

5.4 #3, 4, 9, 10, 19, 25, 28, 33, 34, 38, 44

$$\begin{aligned} 3) \int_0^4 \left(3x - \frac{x^3}{4} \right) dx \\ = \left. \frac{3x^2}{2} - \frac{x^4}{16} \right|_0^4 = \frac{3}{2}(4)^2 - \frac{1}{16}(4)^4 - \frac{3 \cdot 0^2}{2} - \frac{0^4}{16} \\ = \frac{3 \cdot 16}{2} - \frac{(16)^2}{16} \\ = 24 - 16 = \boxed{8} \end{aligned}$$

$$\begin{aligned} 4) \int_{-2}^2 x^3 - 2x + 3 dx \\ = \left. \frac{x^4}{4} - x^2 + 3x \right|_{-2}^2 = \frac{2^4}{4} - 2^2 + 3(2) - \left[\frac{(-2)^4}{4} - (-2)^2 + 3(-2) \right] \\ = \frac{16}{4} - 4 + 6 - \left[\frac{16}{4} - 4 - 6 \right] = \boxed{12} \end{aligned}$$

$$\begin{aligned} 9) \int_0^\pi \sin(x) dx = -\cos x \Big|_0^\pi \\ = -(\cos(\pi)) - (-\cos(0)) \\ = -(-1) - (-1) = \boxed{2} \end{aligned}$$

$$\begin{aligned} 10) \int_0^\pi 1 + \cos x dx = x + \sin x \Big|_0^\pi \\ = \pi + \sin \pi - (0 + \sin 0) \\ = \pi + 0 - 0 = \boxed{\pi} \end{aligned}$$

$$19) \int_1^{-1} (r+1)^2 dr$$

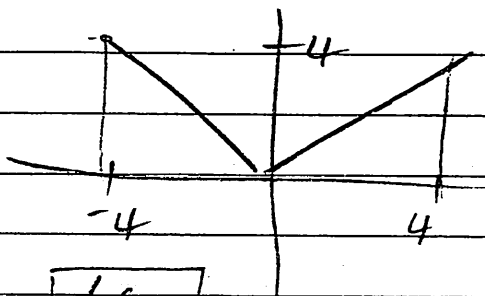
$$= \int_1^{-1} r^2 + 2r + 1 dr$$

$$= \left[\frac{r^3}{3} + r^2 + r \right]_1^{-1}$$

$$= \frac{(-1)^3}{3} + (-1)^2 + 1 - \left(\frac{1^3}{3} + 1 + 1 \right)$$

$$= -\frac{1}{3} - \frac{1}{3} - 2 = \boxed{-\frac{8}{3}}$$

$$25) \int_{-4}^4 |x| dx$$



$$= 2 \left(\frac{1}{2} 4 \cdot 4 \right) = \boxed{16}$$

$$28) \frac{d}{dx} \int_1^{\sin x} 3t^2 dt$$

$$a) \int_1^{\sin x} 3t^2 dt = t^3 \Big|_1^{\sin x} \\ = (\sin x)^3 - 1$$

$$\frac{d}{dx} \int_1^{\sin x} 3t^2 dt = \frac{d}{dx} (\sin^3 x - 1) \\ = 3 \sin^2 x \cdot \left(\frac{d}{dx} \sin x \right) \\ = \boxed{3 \sin^2 x \cos x}$$

$$b) \frac{d}{dx} \int_1^{\sin x} 3t^2 dt \\ = 3 \sin^2 x \cdot \frac{d}{dx} \sin x \\ = \boxed{3 \sin^2 x \cos x}$$

$$33) y = \int_{\sqrt{x}}^0 \sin(t^2) dt = - \int_0^{\sqrt{x}} \sin(t^2) dt$$

$$\frac{dy}{dx} = - \sin((\sqrt{x})^2) \cdot \frac{d}{dx} \sqrt{x} \\ = - \sin(x) \frac{x^{-1/2}}{2} = \boxed{\frac{-\sin x}{2\sqrt{x}}}$$

