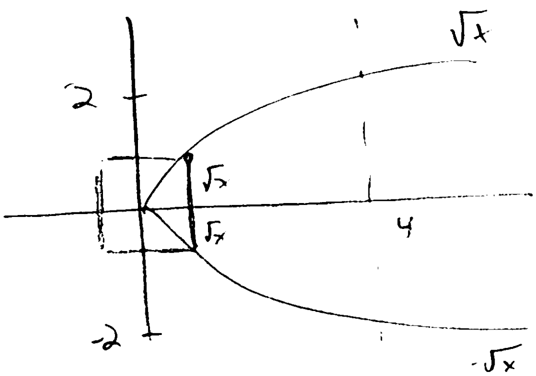
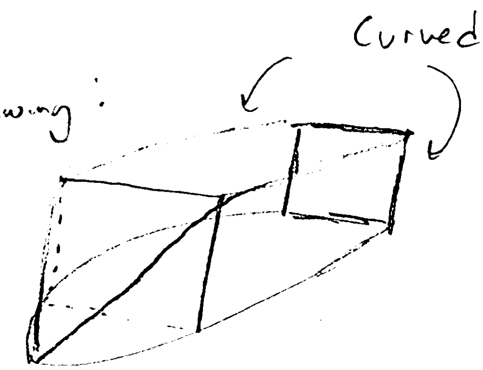
Math 241 Week 9 Quiz

Name: Answer Key

1.) A 3D solid is described as follows. The solid lies between planes perpendicular to the x-axis at x = 0 and x = 4. The cross-sections perpendicular to the axis on the interval $0 \le x \le 4$ are squares, whose side lengths run from the curve $y = \sqrt{x}$ to the curve $y = -\sqrt{x}$. Draw the solid, and calculate its volume.



3D drawing:



$$V = \int_0^4 A(x)\,dx = = \int_0^4 (2\sqrt{x})^2\,dx = 4\int_0^4 x\,dx$$

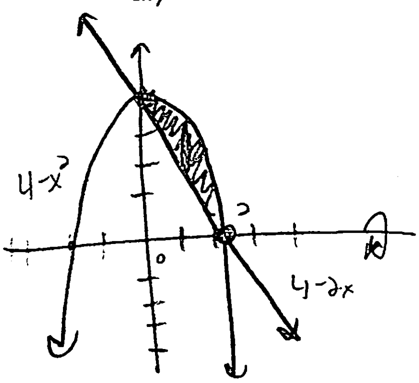
$$= 4\frac{x^2}{2}\Big|_0^4 = 2\cdot 16 = 32$$

2.) A region is bound by the curves $y = 4 - x^2$ and $y = 4 - 2x$. Sketch and calculate the volume of the solid generated by revolving the region about:

$(2-x)(2+x)=0$ $\quad$ $2(2-x)=0$

$x=2 \quad x=-2 \qquad x=2$

a.) The x-axis

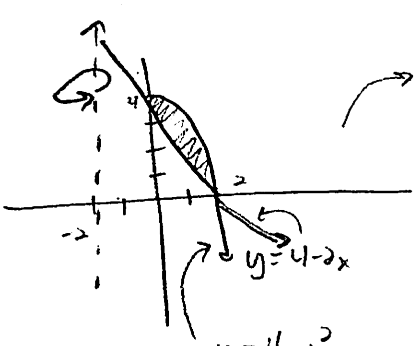


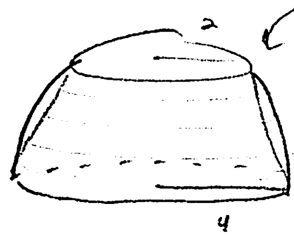
washer method: $\pi\int_0^2 (4-x^2)^2-(4-2x)^2\,dx = \pi\int_0^2 16-8x^2+x^4-(16-16x+4x^2)\,dx$

$= \pi\int_0^2 16x-4x^2-8x^2+x^4\,dx = \pi\int_0^2 16x-12x^2+x^4\,dx$

$= \pi\left(8x^2-4x^3+\frac{x^5}{5}\right)_0^2 = \pi\left(32-32+\frac{32}{5}\right) = 32\pi/5$ units^3

b.) The line x = -2





Shell method

Shell height: $4-x^2-(4-2x) = 2x-x^2$

radius: $x+2$

$2\pi\int_0^2 (x+2)(2x-x^2)\,dx = 2\pi\int_0^2 2x^2-x^3+4x-2x^2\,dx$

$= 2\pi\left[-\frac{x^4}{4}+2x^2\right]_0^2 = 2\pi[-4+8]$

$= 8\pi$

Washer method

$y=4-x^2 \rightarrow x=\sqrt{4-y}$

$y=4-2x \rightarrow x=2-y/2$

$\int_0^4 (\sqrt{4-y}+2)^2-(2-y/2+2)^2\,dy$

b/c rotated about $x=-2$

Integral is solvable, but long. Use shell method.