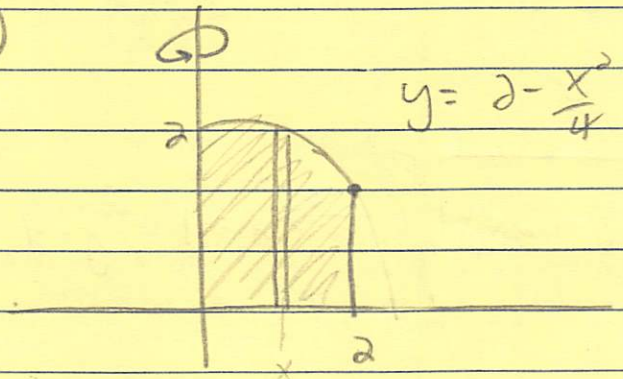
6.2 # 2, 3, 11, 13

2)



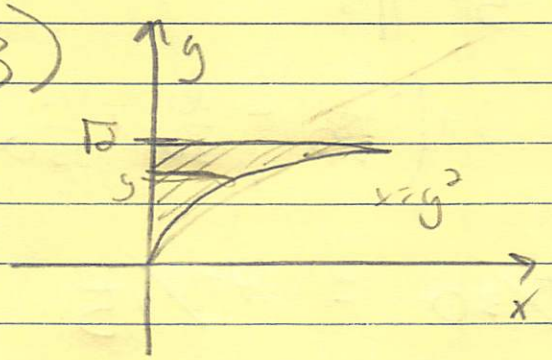
Shell height: $2 - \frac{x^2}{4}$

Shell radius: x

$$V = 2\pi \int_0^2 2x - \frac{x^3}{4}\, dx$$

$$= 2\pi \left[x^2 - \frac{x^4}{16} \right]_0^2 = 2\pi \left[4 - 1 \right] = 6\pi$$

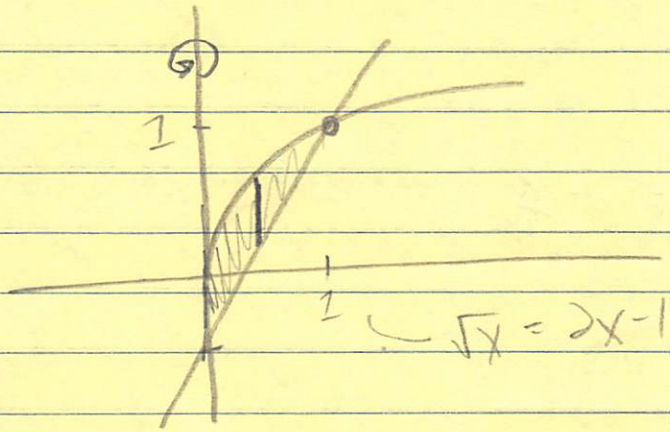
3)



Shell height: y^2 radius y

$$V = 2\pi \int_0^{\sqrt{2}} y^3\, dy = 2\pi \left[\frac{y^4}{4} \right]_0^{\sqrt{2}} = 2\pi$$

11) $y = 2x - 1$, $y = \sqrt{x}$, $x = 0$



$\therefore \sqrt{x} = 2x - 1 \Rightarrow x = 1, y = 1$

Shell height: $\sqrt{x} - (2x - 1) = \sqrt{x} - 2x + 1$

radius: x

$$V = 2\pi \int_0^1 x - 2x^2 + x^{3/2}\, dx$$

$$= 2\pi \left(\frac{x^2}{2} - \frac{2}{3}x^3 + \frac{2x^{5/2}}{5} \right) \Big|_0^1$$

$$= 2\pi \left(\frac{1}{2} - \frac{2}{3} + \frac{2}{5} \right) = 2\pi \left(\frac{15}{30} - \frac{20}{30} + \frac{12}{30} \right)$$

$$= 2\pi \left(\frac{7}{30} \right) = 14\pi/30 = 7\pi/15$$

13) $f(x) = \begin{cases} \sin(x)/x, & 0 < x \leq \pi \\ 1, & x = 0 \end{cases}$

a)

case 1, $x = 0$. Then

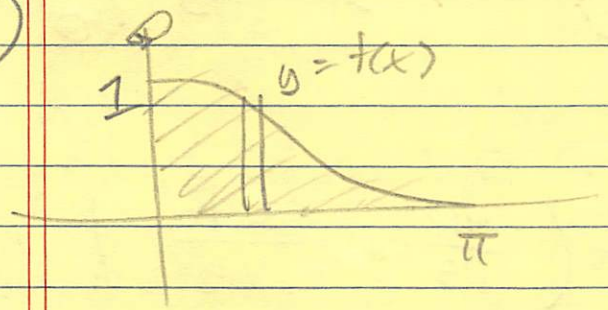
$f(x) \cdot x = 1 \cdot 0 = 0 = \sin(0) = \sin x$ ✓

case 2, $x \neq 0$, $0 < x < \pi$

$f(x) \cdot x = \frac{\sin(x)}{x} \cdot x = \sin(x)$ ✓

So on $0 \leq x \leq \pi$, $x f(x) = \sin x$

b)



Shell height: $f(x)$

radius: x

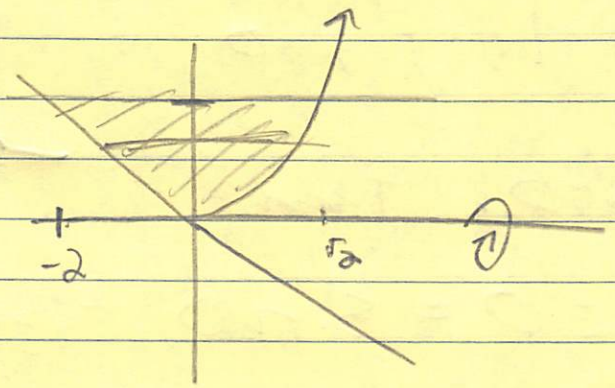
$$V = 2\pi \int_0^{\pi} x f(x)\,dx = 2\pi \int_0^{\pi} \sin x\,dx$$

$$= 2\pi (-\cos x)\Big|_0^{\pi}$$

$$= 2\pi\left(-(-1) - (-1)\right)$$

$$= 2\pi(1+1) = 4\pi$$

15) $x = \sqrt{y}$, $-y = x$, $y = 2$, about x-axis



Shell height : $\sqrt{y} - (-y) = y + \sqrt{y}$

radius : y

$$2\pi \int_0^{\sqrt{2}} y^2 + y^{3/2}\, dy = 2\pi \left(\frac{y^3}{3} + \frac{2y^{5/2}}{5} \right) \Big|_0^{\sqrt{2}}$$

$$2\pi \left(\frac{8}{3} + \frac{8\sqrt{2}}{5} \right) = 2\pi \left(\frac{40}{15} + \frac{24\sqrt{2}}{15} \right)$$

$$= \frac{16\pi}{15} \left(5 + 3\sqrt{2} \right)$$