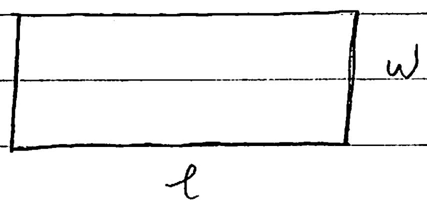
HW #3.7 11, 12, 21, 24

11)



$\frac{dl}{dt} = -2 \text{ cm/sec}$ $\frac{dw}{dt} = 2 \text{ cm/sec}$

At this time, $l = 12\text{cm}$ $w = 5\text{cm}$

a) what is $\frac{dA}{dt}$ at this time?

$A = wl \implies \frac{dA}{dt} = \frac{d}{dt}(wl) = w\frac{dl}{dt} + l\frac{dw}{dt}$

$\frac{dA}{dt} = 5(-2) + 12(2) = -10 + 24 = 14$

note: $(5\text{cm})(-2\text{ cm/sec}) + (12\text{cm})(\frac{2\text{cm}}{\text{sec}})$

$-10 \text{ cm}^2/\text{sec} + 24 \text{ cm}^2/\text{sec} = 14 \text{ cm}^2/\text{sec}$

increasing

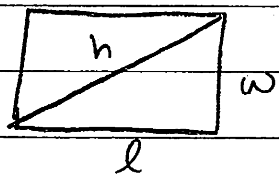
b) what is $\frac{dP}{dt}$ at this time?

$P = 2l + 2w$ $\frac{dP}{dt} = 2\frac{dl}{dt} + 2\frac{dw}{dt}$

$\frac{dP}{dt} = 2(-2\text{ cm/sec}) + 2(2\text{ cm/sec}) = 0 \text{ cm/sec}$

constant

c) What is $\frac{dh}{dt}$ at this time



$$l^2 + w^2 = h^2$$

$$\frac{d}{dt}(l^2 + w^2) = \frac{d}{dt}(h^2)$$

$$2l\frac{dl}{dt} + 2w\frac{dw}{dt} = 2h\cdot\frac{dh}{dt}$$

$$l\frac{dl}{dt} + w\frac{dw}{dt} = h\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{1}{h}\left(l\frac{dl}{dt} + w\frac{dw}{dt}\right)$$

$$= \frac{1}{\sqrt{(12\,cm)^2 + (5\,cm)^2}}\left(12\,cm\left(-2\,cm/sec\right) + 5\,cm\left(2\,cm/sec\right)\right)$$

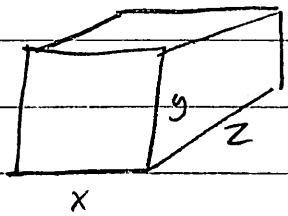
$$= \frac{1}{\sqrt{144\,cm^2 + 25\,cm^2}}\left(-24\frac{cm^2}{sec} + 10\,cm^2/sec\right)$$

$$= \frac{1}{\sqrt{169\,cm^2}}\left(-14\frac{cm^2}{sec}\right)$$

$$= \frac{1}{13\,cm}\left(-14\,cm^2/sec\right) = -\frac{14}{13}\,cm/sec$$

decreasing

12)



$\frac{dx}{dt} = 1\ m/s \qquad \frac{dy}{dt} = -2\ m/s, \quad \frac{dz}{dt} = 1\ m/s$

$x = 4, \quad y = 3, \quad z = 2$

a) $\frac{dV}{dt}$? $\qquad V = xyz \rightarrow \frac{d}{dt}(V) = \frac{d}{dt}(xyz)$

$$\frac{dV}{dt} = xy\frac{dz}{dt} + xz\frac{dy}{dt} + yz\frac{dx}{dt}$$

$$\frac{dV}{dt} = (4m)(3m)(1\ m/s) + (4m)(2m)(-2\ m/s) + (3m)(2m)(1\ m/s)$$

$$= 12m^3/s - 16m^2/s + 6m^2/s$$
$$= 2m^3/s$$

b) $\frac{dS}{dt}$? $\qquad S = 2(xy + yz + zx)$

$$\frac{dS}{dt} = \frac{d}{dt}\left[2(xy + yz + zx)\right]$$

$$= 2\left(x\frac{dy}{dt} + y\frac{dx}{dt} + y\frac{dz}{dt} + z\frac{dy}{dt} + z\frac{dx}{dt} + x\frac{dz}{dt}\right)$$

$$= 2\left(4m\left(-2\frac{m}{s}\right) + 3m\left(1\frac{m}{s}\right) + 3m\left(1\frac{m}{s}\right) + 2m\left(-2\frac{m}{s}\right) + 2m\left(1\frac{m}{s}\right) + 4m\left(2\frac{m}{s}\right)\right)$$

$$= 2\left(-8\frac{m^2}{s} + 3m^2/s + 3m^2/s - 4m^2/s + 2m^2/s + 8m^2/s\right) = 4m^2/s$$

c) $s = \sqrt{x^2+y^2+z^2} \rightarrow s^2 = x^2+y^2+z^2$

$$2s\frac{ds}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt} + 2z\frac{dz}{dt}$$

