Constant-rate Voting

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Let $x(t), y(t)$ be the number of votes for the two candidates as a function of time. Since voting at the beginning is a bit random, we'll pick the point at which it stabilizes to be $t = 0$. We'll label $x(0) = x_0$ and $y(0) = y_0$.

Assume that from $t = 0$ and onwards x generates votes at a constant rate r_x and y at a constant rate r_y.
So

$$x(t) = x_0 + r_x \cdot t$$

and

$$y(t) = y_0 + r_y \cdot t$$

Let $p_x(t)$ be the percentage of x's votes as a function of time.
Then

$$p_x(t) = \frac{x}{x+y} = \frac{x_0 + r_x \cdot t}{x_0 + y_0 + (r_x + r_y) \cdot t}$$

If we take the limit as $t \to \infty$, then we get a horizontal asymptote of

$$\lim_{t \to \infty} p_x(t) = \frac{r_x}{r_x + r_y}$$

This means that given a constant voting rate for each side, the vote will tend to an equilibrium as time goes on. We'll label this equilibrium e_x.

As an example: If Dr. Haran gets 100k votes an hour, and O'Brien gets 40k votes an hour than the resulting equilibrium would be 71.43% in Dr. Haran's favor.

Define F_x to be the voting force of x with respect to y i.e. $F_x = \frac{r_x}{r_y}$.
Then

$$e_x = \frac{r_x}{r_x + r_y} = \frac{F_x}{F_x + 1}$$

or, in reverse:

$$F_x = \frac{e_x}{1 - e_x}$$

So, given a winning percentage of 81.83% for Dr. Haran, this means that in a simplified model of the vote, Dr. Haran's followers were voting at about 4.5 times the rate of O'Brien's followers.