3.5 | 1, 2, 10, 13, 19-43 (odd), 49, 60

1) $y = 6u - 9$, $u = \frac{1}{2}x^4$

$= f(u)$ $\quad = g(x) \quad g'(x) = 2x^3$

$f'(u) = 6$

$\frac{dy}{dx} = f'(g(x))\, g'(x)$

$= 6 \cdot 2x^3 = 12x^3$

2) $y = 2u^3 = f(u)$, $u = 8x - 1 = g(x)$

$f'(u) = 6u^2$ $\quad g'(x) = 8$

$\frac{dy}{dx} = f'(g(x))\, g'(x)$

$= 6(g(x))^2 \cdot 8 = 6(8x-1)^2 \cdot 8$

$= 48(8x-1)^2$

10) $y = (4 - 3x)^9$

$y = f(g(x))$, $y = f(u) = u^9$, $g(x) = 4 - 3x = u$

$f'(u) = 9u^8$ $\quad u' = g'(x) = -3$

$y' = 9u^8 u'$

$= 9(4-3x)^8 \cdot (-3)$

13) $y = \left(\frac{x^2}{8} + x - \frac{1}{x}\right)^4$ $\quad y = f(u) = u^4$, $f'(u) = 4u^3$

$u = g(x) = \frac{x^2}{8} + x - \frac{1}{x}$

$u' = g'(x) = \frac{x}{4} + 1 + x^{-2}$

$y' = f'(u)u' = 4u^3\left(\frac{x}{4} + 1 + x^{-2}\right) = 4\left(\frac{x^2}{8} + x - \frac{1}{x}\right)^3\left(\frac{x}{4} + 1 + x^{-2}\right)$

19) $p = \sqrt{3-t} = (3-t)^{1/2}$

$p' = \frac{1}{2}(3-t)^{-1/2}(-1)$

21) $s = \frac{4}{3\pi}\sin 3t + \frac{4}{5\pi}\cos 5t$

$s' = \frac{4}{3\pi}(\cos 3t)(3) - \frac{4}{5\pi}(\sin 5t)(5)$

23) $r = (\csc\theta + \cot\theta)^{-1}$

$r' = -1(\csc\theta + \cot\theta)^{-2}\left(-\csc\theta\cot\theta - \csc^2\theta\right)$

25) $y = x^2\sin^4 x + x\cos^{-2} x$

$y' = \frac{d}{dx}(x^2\sin^4 x) + \frac{d}{dx}(x\cos^{-2} x)$

$= x^2(4\sin^3 x\cos x) + 2x(\sin^4 x)$

$+ \left(x(-2\cos^{-3} x)(-\sin x) + \cos^{-2} x\right)$

27) $y = \frac{1}{21}(3x-2)^7 + \left(4 - \frac{1}{2x^2}\right)^{-1}$

$y' = \frac{1}{3}(3x-2)^6 \cdot 3 - \left(4 - \frac{1}{2x^2}\right)^{-2}\left(- \; x^{-3}\right)$

$= 3x-2 - 2$

24) $y = (4x+3)^4 (x+1)^{-3} = \dfrac{(4x+3)^4}{(x+1)^3}$

$$y' = \frac{4(4x+3)^3 \cdot 4 \cdot (x+1)^3 - (4x+3)^4 \cdot 3(x+1)^2}{(x+1)^6}$$

$$= \frac{16(4x+3)^3(x+1) - 3(4x+3)^4}{(x+1)^4}$$

$$= \frac{(4x+3)^3\left[16(x+1) - 3(4x+3)\right]}{(x+1)^4}$$

$$= \frac{(4x+3)^3(16x+16-12x-9)}{(x+1)^2}$$

$$= \frac{(4x+3)^3(4x+7)}{(x+1)^2}$$

31) $h(x) = x\tan(2\sqrt{x}) + 7$

$$h'(x) = x\sec^2(2\sqrt{x}) \cdot \left(\frac{1}{\sqrt{x}}\right)$$

$$+ \tan(2\sqrt{x})$$

33) $f(\theta) = \left(\frac{\sin\theta}{1+\cos\theta}\right)^2$

$$\frac{d}{d\theta}\frac{\sin\theta}{1+\cos\theta} = \frac{\cos\theta(1+\cos\theta) - \sin\theta(-\sin\theta)}{(1+\cos\theta)^2}$$

$$= \frac{\cos\theta + \cos^2\theta + \sin^2\theta}{(1+\cos\theta)^2}$$

$$= \frac{\cos\theta + 1}{(1+\cos\theta)^2} = \frac{1}{1+\cos\theta}$$

So $f'(\theta) = 2\left(\frac{\sin\theta}{1+\cos\theta}\right) \cdot \frac{1}{1+\cos\theta}$

$$= \frac{2\sin\theta}{(1+\cos\theta)^2}$$

35) $r = \sin(\theta^2)\cos(2\theta)$

$r' = \cos(\theta)^2 \cdot 2\theta \cdot \cos 2\theta + \sin(\theta^2)(-\sin(2\theta))2$

