A new life reinsurance product

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This document describes the key ideas of a new product for life reinsurance. The first section explains what is the one-year risk in classical life insurance and how is it possible to protect it. However, we will see that this protection is difficult to implement in practice. That's why we will make some simplifications in order to obtain a realistic reinsurance product (second section). Finally, the third section will give all the advantages and disadvantages of this new product.

Keywords : life reinsurance, high diversification effect, actuarial and financial option, aggregate excess of loss and capital optimization through Solvency II.

1 One-year risk in life insurance

In this section, we will answer the following question : what is the one-year risk for an insurance company on a classical life portfolio ? To answer to this question, let's start to introduce some notations.

First, since we are interested in a one-year risk, we will consider the time interval $[0, 1]$ where time $t = 0$ denotes the current time and time $t = 1$ corresponds to the next year. Then, since we are working on a life portfolio, we will denote Γ the set of policyholders belonging to this portfolio at time $t = 0$. For a policyholder $i \in \Gamma$, we will also use the following notations :

— $R_i(t)$ is his reserve at time t (in the first order basis) ;

— τ_i is the random variable describing his remaining lifetime from $t = 0$;

— $c_i(t)$ is the amount to pay if he dies at time t ;

— $B_i(t)$ is a deterministic function giving the cumulative benefits to pay over the period $[0, t]$ if he is alive at time t ;

— $\Pi_i(t)$ is a deterministic function giving the cumulative premium to receive over the period $[0, t]$ if he is alive at time t.

Finally, we will denote $F(t)$ the value of the fund at time t in which reserves and premiums from Γ are invested.

Hence, the one-year risk for the life portfolio Γ comes from the fact that the initial reserves and all the premiums received over $[0, 1]$ are not sufficient to cover all the life and death benefits over $[0, 1]$ plus the next year reserves (for policyholders alive at time $t = 1$), over the global portfolio Γ and by taking into account the financial investment in the fund F. Mathematically [1], it means that the risk comes from the fact that the aggregate one-year cash-inflows

$$\sum_{i\in\Gamma} CF_i^{in}(1,\omega) = \sum_{i\in\Gamma}\left(R_i(0)\frac{F(1,\omega)}{F(0)} + \int_0^1 I_{\{\tau_i(\omega)>t\}}\frac{F(1,\omega)}{F(t,\omega)}d\Pi_i(t)\right)$$

are smaller than the aggregate one-year cash-outflows

$$\sum_{i\in\Gamma} CF_i^{out}(1,\omega) = \sum_{i\in\Gamma}\left(c_i(\tau_i(\omega))I_{\{\tau_i(\omega)<1\}}\frac{F(1,\omega)}{F(\tau_i(\omega),\omega)} + \int_0^1 I_{\{\tau_i(\omega)>t\}}\frac{F(1,\omega)}{F(t,\omega)}dB_i(t) + I_{\{\tau_i(\omega)>1\}}R_i(1,\omega)\right).$$

1. for the following, we will denote I_A the indicator function which is equal to 1 if condition A is satisfied and 0 otherwise, and we will indicate by an "ω" the elements that are stochastic in order to have a clear view of what is deterministic and what is stochastic in the formulas

By consequence, the insurer can protect its life portfolio Γ over a one-year horizon by buying at time $t=0$ a product that gives the following payoff at time $t=1$:

$$\varphi(1,\omega;\Gamma)=\left(\sum_{i\in\Gamma}CF_i^{out}(1,\omega)-\sum_{i\in\Gamma}CF_i^{in}(1,\omega)\right)_+ .$$

We can interpret this product as an european option with a maturity of one year, mixing financial and actuarial risks and with a set Γ which can be regarded as a diversification parameter. Moreover, this option is very interesting in the context of Solvency II. Indeed, regardless the scenario that occurs during the year, the insurance company will always be able to pay all its benefits and to constitute all its reserves at the end of the year. It means that such a product can replace the mortality, longevity, life CAT and life market risk by a simple default risk ! We can therefore see this product as an option to transfer the risk of some Solvency II modules to the default module. An additional advantage for the insurer is the diversification through the Γ parameter. This diversification allows an optimal protection at the lowest cost.

We can therefore consider this option as a reinsurance product which is able to protect the portfolio Γ against all classical life risks and in a more optimal way than traditional life reinsurance products which are currently used. However, this option is very complex and seems too difficult to price. The complexity comes from the following elements :

— the fund F is difficult to model and involves a risk neutral pricing ;

— the next year's reserves $R_i(1)$ are also difficult to model since they depend on future update of actuarial assumptions ;

— we are working on a continuous basis.

