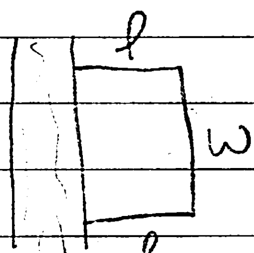
HW #4.5, 7, 8, 12, 16 (a-c), 26, 31, 36, 44, 57

7) 

$2l + w = 800$

maximize $A = lw$

$w = 800 - 2l$

$A(l) = l(800 - 2l) = 800l - 2l^2$

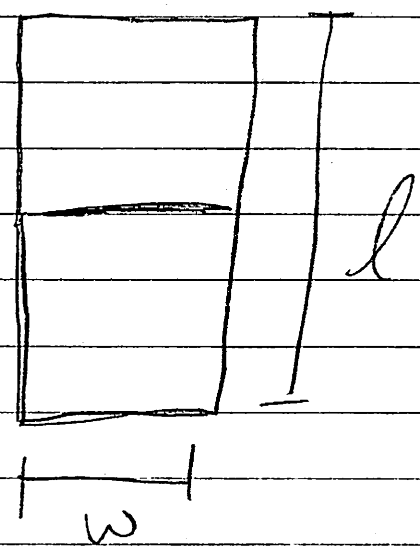
$A'(l) = 800 - 4l$

$A'(l) = 0 \Rightarrow 4l = 800 \rightarrow l = 200$

So max A is $A(200) = 160000 - 2(200)^2$
$= 160000 - 80000$
$= 80{,}000$

$l = 200$, $w = 400$

8)



$lw = 216$

$P = 2l + 3w$

minimize P

$w = \frac{216}{l}$ $\quad P(l) = 2l + 3 \cdot \frac{216}{l}$

$= 2l + 648\, l^{-1}$

$P'(l) = 2 - 648\, l^{-2}$

$P'(l) = 0 \Rightarrow 2 = \frac{648}{l^2} \Rightarrow l^2 = 324$

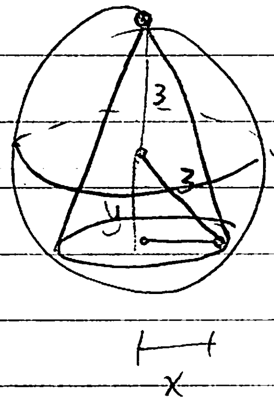
$l = \pm\sqrt{324} = \pm 18$

know 18 is our max, -18 length doesn't make sense

$l = 18 \Rightarrow \quad w = \frac{216}{18} = 12,$

$P(18) = 2 \cdot 18 + 3 \cdot \frac{216}{18} = 36 + 36 = 72$

12)



$y = \sqrt{9-x^2}$, $x = \sqrt{9-y^2}$

$V = \frac{1}{3}\pi r^2 h$

$\uparrow$ $\uparrow$

x $y+3$

$$V = \frac{1}{3}\pi x^2 (y+3)$$

Maximize V

$V(y) = \frac{1}{3}\pi \left(\sqrt{9-y^2}\right)^2 (y+3)$

$= \frac{1}{3}\pi (9-y^2)(y+3)$

$= \frac{1}{3}\pi \left(9y + 18 - y^3 - 3y^2\right)$

$V'(y) = \frac{1}{3}\pi \left(9 - 3y^2 - 6y\right)$

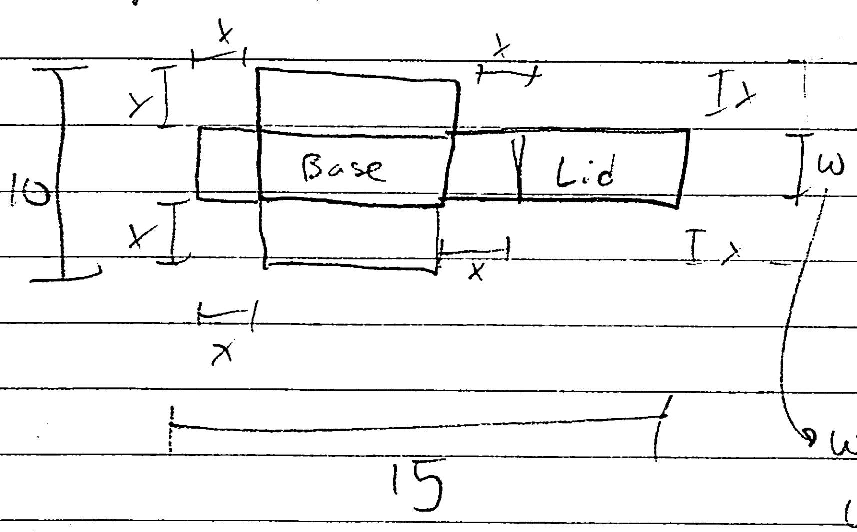
$= -\pi\left(y^2 + 2y - 3\right) = -\pi(y-1)(y+3)$