37) $q = \sin\left(\frac{t}{\sqrt{t+1}}\right)$

$q' = \cos\left(\frac{t}{\sqrt{t+1}}\right) \frac{d}{dt}\left(\frac{t}{\sqrt{t+1}}\right)$

$= \cos\left(\frac{t}{\sqrt{t+1}}\right)\left(\frac{\sqrt{t+1} - t \cdot \frac{1}{2}\frac{1}{\sqrt{t+1}}}{t+1}\right)$

$= \cos\left(\frac{t}{\sqrt{t+1}}\right)\left(\frac{2t+2-t}{2\sqrt{t+1}(t+1)}\right)$

$= \cos\left(\frac{t}{\sqrt{t+1}}\right)\left(\frac{t+2}{2(t+1)\sqrt{t+1}}\right)$

39) $y = \sin^2(\pi t - 2)$

$y' = 2\sin(\pi t - 2) \cdot \cos(\pi t - 2) \cdot \pi$

41) $y = (1+\cos 2t)^{-4}$

$y' = -4\left(1+\cos 2t\right)^{-5}\left(-2\sin 2t\right)$

$= 8\,\frac{\sin 2t}{(\cos 2t)^5}$

43) $y = \sin\left(\cos\left(2t-5\right)\right)$

$y' = \cos\left(\cos\left(2t-5\right)\right)\frac{d}{dt}\left(\cos(2t-5)\right)$

$= -\cos\left(\cos(2t-5)\right)\sin(2t-5)\cdot 2$

49) $y = \left(1+\frac{1}{x}\right)^3$

$y' = 3\left(1+\frac{1}{x}\right)^2\left(-x^{-2}\right)$

$y'' = 6\left(1+\frac{1}{x}\right)^1\left(-x^{-2}\right)\left(-x^{-2}\right)$

$+ 3\left(1+\frac{1}{x}\right)^2\left(2x^{-3}\right)$

60)

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	1	1	5	1/3
1	3	-4	-1/3	-8/3

a) $\bar{g} = 5f(x) - g(x)$, $x = 1$

$y' = 5f'(x) - g'(x)$

$y'(1) = 5f'(1) - g'(1)$

$= 5 \cdot \left(-\frac{1}{3}\right) - \left(-\frac{8}{3}\right)$

$= -\frac{5}{3} + \frac{8}{3} = \underline{1}$

b) $y = f(x)g^3(x)$, $x = 0$

$y' = f'(x)g^3(x) + f(x) \cdot 3g^2(x)\,g'(x)$

$g'(0) = f'(0)g^3(0) + f(0) \cdot 3 \cdot g^2(0) \cdot g'(0)$

$= 5 \cdot 1^3 + 1 \cdot 3 \cdot 1^2 \cdot 1/3$

$= 6$

c) $y = \dfrac{f(x)}{g(x)+1}$, $x=1$

$$y' = \frac{F'(x)(g(x)+1) - g'(x)f(x)}{(g(x)+1)^2}$$

$$y'(1) = \frac{F'(1)(g(1)+1) - g'(1)f(1)}{(g(1)+1)^2}$$

$$= \frac{-\frac{1}{3}(-4+1) - \left(\frac{-8}{3}\right)(3)}{(-4+1)^2}$$

$$= \frac{1+8}{(-3)^2} = 1$$

d) $y = f(g(x))$, $x=0$

$$y' = F'(g(x))\,g'(x)$$

$$y'(0) = F'(g(0))\,g'(0)$$

$$= F'(1)\cdot\frac{1}{3} = -\frac{1}{3}\cdot\frac{1}{3} = -1/9$$

60

e) $y = g(f(x))$ $x=0$

$y' = g'(f(x))\,f'(x)$

$y'(0) = g'(f(0))\,f'(0)$

$= g'(1)\cdot 5 = -\frac{8}{3}\cdot 5 = -\frac{40}{3}$

f) $y = [x^{11} + f(x)]^{-2}$, $x=1$

$y' = -2(x^{11}+f(x))^{-3}(11x^{10}+f'(x))$

$y'(1) = -2(1^{11}+f(1))^{-3}(11(1)^{10}+f'(1))$

$= -2(1+3)^{-3}(11+(-\frac{1}{3}))$

$= -2\frac{1}{4^3}(\frac{33}{3}-\frac{1}{3})$

$= -2\frac{1}{64}(\frac{32}{3}) = -\frac{1}{3}$

g) $y = f(x + g(x))$, $x = 0$

$y' = f'(x+g(x))(1+g'(x))$

$y'(0) = f'(0+g(0))(1+g'(0))$

$= f'(1)(1+\frac{1}{3})$

$= -\frac{1}{3}(\frac{4}{3}) = -4/9$