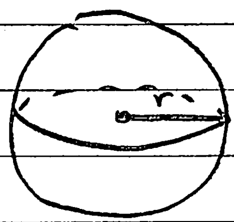
$$\frac{ds}{dt} = \frac{1}{\sqrt{x^2+y^2+z^2}}\left(x\frac{dx}{dt} + y\frac{dy}{dt} + z\frac{dz}{dt}\right)$$

$$= \frac{1}{(4m^2+3m^2+2m^2)^{1/2}}\left(4m\left(1\frac{m}{s}\right) + 3m\left(\frac{-2m}{s}\right) + 2m\left(1\frac{m}{s}\right)\right)$$

$$= \frac{1}{(16m^2+9m^2+4m^2)^{1/2}}\left(4\frac{m^2}{s} - 6\frac{m^2}{s} + 2\frac{m^2}{s}\right)$$

$$= \frac{1}{(29m^2)^{1/2}} \cdot 0\frac{m^2}{s} = 0\frac{m}{s}$$

21)



$$\frac{dV}{dt} = 100\pi \text{ ft}^3/\text{min}$$

at this time, $r = 5\text{ft}$

what is $\frac{dr}{dt}$? $\frac{dS}{dt}$?

$$V = \frac{4}{3}\pi r^3 \rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\rightarrow 100\pi \frac{\text{ft}^3}{\text{min}} = 4\pi (5\text{ft})^2 \frac{dr}{dt}$$

$$\rightarrow 100\pi \frac{\text{ft}^3}{\text{min}} = 4\pi \cdot 25 \text{ ft}^2 \frac{dr}{dt}$$

$$\rightarrow \cancel{100\pi} \frac{\text{ft}^3}{\text{min}} = \cancel{100\pi} \text{ ft}^2 \frac{dr}{dt}$$

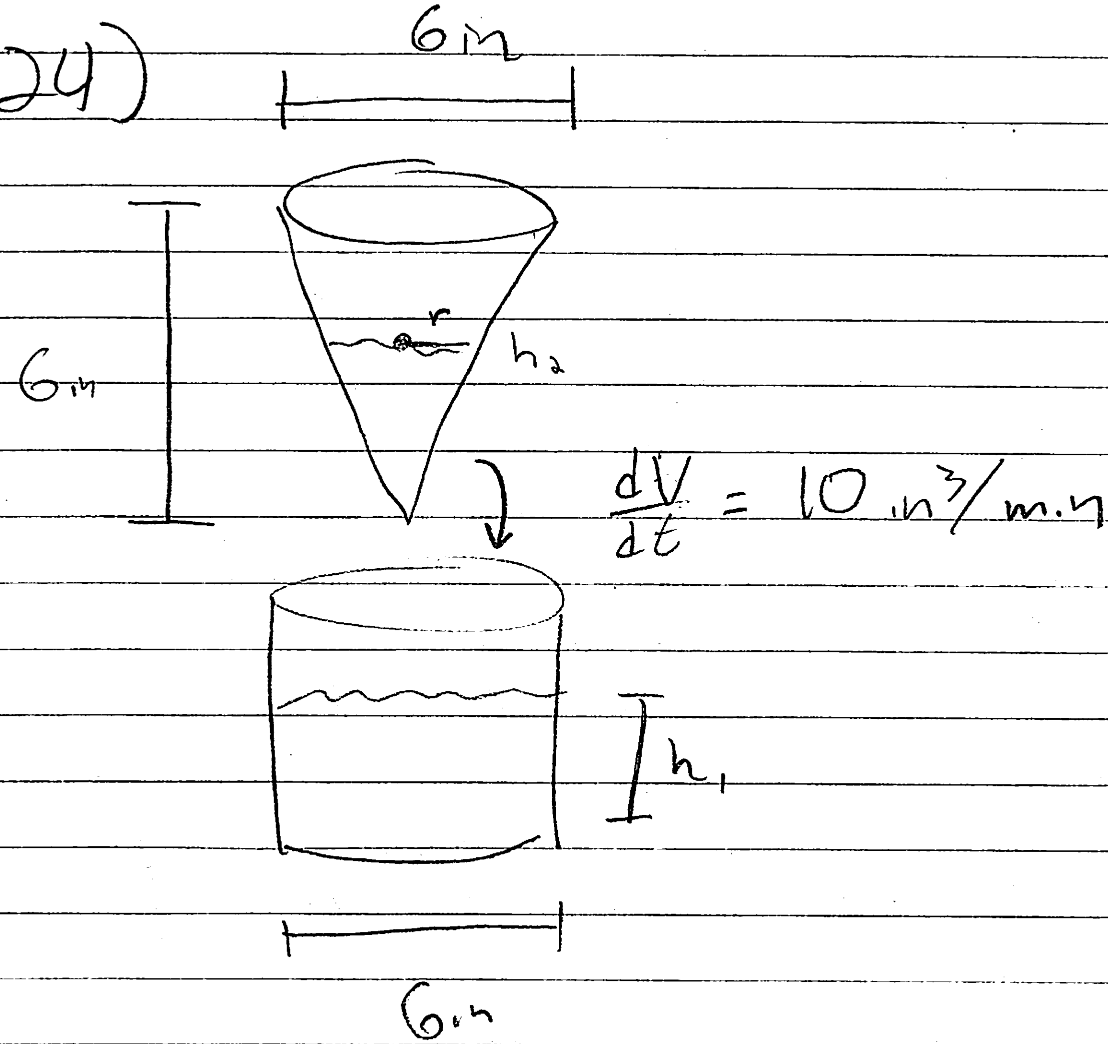
$$\rightarrow 1 \frac{\text{ft}}{\text{min}} = \frac{dr}{dt}$$

