3.6 3, 17, 21, 40

3) $2xy + y^2 = x + y$

$\frac{d}{dx}\left(2xy + y^2\right) = \frac{d}{dx}\left(x + y\right)$

$2\left(y + xy'\right) + 2yy' = 1 + y'$

$2y + 2xy' + 2yy' = 1 + y'$

$2xy' + 2yy' - y' = 1 - 2y$

$y'(2x + 2y - 1) = 1 - 2y$

$y' = \frac{1 - 2y}{2x + 2y - 1}$

17) $\sin(r\theta) = \frac{1}{2}$

$\frac{d}{d\theta}\left(\sin(r\theta)\right) = \frac{d}{d\theta}\frac{1}{2}$

$\cos(r\theta)\left(\theta r' + r\right) = 0$

$\theta r' + r = 0$ dividing by $\cos(r\theta)$

$r' = \frac{-r}{\theta}$

21) $y^2 = x^2 + 2x$

$$\frac{d}{dx}(y^2) = \frac{d}{dx}(x^2 + 2x)$$

$$2y\,y' = 2x + 2$$

$$\{ y y' = x + 1$$

$$y' = \frac{x+1}{y} = \frac{x+1}{\sqrt{x^2+2x}}$$

$$\frac{d}{dx}(yy') = \frac{d}{dx}(x+1)$$

$$yy'' + y'y' = 1$$

$$y'' = \frac{1-(y')^2}{y} = \frac{1-\left(\frac{x+1}{\sqrt{x^2+2x}}\right)^2}{\sqrt{x^2+2x}}$$

40) prove $\frac{d}{dx} x^{p/q} = \frac{p}{q} x^{p/q - 1}$, p, q integers.

We know $\frac{d}{dx} x^p = p x^{p-1}$ for p integers.

Say $y = x^{p/q}$. Then $y^q = x^p$

$\Rightarrow \frac{d}{dx}(y^q) = \frac{d}{dx}(x^p) = p x^{p-1}$, by above fact

now $\frac{d}{dx}(y^q) = q y^{q-1} y'$ by implicit differentiation

So $q\,y^{q-1}y' = p\,x^{p-1}$

$$y' = \frac{p x^{p-1}}{q y^{q-1}}$$

but $y = x^{p/q}$, so

$$y' = \frac{p x^{p-1}}{q\left(x^{p/q}\right)^{q-1}} = \frac{p}{q}\,\frac{x^{p-1}}{x^{p-p/q}}$$

$$= \frac{p}{q}\,x^{p-1-(p-p/q)} = \frac{p}{q}\,x^{p-1-p+p/q}$$

$$= \frac{p}{q}\,x^{p/q-1}$$

So $y' = \frac{p}{q}x^{p/q-1}$, as needed.