That's why in the next section, we will make some simplifications in order to obtain a realistic reinsurance product (i.e. easy to price and easy to monitor).

2 To a realistic life reinsurance product

2.1 Simplified model

The first simplification consists to assume that the fund F is not stochastic but gives a deterministic and constant return. It means that we can replace the stochastic ratios $\frac{F(1,\omega)}{F(t,\omega)}$ by the deterministic factor $(1+r)^{1-t}$ where r is the expected yearly return of the fund over $[0,1]$.

The second simplification is to replace the reserve of policyholder i at time $t=1$ by a deterministic estimation. It means that we replace $R_i(1,\omega)$ by $\hat{R}_i(1)$.

And finally, the last simplification consists to assume that all payments (premiums and benefits) are paid at the end of each month. In other words, we will replace the continuous interval $[0,1]$ by the discrete set $\{\frac{k}{12}\mid k=1,\ldots,12\}$.

Under these three assumptions, it means that $CF_i^{in}(1,\omega)$ and $CF_i^{out}(1,\omega)$ will respectively become

$$\overline{CF}_i^{in}(1,\omega)=R_i(0)(1+r)+\sum_{k=1}^{12}\overline{\pi}_i\left(\frac{k}{12}\right)(1+r)^{1-\frac{k}{12}}I_{\{\overline{\tau}_i(\omega)>\frac{k}{12}\}}$$

and

$$\overline{CF}_i^{out}(1,\omega)=\overline{c}_i(\overline{\tau}_i(\omega))(1+r)^{1-\overline{\tau}_i(\omega)}I_{\{\overline{\tau}_i(\omega)\leq 1\}}+\sum_{k=1}^{12}\overline{b}_i\left(\frac{k}{12}\right)(1+r)^{1-\frac{k}{12}}I_{\{\overline{\tau}_i(\omega)>\frac{k}{12}\}}+\hat{R}_i(1)I_{\{\overline{\tau}_i(\omega)>1\}}$$

where

- $\overline{\tau}_i$ is a discrete random variable taking the value $\frac{k}{12}$ if policyholder i dies on $\left(\frac{k-1}{12}, \frac{k}{12}\right)$;
- $\overline{c}_i(\frac{k}{12})$ is the amount to pay if policyholder i dies on $\left(\frac{k-1}{12}, \frac{k}{12}\right)$;
- $\overline{b}_i(\frac{k}{12})$ is the benefit to pay if policyholder i is alive at time $\frac{k}{12}$;
- $\overline{\pi}_i(\frac{k}{12})$ is the premium to receive if policyholder i is alive at time $\frac{k}{12}$.

And the payoff paid at time $t = 1$ is therefore given by

$$\bar{\varphi}(1, \omega; \Gamma, K) = \left(\sum_{i \in \Gamma} \overline{CF}_i^{out}(1, \omega) - \sum_{i \in \Gamma} \overline{CF}_i^{in}(1, \omega) - K \right)_+$$

where $K \geq 0$ is an annual aggregate deductible.

We can now interpret this product in two different ways. In one hand, we can say that this is an actuarial option (i.e. comporting only actuarial risks) with a maturity of one year and diversification through the set Γ. And in the other hand, we can say that this product looks like to a generalization of multiline aggregate excess of loss (with an unlimited cover and a stochastic priority equal to $\sum_{i\in\Gamma} \overline{CF}_i^{in}(1,\omega)+K$) where the only uncertainty comes from the random vector $(\overline{\tau}_1, \ldots, \overline{\tau}_{|\Gamma|})$.

2.2 Administration

In this section, we will discuss the simple administration of this product. The inputs required for this product are the interest rate r and the following table :

i	$R_i(0)$	$\hat{R}_i(1)$	$\overline{\pi}_i(\frac{1}{12})$	$\ldots$	$\overline{\pi}_i(\frac{12}{12})$	$\overline{b}_i(\frac{1}{12})$	$\ldots$	$\overline{b}_i(\frac{12}{12})$	$\overline{c}_i(\frac{1}{12})$	$\ldots$	$\overline{c}_i(\frac{12}{12})$
$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$		$\vdots$	$\vdots$		$\vdots$

Since the product takes into account 13 different cases for each policyholder :

$$\overline{\tau}_i = \frac{1}{12} \quad , \quad \overline{\tau}_i = \frac{2}{12} \quad , \quad \ldots \quad , \quad \overline{\tau}_i = \frac{11}{12} \quad , \quad \overline{\tau}_i = \frac{12}{12} \quad \text{and} \quad \overline{\tau}_i > 1,$$

we can build the table of conditional cash-flows for each policyholder :

