

Math 241 Week 6 Quiz

Name: Answer Key

1. Find the intervals of increasing/decreasing and intervals of concavity, local extrema and inflection points of the function $f(x) = x^3 - 6x^2 + 9x$. Then sketch its graph based on this information.

$$f(x) = x(x^2 - 6x + 9) = x(x-3)^2 \quad \text{zeros at } x=0, 3$$

$$f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x-3)(x-1)$$

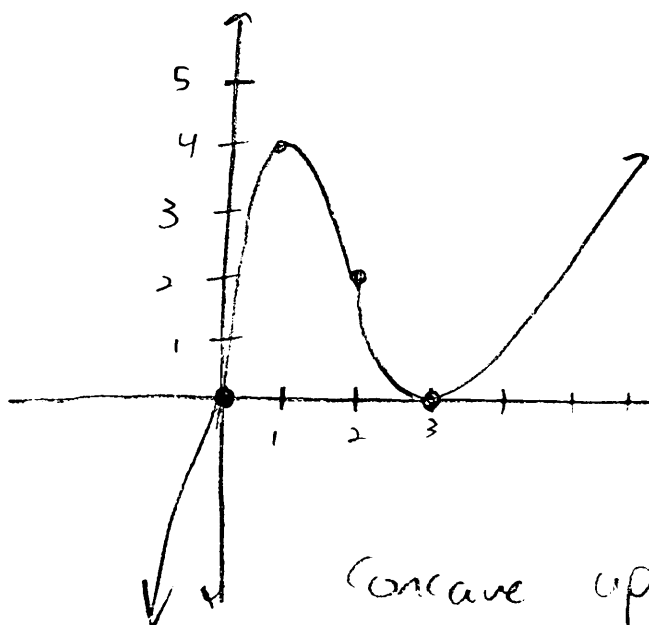
critical points: $x=3, x=1$

$$f''(x) = 6x - 12$$

inflection point: $x=2$ $f(2) = 2(2-3)^2 = 2$

$f''(1) = -6 < 0 \rightarrow x=1$ gives local max
 $f(1) = 1(1-3)^2 = 4$

$f''(3) = 6 > 0 \rightarrow x=3$ gives local min
 $f(3) = 3(3-3)^2 = 0$

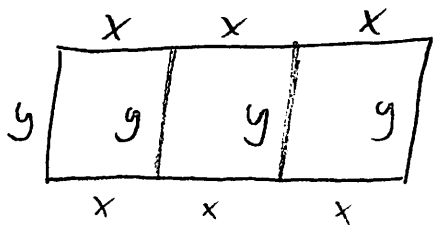


Increasing on
 $(-\infty, 1), (3, \infty)$

Decreasing on
 $(1, 3)$

concave up on $(2, \infty)$
 concave down on $(-\infty, 2)$

2. A farmer has 240 ft of fence to make 3 side by side identical rectangular pig pens. What dimensions would maximize the total area?



$$4y + 6x = 240 \rightarrow 2y + 3x = 120$$

$$A = 3xy$$

maximize A . Write A in terms of x or y

$$A(x) : \quad 2y + 3x = 120 \rightarrow 2y = 120 - 3x$$

$$y = 60 - \frac{3}{2}x$$

$$A(x) = 3x \left(60 - \frac{3}{2}x \right)$$

$$= 180x - \frac{9}{2}x^2$$

$$A'(x) = 180 - 9x \quad A'(x) = 0 \Rightarrow 9x = 180$$

$$x = 20$$

$$\Rightarrow y = 60 - \frac{3}{2}(20) = 60 - 30 = 30$$

$$x = 20, y = 30$$

$$A(y) : \quad 2y + 3x = 120 \rightarrow 3x = 120 - 2y$$

$$x = 40 - \frac{2}{3}y$$

$$A(y) = 3 \left(40 - \frac{2}{3}y \right) y = 120y - 2y^2$$

$$A'(y) = 120 - 4y \quad A'(y) = 0 \Rightarrow y = 30$$

$$\Rightarrow x = 40 - \frac{2}{3}(30) = 40 - 20 = 20$$