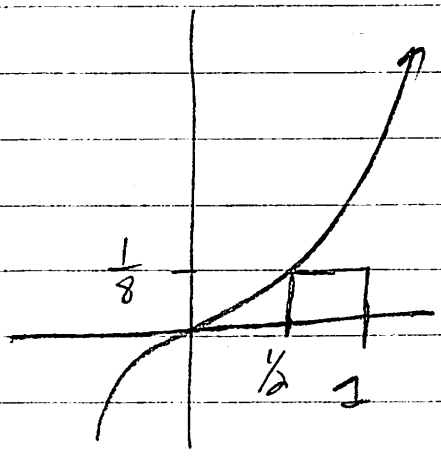
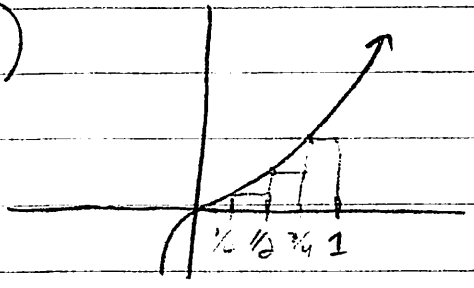
5.1 & 5.2

5.1 # 2 $f(x) = x^3$ over $[0,1]$



a) $A = 0 \times \frac{1}{2} + \frac{1}{8} \times \frac{1}{2} = \boxed{1/16}$

b)



$A = \frac{1}{4}\left(0 + \left(\frac{1}{4}\right)^3 + \left(\frac{1}{2}\right)^3 + \left(\frac{3}{4}\right)^3\right)$

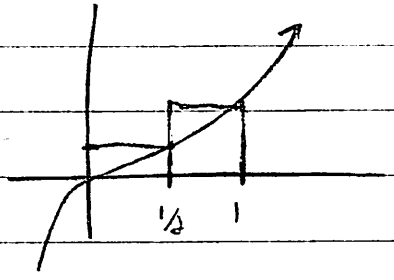
$= \frac{1}{4}\left(\left(\frac{1}{4}\right)^3 + \frac{1}{2}\left(\frac{1}{4}\right) + 27\left(\frac{1}{4}\right)^3\right)$

$= \frac{1}{16}\left(\left(\frac{1}{4}\right)^2 + \frac{1}{2} + 27\left(\frac{1}{4}\right)^2\right)$

$= \frac{1}{16}\left(\frac{1}{16} + \frac{1}{2} + \frac{27}{16}\right) = \frac{1}{16}\left(\frac{9}{4}\right)$

$= \boxed{9/64}$

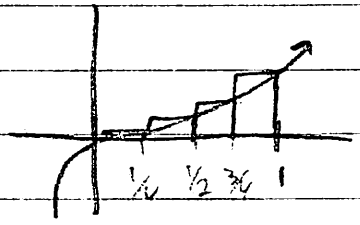
c)



$A = \frac{1}{2}\left(\left(\frac{1}{2}\right)^3 + 1^3\right)$

$= \frac{1}{2}\left(\frac{1}{8} + 1\right) = \frac{1}{2}\frac{9}{8} = \boxed{\frac{9}{16}}$

d)



$A = \frac{1}{4}\left(\left(\frac{1}{4}\right)^3 + \left(\frac{1}{2}\right)^3 + \left(\frac{3}{4}\right)^3 + 1^3\right)$

$= \frac{1}{4}\left(\frac{1}{64} + \frac{1}{8} + \frac{27}{64} + 1\right) = \frac{1}{4}\left(\frac{25}{16}\right) = \boxed{\frac{25}{64}}$

5.2 #1

$$\sum_{k=1}^{2} \frac{6k}{k+1} = \frac{6}{2} + \frac{12}{3} = 3 + 4 = 7$$

#2

$$\sum_{k=1}^{3} \frac{k-1}{k} = \frac{0}{1} + \frac{1}{2} + \frac{2}{3} = \frac{3}{6} + \frac{4}{6} = \frac{7}{6}$$

8)

a) $$\sum_{k=1}^{6} (-2)^{k-1} = (-2)^0 + (-2)^1 + (-2)^2 + (-2)^3 + (-2)^4 + (-2)^5$$

$$= 1 - 2 + 4 - 8 + 16 - 32$$

yes

b) $$\sum_{k=0}^{5} (-1)^k 2^k = \sum_{k=0}^{5} (-2)^k$$

$$= (-2)^0 + (-2)^1 + (-2)^2 + (-2)^3 + (-2)^4 + (-2)^5$$

$$= 1 - 2 + 4 - 8 + 16 - 32$$ ✓

yes

c) $$\sum_{k=-2}^{3} (-1)^{k+1} 2^{k+2} = \sum_{k=-2}^{3} (-2)^{k+1} \cdot 2$$

$$= 2 \sum_{k=-2}^{3} (-2)^{k+1}$$

$$2\left((-2)^{-1} + (-2)^0 + (-2)^1 + (-2)^2 + (-2)^3 + (-2)^4\right)$$

$$= -1 + 2 - 4 + 8 - 16 + 32, \quad \text{so no}$$

10) a $\sum_{k=1}^{4} (k-1)^2 = 0^2+1^2+2^2+3^2$

$= 1+4+9 = 14$

b $\sum_{k=-1}^{3} (k+1)^2 = 0^2+1^2+2^2+3^2+4^2$

$= 1+4+9+16 = 30$

c $\sum_{k=-3}^{-1} k^2 = (-3)^2+(-2)^2+(-1)^2$

$= 9+4+1 = 14$

b isn't the same

13) $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16} = \sum_{k=1}^{4} \left(\frac{1}{2}\right)^k$

14) $2+4+6+8+10$

$= \sum_{k=1}^{5} 2k$