	$\overline{CF}_i^{in}$ given				$\overline{CF}_i^{out}$ given			
i	$\overline{\tau}_i = \frac{1}{12}$	$\ldots$	$\overline{\tau}_i = 1$	$\overline{\tau}_i > 1$	$\overline{\tau}_i = \frac{1}{12}$	$\ldots$	$\overline{\tau}_i = 1$	$\overline{\tau}_i > 1$
$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$

where

$$\overline{CF}_i^{in} = \begin{cases} R_i(0)(1+r) + \sum_{k=1}^{l-1} \overline{\pi}_i(\frac{k}{12})(1+r)^{1-\frac{k}{12}} & \text{given} \quad \overline{\tau}_i = \frac{l}{12} \\ R_i(0)(1+r) + \sum_{k=1}^{12} \overline{\pi}_i(\frac{k}{12})(1+r)^{1-\frac{k}{12}} & \text{given} \quad \overline{\tau}_i > 1 \end{cases}$$

and

$$\overline{CF}_i^{out} = \begin{cases} \overline{c}_i(\frac{l}{12})(1+r)^{1-\frac{l}{12}} + \sum_{k=1}^{l-1} \overline{b}_i(\frac{k}{12})(1+r)^{1-\frac{k}{12}} & \text{given} \quad \overline{\tau}_i = \frac{l}{12} \\ \sum_{k=1}^{12} \overline{b}_i(\frac{k}{12})(1+r)^{1-\frac{k}{12}} + \hat{R}_i(1) & \text{given} \quad \overline{\tau}_i > 1. \end{cases}$$

At the end of the year, we know in which of the 13 cases we are for each policyholder. Hence, we can easily compute the payoff of the product by using the table of conditional cash-flows (very simple administration).

2.3 Pricing

Because this product works on a one-year horizon, we are only interested in the survival probabilities for next year. This is a big advantage for the reinsurers since they don't need to project the survival probabilities on a long term horizon. That's why we will consider the random vector $(q_1, \ldots, q_{|\Gamma|})$ where the random variable q_i corresponds to the one-year probability of death of policyholder i. Note that these random variables are not independent if we consider CAT events (including pandemics which represent the biggest risk). Hence, we can define the death indicator I_i for policyholder i such that

$$I_i|q_i \sim Be(q_i) \quad \text{and} \quad I_i|q_i \quad \text{are independent for all } i \in \Gamma.$$

By consequence, we can compute the payoff distribution through a Monte Carlo method applied on the following algorithm :

1. Simulate the random vector $(q_1, \ldots, q_{|\Gamma|})$.
2. Simulate which policyholders will die on $[0, 1]$ by using the death indicators I_i.
3. For each of these policyholders, simulate the month of death.
4. Deduce the payoff of the product by using the table of conditional cash-flows.

Knowing this distribution, the reinsurer can apply a traditional pricing method.

3 Conclusion

To conclude, let's discuss the advantages and disadvantages of this new product for life reinsurance. The advantages are the following :

- Scope : the insurer can put in the set Γ every kind of life contracts (pure endowment, annuity and term insurance). Hence, he can buy only one single reinsurance treaty to protect his global life portfolio.
- Diversification effect : the insurer benefits from a very high diversification effect by buying this product. It means that the product offers a very efficient cover for a very small price. For an international insurance company, we can even imagine that this set Γ be extended to the group level (instead to be limited at an entity level) in order to benefit from a better diversification.
- Administration : this product is very easy to monitor.
- Natural generalization of aggregate excess of loss used in non-life reinsurance.
- Easily understandable by using a cash inflow/outflow interpretation.
- One-year risk : the risk for the reinsurer is really limited since the financial risk is not taken into account and the life risk is only based on the next year (not on a long term horizon). It means that the only uncertainty for the reinsurer comes from the one-year probabilities of death.
- Solvency II context : this product is able to replace the mortality, longevity and CAT life risk by the default risk of reinsurers. The insurance company can therefore perform a capital optimization through this product.

However, this product is more difficult to price than traditional life reinsurance products. The main difficulty comes from the modelisation of the random vector $(q_1, \ldots, q_{|\Gamma|})$, especially if the reinsurer wants to take into account CAT risk in an explicit way. Moreover, the disadvantages for the insurers are the deterministic estimation of the return r of the fund and the deterministic estimation of next year reserves $\hat{R}_i(1)$. It means that a bad deviation from these estimations shall be borne by the insurer.

Finally, notice that we can imagine a lot of clauses for this product and that it may also be generalized to SLT (Similar to Life Techniques) business.