$V'(y) = 0 \rightarrow y = 1, -3$ $y = -3 \rightarrow$ there is no cone

so $y=1$ gives max

$V(1) = \frac{1}{3}\pi(9-1)(1+3) = \frac{1}{3}\pi \cdot 8 \cdot 4 = \frac{32\pi}{3}$

16)



a) $V(x) = x\left(\frac{15-2x}{2}\right)(10-2x)$

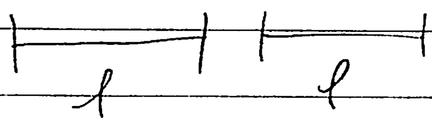
$= x(15-2x)(5-x)$

$= x(75-15x-10x+2x^2)$

$= 2x^3 - 25x^2 + 75x$

$w + 2x = 10$

$w = 10 - 2x$



b) Domain $(0,5)$

$l + l + x + x = 15$

$\rightarrow l = \frac{15-2x}{2}$

c) —

d) maximize $V(x)$. $V'(x) = 6x^2 - 50x + 75$

$V'(x) = 0 \longrightarrow x = \frac{50 \pm \sqrt{50^2 - 4(6)(75)}}{-12}$

$= \frac{50 \pm \sqrt{2500 - 1800}}{-12} = \frac{50 \pm \sqrt{700}}{12} = \frac{50 \pm 10\sqrt{7}}{12}$

$= \frac{25}{6} \pm \frac{5\sqrt{7}}{6}$ $\quad \frac{25}{6} + \frac{5\sqrt{7}}{6}$ out of $(0,5)$

So $x = \frac{25}{6} - \frac{5\sqrt{7}}{6}$ gives max.

$V\left(\frac{25}{6} - \frac{5\sqrt{7}}{6}\right)$ is max, whatever that is

26)

b) $2(x+y)=36 \quad x+y=18 \quad y=18-x$

$V=\pi x^2 y = \pi x^2(18-x) = 18\pi x^2 - \pi x^3$

$V'(x) = 36\pi x - 3\pi x^2 = \pi x(36-3x)$

$= 3\pi x(12-x)$

c.p's $x=0, \underline{\underline{12}}$ max

$x=12, \ y=6$

a) $2(x+y)=36, \quad x+y=18 \rightarrow y=18-x$

$V=\pi r^2 h$

$h = y$, $x=2\pi r \rightarrow r=\frac{x}{2\pi}$

$V=\pi \frac{x^2}{4\pi^2} y = \frac{x^2}{4\pi}(18-x) = \frac{9}{2\pi}x^2 - \frac{x^3}{4\pi}$

$V'(x) = \frac{9}{\pi}x - \frac{3}{4\pi}x^2 = \frac{3x}{\pi}\left(3-\frac{x}{4}\right)$

$V'(x)=0 \Longrightarrow x=0, 12$

same, $x=12, \ y=6$

31) $s = -16t^2 + 96t + 112$

a) $v = s' = -32t + 96$

$v(0) = 96$ ft/sec

b) maximize $s(t)$

$v(t) = 0 \Rightarrow 96 = 32t \rightarrow t = 3$

$s(3) = -16(3)^2 + 96(3) + 112$

$= -16 \cdot 9 + 96 \cdot 3 + 112$

$= -144 + 288 + 112$

$= 256$ ft

c) $s(t) = 0 = -16(t^2 - 6t - 7) = -16(t-7)(t+1)$

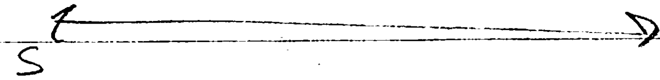
$\rightarrow t = -1,\ t = 7$

negative time not in model

$s'(7) = -32(7) + 96 = -224 + 96 = -128$

-128 ft/sec down

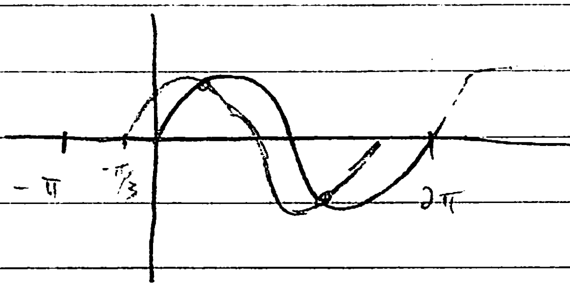
36)