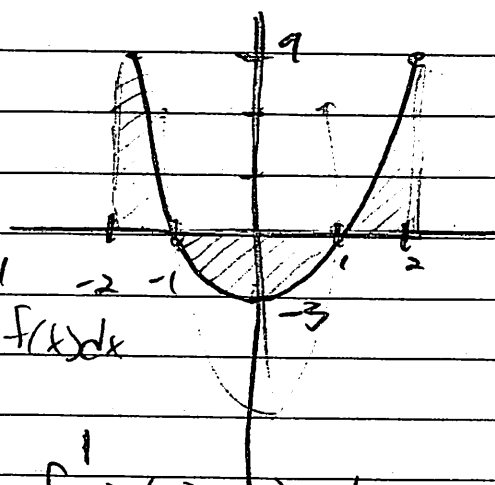
$$34) \quad y = \int_0^{x^2} \cos \sqrt{t} \, dt$$

$$\frac{dy}{dx} = \cos \sqrt{x^2} \cdot \frac{d}{dx}(x^2)$$

$$= \cos(x) \cdot 2x = \boxed{2x \cos x}$$

$$38) \quad y = 3x^2 - 3, \quad -2 \leq x \leq 2$$

$$= 3(x+1)(x-1)$$



$$\text{Total area: } 2 \int_1^2 f(x) dx - \int_{-1}^1 f(x) dx$$

$$= 2 \int_1^2 3(x^2 - 1) dx - \int_{-1}^1 3(x^2 - 1) dx$$

$$= 6 \int_1^2 x^2 - 1 dx - 3 \int_{-1}^1 x^2 - 1 dx$$

$$= 6 \left(\frac{x^3}{3} - x \right) \Big|_1^2 - 3 \left(\frac{x^3}{3} - x \right) \Big|_{-1}^1$$

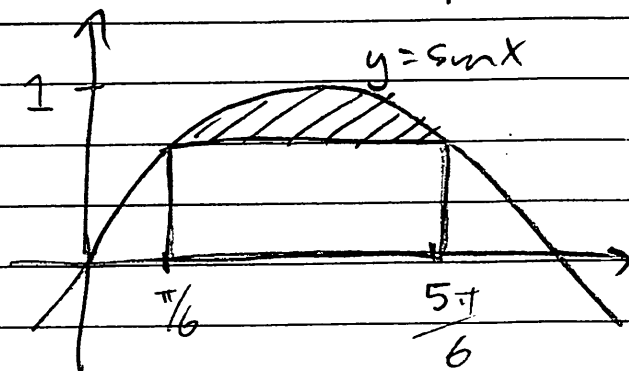
$$= 6 \left(\frac{8}{3} - 2 - \left(\frac{1}{3} - 1 \right) \right) - 3 \left(\frac{1}{3} - 1 - \left(-\frac{1}{3} + 1 \right) \right)$$

$$= 6 \left(\frac{7}{3} - 1 \right) - 3 \left(\frac{2}{3} - 2 \right) = 14 - 6 - 2 + 6$$

$$= \boxed{12}$$

$$l = \frac{5\pi}{6} - \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$$

44)



$$| w = \sin \frac{\pi}{6} = \frac{1}{2}$$

$$\text{Area} = \int_{\pi/6}^{5\pi/6} \sin x \, dx - \frac{2\pi}{3} \cdot \frac{1}{2}$$

$$= -\cos x \Big|_{\pi/6}^{5\pi/6} - \frac{\pi}{3}$$

$$= -\cos\left(\frac{5\pi}{6}\right) - \left(-\cos\frac{\pi}{6}\right) - \frac{\pi}{3}$$

$$= -\left(-\frac{\sqrt{3}}{2}\right) - \left(-\frac{\sqrt{3}}{2}\right) - \frac{\pi}{3}$$

$$= \boxed{\sqrt{3} - \frac{\pi}{3}}$$

