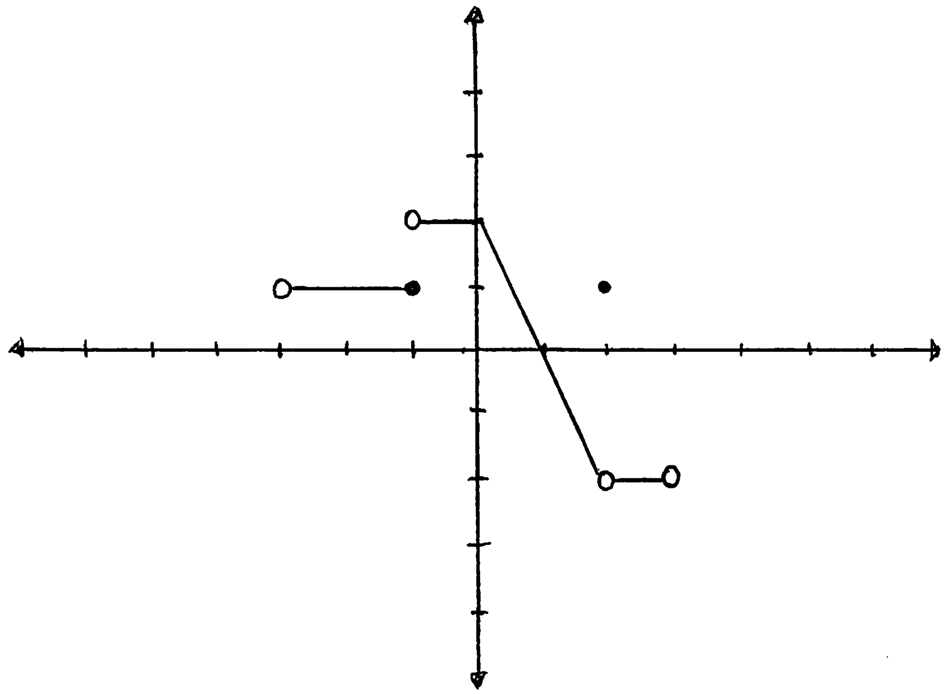
have any horizontal asymptotes? If so, what are they?

$$\lim_{x\to\infty}\frac{x^{\frac{1}{3}}-x^{\frac{1}{2}}}{x^{\frac{1}{3}}+x^{\frac{1}{2}}} = \lim_{x\to\infty}\frac{x^{\frac{1}{3}}-x^{\frac{1}{2}}}{x^{\frac{1}{3}}+x^{\frac{1}{2}}}\frac{\left(\frac{1}{x^{\frac{1}{2}}}\right)}{\left(\frac{1}{x^{\frac{1}{2}}}\right)}$$

$$= \lim_{x\to\infty}\frac{x^{\frac{1}{3}-\frac{1}{2}}-1}{x^{\frac{1}{3}-\frac{1}{2}}+1} \qquad \frac{1}{3}-\frac{1}{2} = \frac{2}{6}-\frac{3}{6} = -\frac{1}{6}$$

$$= \lim_{x\to\infty}\frac{\overset{\to 0}{\frac{1}{\sqrt[6]{x}}}-1}{\underset{\to 0}{\frac{1}{\sqrt[6]{x}}}+1} = \frac{-1}{1} = -1$$

4. (15 points) The graph of a function $g(x)$ is given below.



(a) (8 points) What is/are the interval(s) of continuity? For all values of x where $g(x)$ is discontinuous, state why.

$(-3,-1)$, $(-1,2)$, $(2,3)$

discontinuous at $x=-1$: $\lim_{x\to -1^-} g(x) \neq \lim_{x\to -1^+} g(x)$

$x = 2$: $\lim_{x\to 2} g(x) \neq g(2)$

(b) (7 points) What is/are the interval(s) of differentiability?

$(-3,-1)$, $(-1,0)$, $(0,2)$, $(2,3)$

5. (5 points) Give the formal definition of what

$$\lim_{x \to c^-} f(x) = L$$

means.

$\lim_{x \to c^-} f(x) = L$ if for all $\varepsilon > 0$,

There is some $\delta > 0$ s.t. for all

$c - \delta < x < c$, $|f(x) - L| < \varepsilon$

6. (10 points) Find y' and y'', given $y = 2x + x^5 + \sqrt[3]{x} + \frac{1}{x^4} + cos(x)$

$$y' = 2 + 5x^4 + \frac{1}{3}x^{-2/3} - 4x^{-5} - \sin(x)$$

$$y'' = 20x^3 - \frac{2}{9}x^{-5/3} + 20x^{-6} - \cos(x)$$

7. (5 points) Differentiate one of the following functions. Clearly indicate which problem you would like graded; only one will be graded.

$$k(u) = (u^2 - 2u + 1)\,(\sqrt[5]{u} - sec(u)) \qquad h(t) = \frac{sin(h) - h}{\sqrt{h} + h}$$

$$k'(u) = (2u-2)(\sqrt[5]{u} - \sec(u)) + (u^2 - 2u + 1)\left(\frac{1}{5}u^{-4/5} - \sec(u)\tan(u)\right)$$

$$h'(t) = \frac{(\cos(h) - 1)(\sqrt{h} + h) - (\sin(h) - h)(\frac{1}{2}h^{-1/2} + 1)}{(\sqrt{h} + h)^2}$$

8. (5 points) Differentiate the following function.

$$z = \theta \, tan \left(1 + \sqrt{\theta^2 + 1}\right)$$

$$z' = \theta \frac{d}{d\theta}\left(\tan\left(1+\sqrt{\theta^2+1}\right)\right) + \tan\left(1+\sqrt{\theta^2+1}\right)$$

$$= \theta\left(\sec^2\left(1+\sqrt{\theta^2+1}\right) \cdot \frac{d}{d\theta}\left(1+\sqrt{\theta^2+1}\right) + \tan\left(1+\sqrt{\theta^2+1}\right)\right.$$

$$= \theta \sec^2\left(1+\sqrt{\theta^2+1}\right) \frac{1}{2}(\theta^2+1)^{-1/2} \frac{d}{d\theta}(\theta^2+1) + \tan\left(1+\sqrt{\theta^2+1}\right)$$

$$= \theta \sec^2\left(1+\sqrt{\theta^2+1}\right) \frac{1}{2}(\theta^2+1)^{-1/2} \cdot 2\theta + \tan\left(1+\sqrt{\theta^2+1}\right)$$

$$= \theta^2 \sec^2\left(1+\sqrt{\theta^2+1}\right) + \tan\left(1+\sqrt{\theta^2+1}\right)$$