$$S = 4\pi r^2 \rightarrow \frac{dS}{dt} = 8\pi r \frac{dr}{dt}$$

$$= 8\pi (5\text{ft})(1\text{ft}/\text{min})$$

$$= 40\pi \text{ ft}^2/\text{min}$$

24)



a) what is $\frac{dh_1}{dt}$ when $h_2 = 5$ in?

h_2 is actually irrelevant here

h_1 depends only on $\frac{dV}{dt}$, which doesn't depend on h_2.

$$V = \pi(3)^2 h_1 = 9\pi h_1$$

$$\frac{dV}{dt} = 9\pi \frac{dh_1}{dt} \rightarrow \frac{dh_1}{dt} = \frac{dV}{dt}\frac{1}{9\pi}$$

$$= \frac{10}{9\pi} \text{ in/min}$$

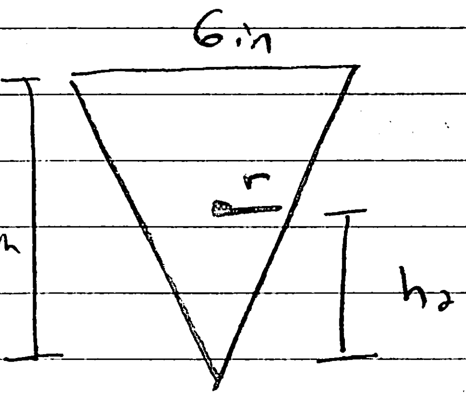
24 b) what is $\frac{dh_2}{dt}$ at this time?

$V = \frac{1}{3}\pi r^2 h_2$

$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{1}{3}\pi r^2 h_2\right) = \frac{\pi}{3}\left(2rh_2\frac{dr}{dt} + r^2\frac{dh_2}{dt}\right)$

we know $h_2 = 5$ in, $\frac{dV}{dt} = -10$ in³/min

need r, $\frac{dr}{dt}$



Similar triangles:

$\frac{6}{2r} = \frac{6}{h_2} \rightarrow 2r = h_2$

so at $h_2 = 5$, $r = 2.5$

$2r = h_2 \Longrightarrow 2\frac{dr}{dt} = \frac{dh_2}{dt} \Longrightarrow \frac{dr}{dt} = \frac{1}{2}\frac{dh_2}{dt}$

So $\frac{\pi}{3}\left(2(2.5)(5)\left(\frac{1}{2}\frac{dh_2}{dt}\right) + (2.5)^2\left(\frac{dh_2}{dt}\right)\right) = -10$

$\frac{\pi}{3}\left(\frac{25}{2}\frac{dh_2}{dt} + \frac{25}{4}\frac{dh_2}{dt}\right) = -10$

$\pi/3\left(\frac{75}{4}\frac{dh_2}{dt}\right) = -10 \rightarrow \frac{dh_2}{dt} = \frac{-120}{75\pi}$ in/min

$= -24/15\pi$ in/min