4.3

3) $f'(x) = (x-1)^2(x+2)$

a) c.p.'s are $x=1, x=-2$

b)

x	-3	-2	0	1	2
$f'(x)$	$-$	0	$+$	0	$+$

increasing $(-2,1), (1,\infty)$

decreasing $(-\infty,-2)$

c) local min at $x=-2$.

6) $f'(x) = (x-7)(x+1)(x+5)$

a) c.p.'s are $x = 7, -1, -5$

b)

x	-6	-5	-2	-1	0	7	8
$f'(x)$	$-$	0	$+$	0	$-$	0	$+$

increasing $(-5,-1), (7,\infty)$

decreasing $(-\infty,-5), (-1,7)$

c) local mins at $-5, 7$
local max at -1

11) $h(x) = -x^3 + 2x^2$

$h'(x) = -3x^2 + 4x = x(4-3x)$

$h'(x) = 0 \Rightarrow x = 0, \frac{4}{3}$

x	-1	0	1	$4/3$	2
$h'(x)$	$-$	0	$+$	0	$-$

a) Increasing $(0, 4/3)$

decreasing $(-\infty, 0), (4/3, \infty)$

b) extreme values:

$x = 0 \Rightarrow h(0) = 0$ local min

$x = 4/3 \Rightarrow h(4/3) = -\frac{64}{27} + 2 \cdot \frac{16}{9} = \frac{-64}{27} + \frac{32}{9}$

$= \frac{-64}{27} + \frac{96}{27} = \frac{32}{27}$ local max

c) no global max or min

(degree 3 polynomial)

12) $h(x) = 2x^3 - 18x$

$h'(x) = 6x^2 - 18$

$h'(x) = 0 = 6x^2 - 18 \rightarrow x^2 = 3 \Rightarrow x = \pm\sqrt{3}$

x	-2	$-\sqrt{3}$	0	$\sqrt{3}$	2
$h'(x)$	$+$	0	$-$	0	$+$

a) Increasing on $(-\infty, -\sqrt{3}), (\sqrt{3}, \infty)$

Decreasing on $(-\sqrt{3}, \sqrt{3})$

b) local extrema:

local max : $x = -\sqrt{3} \longrightarrow h(-\sqrt{3}) = 2(-\sqrt{3})^3 - 18(-\sqrt{3})$
$= 6(-\sqrt{3}) + 18\sqrt{3}$
$= 12\sqrt{3}$

local min : $x = \sqrt{3} \rightarrow h(\sqrt{3}) = 2(\sqrt{3})^3 - 18\sqrt{3}$
$= 6\sqrt{3} - 18\sqrt{3}$
$= -12\sqrt{3}$

c) global extrema: none
(3rd degree polynomial)

23) $f(x) = \dfrac{x^2-3}{x-2}, \quad x \neq 2$

$$f'(x) = \frac{2x(x-2) - (x^2-3)}{(x-2)^2} = \frac{2x^2-4x-x^2+3}{(x-2)^2}$$

$$= \frac{x^2-4x+3}{(x-2)^2} = \frac{(x-1)(x-3)}{(x-2)^2}$$

so $f'(x) = 0 \Rightarrow x = 1, 3$

$f'(x)$ und $\Rightarrow x = 2$

c.p. are $1, 2, 3$

x	0	1	1.5	2	2.5	3	4
$f'(x)$	+	0	−	0	−	0	+

a) increasing $(-\infty, 1), (3, \infty)$

decreasing $(1, 2), (2, 3)$

b) local extrema

local max $x = 1 \rightarrow f(1) = \frac{-2}{-1} = 2$

local min $x = 3 \rightarrow f(3) = 6$

c) no absolute extrema, ($\lim_{x \to \infty} f(x) = \infty$ so no max, $\lim_{x \to 2^-} f(x) = -\infty$, no min)

24) $F(x) = \frac{x^3}{3x^2+1}$

$F'(x) = \frac{3x^2(3x^2+1) - x^3(6x)}{(3x^2+1)^2} = \frac{9x^4+3x^2-6x^4}{(3x^2+1)^2}$

$= \frac{3x^2(x^2+1)}{(3x^2+1)^2}$

$\left.\begin{array}{l} F'(x) = 0 \longrightarrow x = 0 \\ F'(x) \text{ undefined never} \end{array}\right\} \longrightarrow$ c.p. is 0

x	-1	0	1
$f'(x)$	$+$	0	$+$

a) increasing on $(-\infty, 0), (0, \infty)$

decreasing never

b) local extrema: none

c) global extrema: none

57) $f(x) = \frac{x}{2} - 2\sin\frac{x}{2}$, $0 \le x \le 2\pi$

$F'(x) = \frac{1}{2} - \cos\frac{x}{2}$

$F'(x) = 0 \longrightarrow \cos\frac{x}{2} = \frac{1}{2} \Longrightarrow \frac{x}{2} = \frac{\pi}{3}, \frac{5\pi}{3}$

$\rightarrow x = \frac{2\pi}{3}, \frac{10\pi}{3}$

x	0	$2\pi/3$	π	$10\pi/3$	2π
$f'(x)$	$-$	0	$+$	0	$+$

endpoint endpoints

local extrema: min at $x = \frac{2\pi}{3}$

$f\left(\frac{2\pi}{3}\right) = \frac{\pi}{3} - 2\sin\left(\pi/6\right)$

$= \frac{\pi}{3} - \sqrt{3}$

local max at $x=0$ $f(0) = 0$

local max at $x = 2\pi$, $f(2\pi) = \pi -$

38) $f(x) = -2\cos x - \cos^2 x \quad , -\pi \le x \le \pi$

$f'(x) = 2\sin x + 2\cos x \sin x \quad = 2\sin x(1+\cos x)$

$f'(x) = 0 \implies \sin x = 0$ or $\cos x = -1$

$\Downarrow \qquad\qquad \Downarrow$

$x = 0 \qquad\qquad x = \pm\pi$

these are the endpoints anyways

x	$-\pi$	$-\pi/2$	0	$\pi/2$	π
$f'(x)$	0	$-$	0	$+$	0

local max : $x = -\pi \rightarrow f(-\pi) = 2 - 1 = 1$

local max : $x = \pi \rightarrow f(\pi) = 2 - 1 = 1$

local min : $x = 0 \rightarrow f(0) = -2 - 1 = -3$