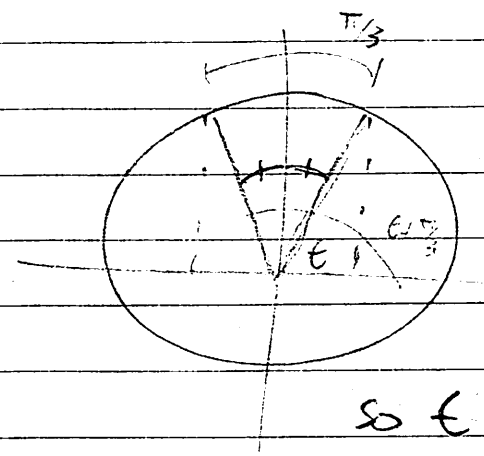


$S_1 = \sin t$

$S_2 = \sin(t + \pi/3)$

a) solve $\sin(t) = \sin(t + \pi/3)$





so $t + \pi/6 = \pi/2$

$\rightarrow t = \pi/2 - \pi/6$

$= \pi/3$

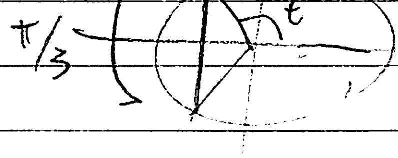
Similarly, $t = 4\pi/3$

b) distance between particles is $S_1 - S_2 = d(t)$

$= \sin t - \sin(t + \pi/3)$

maximize $d(t)$. $d'(t) = \cos t - \cos(t + \pi/3)$

$d'(t) = 0 \longrightarrow \cos t = \cos(t + \pi/3)$



$\pi - \pi/6 = t = 5\pi/6$

$d(5\pi/6) = \sin(5\pi/6) - \sin(5\pi/6 + \pi/3)$

$= \frac{1}{2} - \sin\left(\frac{7\pi}{6}\right)$

$= \frac{1}{2} - \left(-\frac{1}{2}\right) = 1$

c) When is $d(t)$ changing fastest?

i.e when is $d'(t)$ max?

i.e when is $d''(t) = 0$?

$d''(t) = \frac{d}{dt}\left(\cos t - \cos(t + \pi/3)\right)$

$= -\sin t + \sin(t + \pi/3)$

$d''(t) = 0 \implies \sin t = \sin(t + \pi/3)$

$\implies t = \pi/3,\ 4\pi/3$

44)

$m(x) = (50+x)(200-2x)$

$= 10000 - 100x + 200x - 2x^2 = -2x^2 + 100x + 10000$

$c(x) = 6000 + 32(50+x)$

$= 7600 + 32x$

$p(x) = m(x) - c(x) = -2x^2 + 100x + 10000 - 32x - 7600$

$= -2x^2 + 68x + 2400$

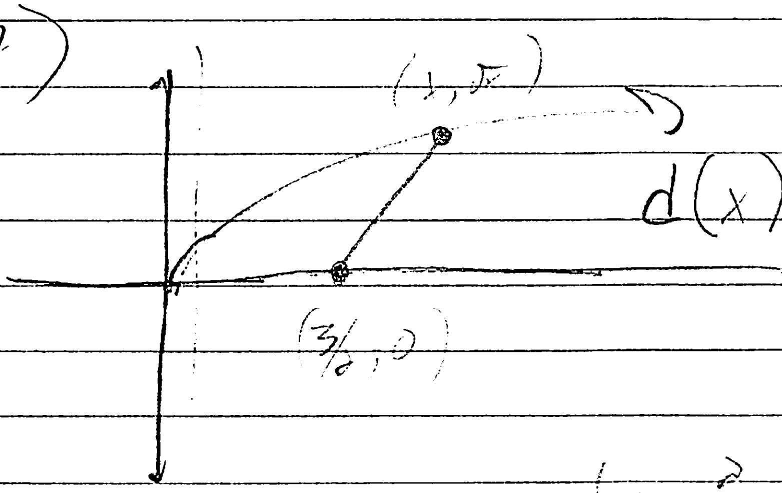
maximize profit

$p'(x) = -4x + 68 \qquad p'(x) = 0$

$\Rightarrow 68 = 4x \longrightarrow x = 17$

200 67 would max profit

57)



$d(x) = \sqrt{(x - 3/2)^2 + (\sqrt{x})^2}$

tip: minimize $d(x)^2 = (x - 3/2)^2 + x$

$= x^2 - 3x + 9/4 + x$

$= x^2 - 2x + 9/4$

$[d(x)^2]' = 2x - 2 \qquad$ min at $x = 1$

$$d(1) = \sqrt{(1-3/2)^2+1} = \sqrt{(-\tfrac{1}{2})^2+1} = \sqrt{5/4}$$

minimal distance

b)